Mid-Term Examination: ESE 558 Digital Image Processing
Date: 3/16/99, Duration: 2 hour 30 mins, Spring 1999
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This test is open-text books. No other reference materials are allowed. Note: RK and GW respectively denote the text books by Rosenfeld & Kak and Gonzalez & Woods. In answering the following questions write down all the important steps to get full credit.

1. *(5+5 points) RK, Ch. 2, Mathematical Preliminaries*
(a) Starting from the definition of (continuous) Fourier transform, derive the Fourier transform of $\mathrm{Sin}(2\pi(ax+by))$ where a, b are scalar constants.

   (b) RK, page 24: Prove the first part of Equation (42), i.e. $F(aM+u, bN+v) = F(u,v)$.

2. *(5+5 points) GW, Ch. 2, Imaging Geometry*
(a) For the camera model presented in Section 2.5.3, page 61, in GW, determine the image coordinates of a point with world coordinates (0.5,0.5,0.5)m for a camera located at (0,0,1.5)m, panned 135 degrees, and tilted 120 degrees. Assume a lens of focal length 0.050m, and let $r_1 = r_2 = r_3 = 0$.

   (b) For the stereo camera shown in Fig. 2.21, page 69 in GW, you are given that the baseline B is 100.0 mm, focal length λ is 50.0 mm, image coordinates on the left camera (x_1, y_1) is (-0.1,0.1) mm, and the disparity $x_2 - x_1$ is -2.0 mm. Find the world coordinates (X_1, Y_1, Z_1) and (X_2, Y_2, Z_2) of the corresponding world point.

3. *5+5 points, Digitization*
(a) An image is specified by $f(\mathbf{r}) = cos 2\pi(\mathbf{w_0} \cdot \mathbf{r})$ where $\mathbf{r} = (x,y)$ and $\mathbf{w_0} = (0.3, 0.9)$. The image is sampled on a lattice $\mathbf{r_{mn}} = m\mathbf{r_1} + n\mathbf{r_2}$ where $\mathbf{r_1} = (1,0)$, and $\mathbf{r_2} = (0,1)$ for $m, n = 0, \pm1, \pm2, \pm3, \cdots$..
(i) Find the reciprocal lattice $\mathbf{w_{mn}} = m\mathbf{w_1} + n\mathbf{w_2}$ (i.e. find $\mathbf{w_1}$ and $\mathbf{w_2}$).
(ii)If $G(\mathbf{w}) = rect(\mathbf{w})$ is the Fourier transform of the interpolation filter $g(\mathbf{r})$ used in reconstructing the image from its samples, find an explicit expression for the reconstructed image $f'(\mathbf{r})$.

   (b) In the conventional optimal quantization problem (see page 109 in RK), the criterion of optimality is defined in the mean-SQUARED-error sense. Here, redifine optimality in the mean-ABSOLUTE-error sense and solve the optimal quantization problem. The problem is defined as follows. An image brightness function $z = f(x,y)$ takes values in the range (z_1, z_{K+1}). This function is to be sampled and quantized to K discrete levels, represented by $q_1, q_2, ..., q_K$. Let $z_1, z_2, ..., z_{K+1}$ represent the corresponding decision levels, i.e. if an input sample z lies in the interval between z_i and z_{i+1}, then it is assigned the value q_i at the output. The error in this case is defined as absolute value of $|z - q_i|$ (note: the error is not squared). The probability density function $p(z)$ for the image brightness is given. The following result may be helpful in solving this problem:
The equation
$$e(\alpha, z) = \int_{-\infty}^{\alpha} f(\alpha, z)\, dz \tag{1}$$

can be written as

$$e(\alpha_1, \alpha_2, z) \;=\; \int_{-\infty}^{\alpha_1} f(\alpha_2, z)\, dz \tag{2}$$

where $\alpha_1 = \alpha_2 = \alpha$, and using chain rule we get

$$\frac{de}{d\alpha} = \frac{\partial e}{\partial \alpha_1} + \frac{\partial e}{\partial \alpha_2} \tag{3}$$

Formulate and solve, to the extent possible, the problem of determining optimal values of $z_2, z_3, ..., z_K$ and $q_1, q_2, ..., q_K$ so as to minimize the quantization error in a mean-ABSOLUTE-error sense.