1. (10 points)
In the conventional optimal quantization problem, the criterion of optimality is defined in the mean-SQUARED-error sense. Here, redifine optimality in the mean-ABSOLUTE-error sense and solve the optimal quantization problem. The problem is defined as follows. An image brightness function $z = f(x, y)$ takes values in the range (z_1, z_{K+1}). This function is to be sampled and quantized to K discrete levels, represented by $q_1, q_2, ..., q_K$. Let $z_1, z_2, ..., z_{K+1}$ represent the corresponding decision levels, i.e. if an input sample z lies in the interval between z_i and z_{i+1}, then it is assigned the value q_i at the output. The error in this case is defined as absolute value of $|z - q_i|$ (note: the error is not squared). The probability density function $p(z)$ for the image brightness is given. The following result may be helpful in solving this problem:
The equation

$$e(\alpha, z) \; = \; \int_{-\infty}^{\alpha} f(\alpha, z) \; dz \tag{1}$$

can be written as

$$e(\alpha_1, \alpha_2, z) \; = \; \int_{-\infty}^{\alpha_1} f(\alpha_2, z) \; dz \tag{2}$$

where $\alpha_1 = \alpha_2 = \alpha$, and using chain rule we get

$$\frac{de}{d\alpha} = \frac{\partial e}{\partial \alpha_1} + \frac{\partial e}{\partial \alpha_2} \tag{3}$$

(a) (6 points) Formulate and solve, to the extent possible, the problem of determining optimal values of $z_2, z_3, ..., z_K$ and $q_1, q_2, ..., q_K$ so as to minimize the quantization error in a mean-ABSOLUTE-error sense.

(b) (4 points) Given $p(z)$, outline a computational algorithm to obtain numerical values of the solution derived above.

2. (10 points)
   In standard image compression techniques such as sequential baseline JPEG, transform coefficients are Huffman coded (after taking run-length of zero coefficients into account) and mean brightnesses ("DC coefficients") are difference coded. In a sense, such techniques combine several compression methods.
   (a) (2.5+2.5+2.5 points) In this context explain (in about half page each) the basic principle behind each of the following methods for image compression:
   (i) Transform compression, (ii) Huffman coding, and (iii) run-length coding.

   (b) (2.5 points) In transform compression method, explain how the compression rate can be varied.