Final Examination: ESE 558 Digital Image Processing

Date: 5/12/98, Maximum Points : 45, Duration: 2 hour 30 mins, Spring 1998

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This test is open-text books and assigned paper. No other reference materials are allowed. Note: RK and GW respectively denote the text books by Rosenfeld & Kak and Gonzalez & Woods.

In answering questions, show all steps to get full credit.

1. *(5+5=10 points) Digitization*

   (a) **Optimal Sampling:**

   The autocorrelation function of a homogeneous random field has the form

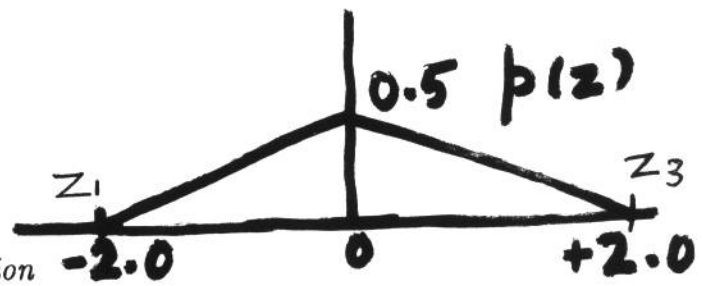
   $$R_{ff}(\alpha, \beta) = [R_{ff}(0,0) - \eta^2] \times exp[-c_1 |\alpha| - c_2 |\beta|] + \eta^2$$

   with $\eta = 0$ (see Eq.(47) on page 97 in R&K book). Discribe an algorithm for the *optimal sampling* of the random field. Assume that a numerical technique is available to solve integral equations of the form

   $$\int_{-T}^{T} k_0 \, exp[-c \, |s - t| \,] \, \phi(t) dt = r\phi(s).$$

   (b) **Optimal Quantization:**

   The brightness of an image has the probability density function shown in Fig. 1. Solve the optimal quantization problem to sample the signal to 2 discrete levels (solve for z_2, q_1, and q_2 in the notation of *R&K*.)



2. *(4+3+3+5=15 points) Image Compression*

   (a) Eight symbols and their probability of occurrence in an image compression method are given below. Divide them into two blocks of 4 symbols each appropriately and construct the best Huffman Shift code. Explain what criteria you have used in dividing into two blocks, and mapping between symbols in the first bloc and the second block, so that average codelength is optimized.

Symbol :	a,	b,	c,	d,	e,	f,	g,	h
Probability :	0.15,	0.05,	0.2,	0.09,	0.1,	0.01,	0.25,	0.15

   (b) Compute the average code length of your Huffman Shift code and compare it with the entropy of the symbols.

   (c) Construct the best B_1 code for the same symbols.

   (d) In the JPEG sequential baseline system of compression, find the encoding of the 1-D DCT block [10, 0, -5, 0, 0, 2, EOB] given that the DC coefficient of the previous block is 20. (Show all steps in deriving the encoding for each entry above to demonstrate your understanding.)

3. *(3+3+4=10 points) Image Restoration :*
The Point Spread Function (PSF) of an imaging system is

$$h(x,y) = \frac{1}{A \cdot B} rect\left(\frac{x}{A}, \frac{y}{B}\right)$$

or

$$= \frac{1}{A} rect1\left(\frac{x}{A}\right) \cdot \frac{1}{B} rect1\left(\frac{y}{B}\right)$$

(rect1(t) is the one-dimensional rect function).
(a) Find the Line Spread Function (LSF) corresponding to a line source on the y-axis (i.e. x=0).
(b) Find the Edge Spread Function (ESF) corresponding to the above LSF.
(c) Explain (i) the Weiner Filter (WF) and (ii) the spatial Domain Convolution/Deconvolution Transform (ST) (with two non-zero terms for a circularly symmetric PSF) (i.e. give expressions for each and define the terms in the expressions). Compare the performance of these two filters for image restoration by listing two relative advantages and disadvantages of each filter.

4. *(3+2+5=10 points) Image Reconstruction :*
(a) The value of a function $f(x,y)$ is 1 inside a circle of radius R centered at the origin. Find the Radon Transform $P_0(t)$ of $f(x,y)$ (i.e. find the projection onto the x-axis.)
(b) State the Fourier Slice Theorem.
(c) for the Filtered Backprojection Algorithm, list the input, output, and the computational steps necessary to transform the input data to output data.