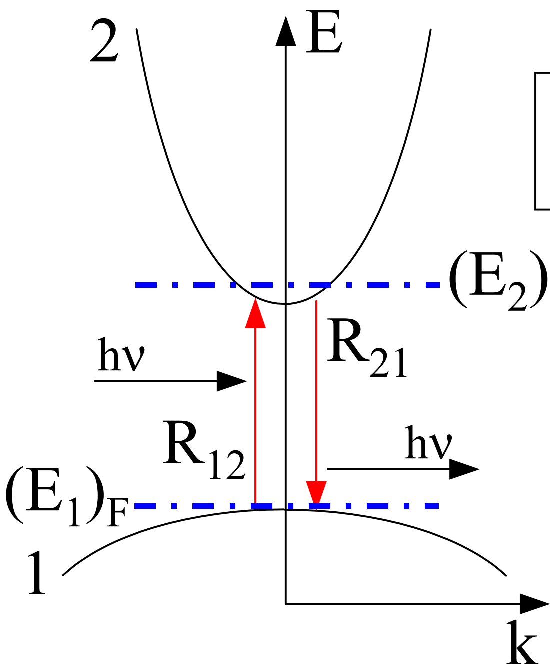
Optical gain in semiconductor lasers

Material Gain: **G=G(n,p)**



$$\boxed{\text{Transition Rate}} = \boxed{\text{Probability of Transition}} * \boxed{\text{Probability of having initial state filled}} * \boxed{\text{Probability of having final state empty}}$$

$$R_{21} = R_r \cdot f_2 \cdot (1 - f_1)$$

$$R_{12} = R_r \cdot f_1 \cdot (1 - f_2)$$

$$f_{2,1}(E_{2,1}) = \frac{1}{1 + \exp\left(\frac{E_{2,1} - (E_{2,1})_F}{kT}\right)}$$

$$R_{21} - R_{12} = R_r \cdot (f_2 - f_1)$$

For stimulated emission to overcome absorption, i.e. for positive material gain, population inversion is required $f_2 - f_1 > 0$ (Here $f_2 = f_e$, $f_1 = 1 - f_h$).

This condition is satisfied if separation of the quasi-fermi levels is larger than the band gap.

$$(E_2)_F - (E_1)_F \geq E_g$$

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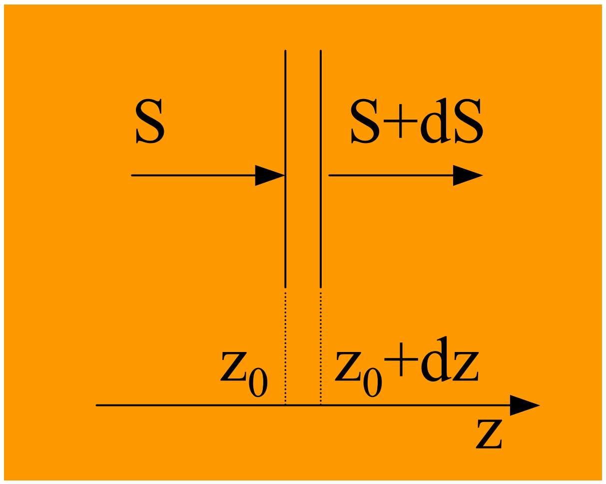


Material gain can be defined as

$$G = \frac{1}{S} \cdot \frac{dS}{dz}$$

S – photon cavity concentration

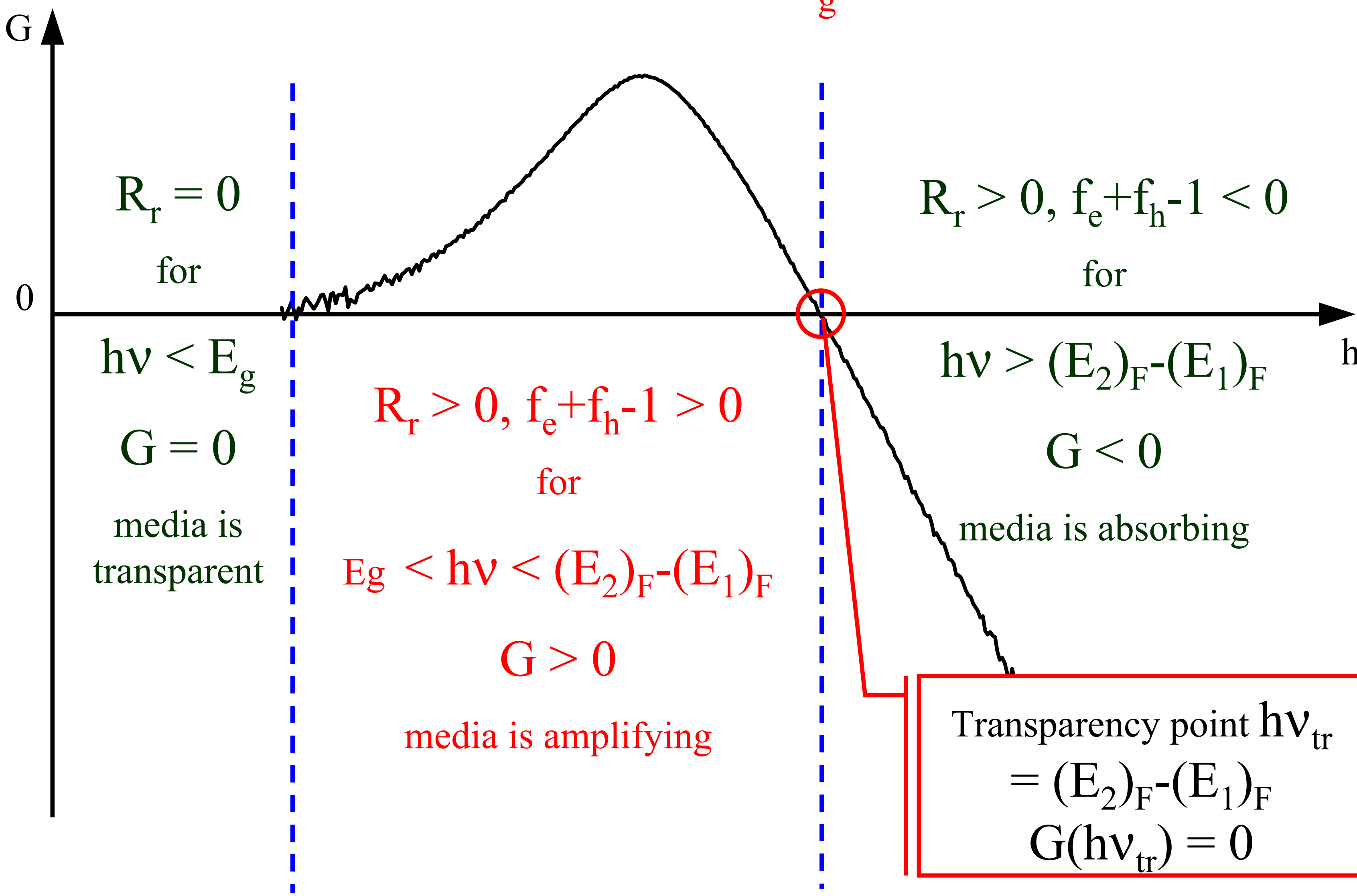
$$^{*}\, dz = \upsilon_g \cdot dt$$

$$\frac{dS}{dt} = R_{21} - R_{12}$$

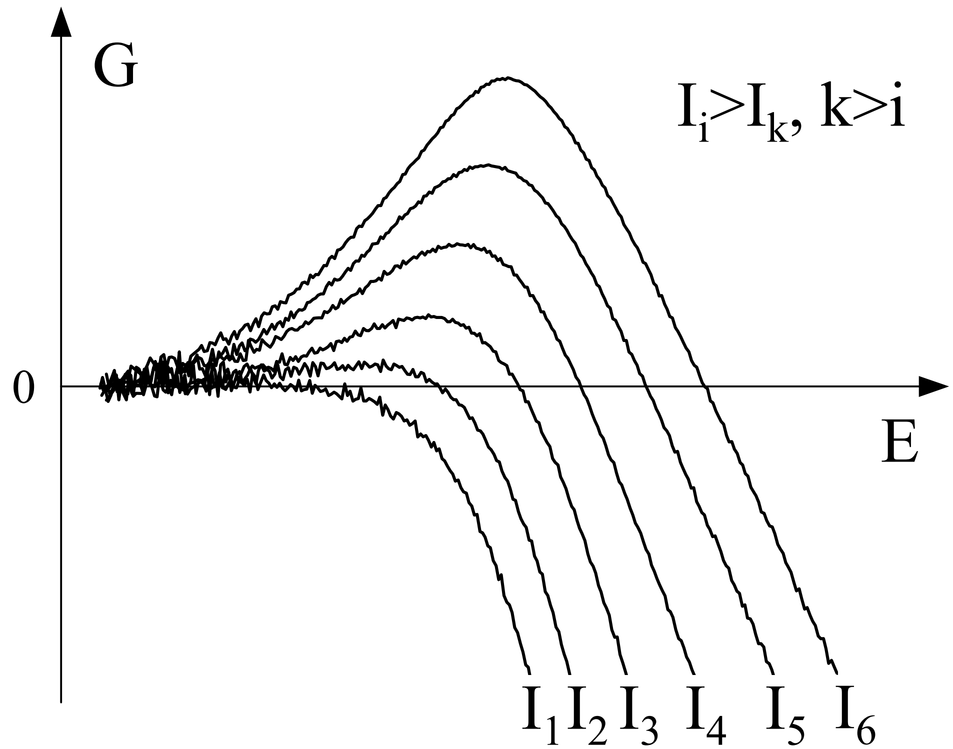
$$G = \frac{R_r}{\upsilon_g} \cdot \left(f_e + f_h - 1\right) \cdot \frac{1}{S}$$

Optical gain depends on material parameters (R_r), injected carrier concentration, temperature (f_e+f_h-1) and photon concentration (S).

Optical gain spectrum: $G(h\nu) = \frac{R_r}{\upsilon_g} \cdot (f_e + f_h - 1) \cdot \frac{1}{S}$



Gain spectrum



At $I=I_1$ $G_{max}=0$, transparent for $h\nu=E_g$

I_1 is called transparency current

For $I > I_1$ $G > 0$

When current (I) increases the carrier concentration in the active region (N) increases, i.e. population inversion (f_e+f_h-1) increases. As a result optical gain in max (G_{max}) becomes larger with current. The rate of G_{max} increase with carrier concentration or injection current is called differential gain (dG/dN or dG/dI).

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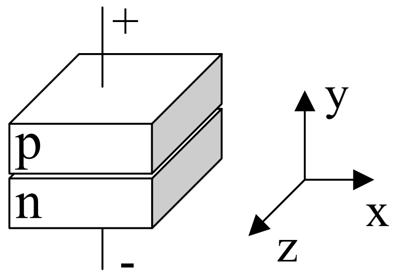
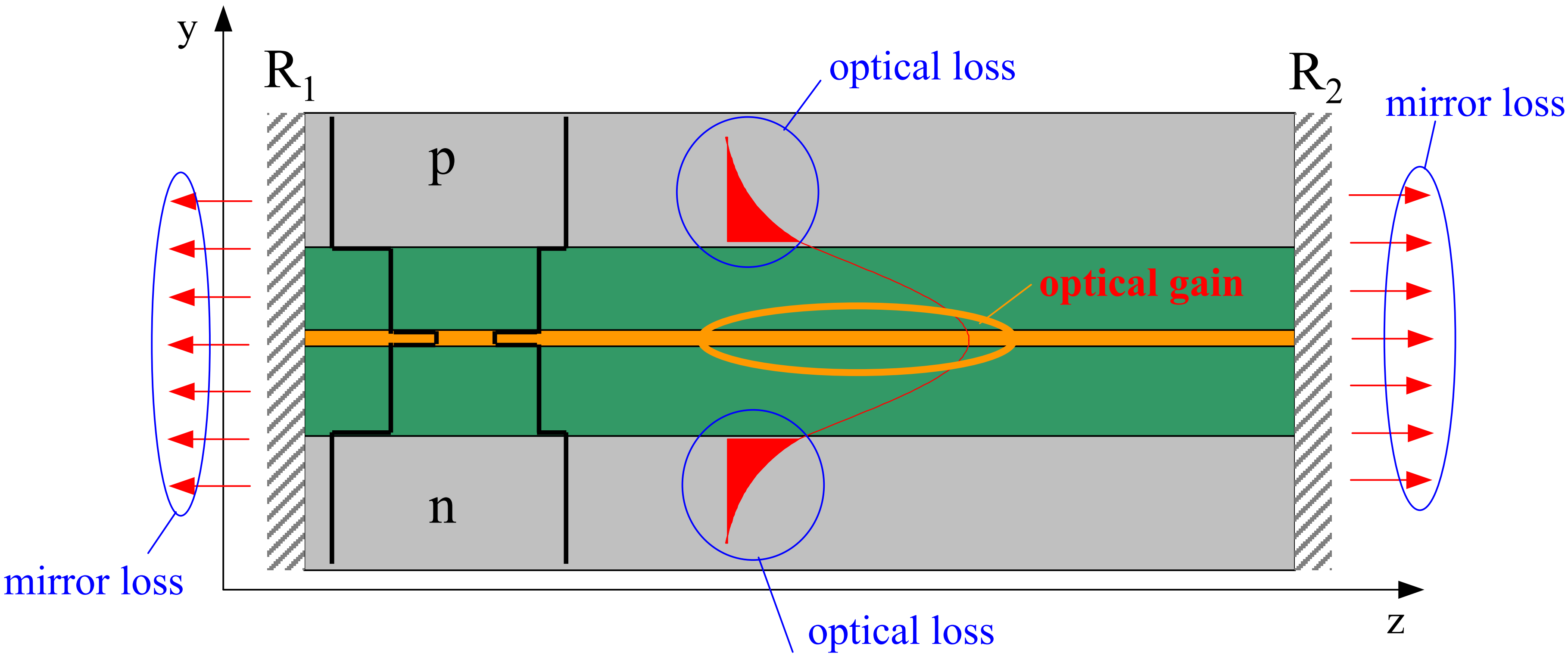


Modal Gain: $\mathbf{g = \Gamma G - \alpha_{tot}}$



Modal gain is much less than the actual material gain in the laser active region.

1. The volume occupied by the optical field is larger than the active region volume and appropriate scaling is necessary (Γ – confinement factor, of the order of V/V_P);

2. With optical loss the net gain is the difference between the scaled material gain and the modal loss.

Threshold condition and modal gain after threshold

$$\mathbf{g(I_{th}) = \Gamma * G(I_{th}) - \alpha_{tot} = 0}$$

Under this condition the optical resonator acquires infinite quality factor, i.e. photon lifetime inside the cavity is infinite, i.e. lasing is on.

For $I > I_{th}$ G should stay equal to its threshold value G_{th} to keep $g = 0$

* The system with $g > 0$ cannot be in equilibrium because S inside the cavity would gradually increase in time.

The carrier concentration in the active region after threshold will stay pinned to its threshold value and all injected carriers will produce photons through stimulated recombination.

Due to the contribution of spontaneous emission to the laser mode, the lasing threshold is reached at G smaller than $G_{th} = a_{tot}/G$. At high current, the role of spontaneous emission is reduced (carrier lifetime decreases) and material gain is almost equal to G_{th}.

Threshold current

$$g(I_{th}) = \Gamma * G(I_{th}) - \alpha_{tot} = 0$$

G > 0 needed in active.
After threshold $G = G_{th}$ and $N = N_{th}$
What I_{th} is needed for N_{th}?

Carrier rate equation

$$\frac{dN}{dt} = \frac{\eta_i \cdot I}{qV} - R_{rec}$$

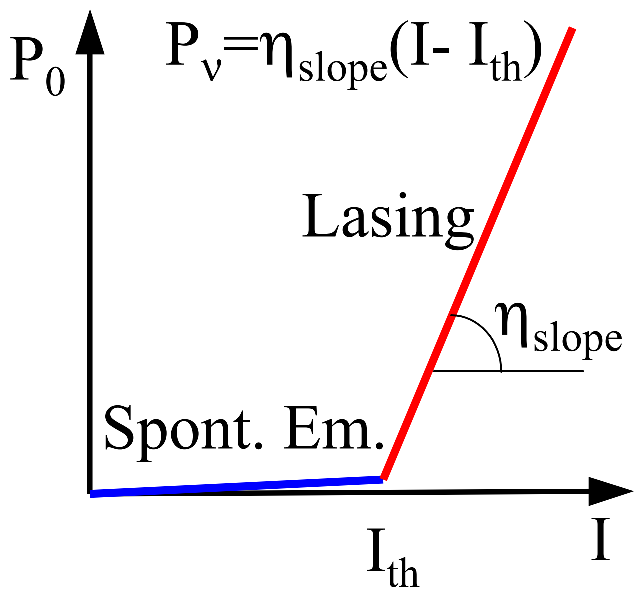
$$R_{rec} = R_{sp} + R_{NR} + R_{st} = \frac{N}{\tau} + R_{st}$$

$$\frac{1}{\tau} = \frac{1}{N} \cdot \left(R_{sp} + R_{NR}\right) = A_{NR} + B \cdot N_{th} + C \cdot N_{th}^2$$

At the threshold $R_{st} \approx 0$ and $\eta_i \cdot I_{th} = qV \cdot R_{rec}\left(N_{th}\right)$

$$\eta_i \cdot I_{th}/qV = A_{NR} \cdot N_{th} + B \cdot N_{th}^2 + C \cdot N_{th}^3$$

Light-current characteristics and rate equations



$$\begin{cases} \dfrac{dN}{dt} = \dfrac{\eta_i \cdot I}{q \cdot V} - \dfrac{N}{\tau} - \upsilon_g \cdot G \cdot S \\ \dfrac{dS}{dt} = \Gamma \cdot \upsilon_g \cdot G \cdot S + \Gamma \cdot \beta_{sp} \cdot R_{sp} - \dfrac{S}{\tau_p} \end{cases}$$

Above threshold

$$\boxed{P_\nu = \left[\eta_i \frac{\alpha_m}{\alpha_i + \alpha_m} \cdot \frac{h\nu}{q}\right] \cdot \left(I - I_{th}\right)}$$

Light-current characteristics - II

Stored optical energy in the cavity: $S \cdot h\nu \cdot V_P$;

Energy loss rate through the mirrors: $\upsilon_g \cdot \alpha_m = 1/\tau_m$;

Output power from the mirrors: $P_\nu = \upsilon_g \cdot \alpha_m \cdot S \cdot h\nu \cdot V_P$;

Threshold gain: $\Gamma \cdot G_{TH} = \alpha_i + \alpha_m = 1/(\upsilon_g \cdot \tau_p)$;

Threshold current: $\dfrac{\eta_i \cdot I_{TH}}{q \cdot V} = (R_{SP} + R_{NR} + R_L)_{TH} = \dfrac{N_{TH}}{\tau}$;

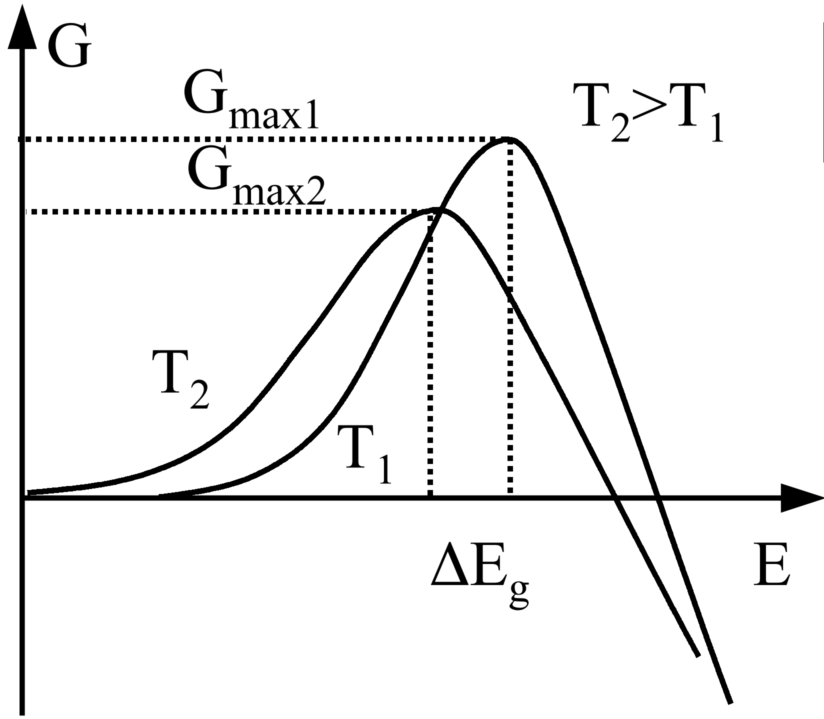
After threshold carrier rate equation: $\dfrac{dN}{dt} = \eta_i \cdot \dfrac{(I - I_{TH})}{q \cdot V} - \upsilon_g \cdot G \cdot S$;

After threshold steady state: $S = \dfrac{\eta_i \cdot (I - I_{TH})}{q \cdot \upsilon_g \cdot G_{TH} \cdot V}$;

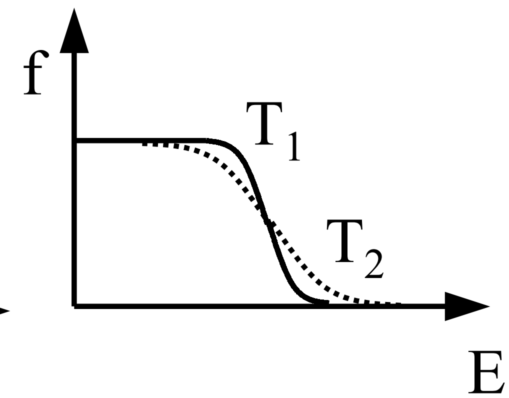
Optical confinement: $\Gamma = V/V_P$;

Output power: $P_\nu = \eta_i \cdot \dfrac{h\nu}{q} \cdot \dfrac{\alpha_m}{\alpha_i + \alpha_m} \cdot (I - I_{TH})$;

Temperature dependence of gain and threshold current



$$\boxed{G \sim f_e + f_h - 1}$$



$I_{th} = I_{th}(T)$

Empirically obtained: $\mathbf{I_{th} \sim exp(T/T_0)}$

Above threshold $\eta_{ext} = \eta_{ext}(T)$, $\eta_{ext} \sim \mathbf{exp(-T/T_1)}$

In order to keep output power at a given level

current through the laser should be increased accordingly.