Rao Transforms: A New Approach to Integral and Differential Equations

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Abstract

Rao Transforms (RTs) provide a novel approach to the century old problem of integral equations. *"It [the approach] is guaranteed to produce doctoral dissertations"* is the overall assessment of an international expert in the field who reviewed the approach critically for a U.S. government research funding agency. *Differential equations* are solved by first converting them to integral equations by incorporating boundary conditions, and then solving the resulting integral equations. Since many fundamental laws of physics are stated using differential equations, RTs are expected to have wide applications in scientific, engineering, and medical applications. RTs are based on a breakthrough strategy of *"Localize, Solve, and Synthesize"* using the simple equation $L(u,v)=G(u+v,u)$ to change a global form integration kernel G to a local form kernel L. This apparently simple idea seems to have eluded researchers until now. RTs were invented by this author while doing research on shift-variant image deblurring which involves solving a Fredholm integral equation of the First Kind, and extending the results of that work to general integral equations. RT*s* are an extension of the Spatial-Domain Convolution/Deconvolution Transform (S Transform) invented by this author in 1989 related to convolution integral equations. S transform has been successfully used in computer vision and image processing applications such as depth-from-defocus and image restoration, and RTs are expected to be similarly useful.

RTs provide both symbolic and numerical solutions. The solution is fully *localized* and therefore offers significant computational savings and permits extremely fine-grained parallel implementation on a computer. The approach is *simple* as it is easy to implement and comprehend, and *unified* as a common framework solves a large class of diverse problems. Therefore RTs have both *computational* and *theoretical* advantages in comparison with existing techniques. RTs can be naturally extended from the case of one dimensional problems to multi-dimensional cases.

The basic theory of RTs, and their application to two practical problems will be presented. The first application example is shift-variant image/signal filtering, and the second is 3D computer vision through inverse optics.

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