Mid-Term Examination: ESE 558 Digital Image Processing
Date: 3/27/2001, Duration: 2 hour 30 mins, Spring 2001
SUNY at Stony Brook, Murali Subbarao, Max. Marks: 25

This test is open-text books. No other reference materials are allowed. Note: RK and GW respectively denote the text books by Rosenfeld & Kak and Gonzalez & Woods. In answering the following questions write down all the important steps to get full credit.

1. *(5 points, RK, Ch. 2, Mathematical Preliminaries)*
   The Fourier transform of $f(x, y)$ is given to be $F(u, v)$. Starting from the definition of the continuous Fourier transform, derive the Fourier transform of $f(a(x-b), c(y-d))$ in terms of $F(u, v)$ where a, b, c, d are scalar constants.

2. *5 points, GW, Ch. 2, Connected component labelling*
   A binary image b[m][n] of size 100x100 is given where b[i][j]=0 for background and b[i][j]=1 for object pixels. There are a maximum of 10 objects and none of them touch the border of the image. Assuming that the image has already been read into a two-dimensional integer array b[100][100], give a step-by-step computational algorithm for connected component labeling using 4-connectedness definition. You can present the algorithm as a procedure/function in a high-level programming language (e.g. C , FORTRAN) or as psuedo-code in English. You can skip the implementation of Equation 2.4-1 on page 45 in GW book (as it is already given there), but you must describe how the array B is assigned values initially. You must also give the algorithm or code for reassiginig a unique label to pixels with equivalent labels.

3. *(5 points GW Ch. 2, Imaging geometry)*
   Excercise 2.17 on page 79 in GW text book.

4. *(5 points, RK, Ch. 4, Digitization and Aliasing)*
   An image is specified by $f(\mathbf{r}) = cos 2\pi(\mathbf{w_0} \cdot \mathbf{r})$ where $\mathbf{r} = (x, y)$ and $\mathbf{w_0} = (0.2, 0.7)$. The image is sampled on a lattice $\mathbf{r_{mn}} = m\mathbf{r_1} + n\mathbf{r_2}$ where $\mathbf{r_1} = (1, 0)$, and $\mathbf{r_2} = (0, 1)$ for $m, n = 0, \pm 1, \pm 2, \pm 3, \cdots$..
   (i) Find the reciprocal lattice $\mathbf{w_{mn}} = m\mathbf{w_1} + n\mathbf{w_2}$ (i.e. find $\mathbf{w_1}$ and $\mathbf{w_2}$).
   (ii)If $G(\mathbf{w}) = rect(\mathbf{w})$ is the Fourier transform of the interpolation filter $g(\mathbf{r})$ used in reconstructing the image from its samples, find an explicit expression for the reconstructed image $f'(\mathbf{r})$.

5. *(5 points, RK, Ch. 4, Optimal quantization)*
   The probability density function of an image signal z is $p(z) = 0.75(1 - z^2)$ for $|z| \leq 1$ and $p(z) = 0$ for $|z| > 1$. We need to optimally quantize the signal to two distinct levels q_1, q_2 with a decision level z_2. Guess a solution for z_2 using common sense and then solve for q_1, q_2.