Blackbody Radiation

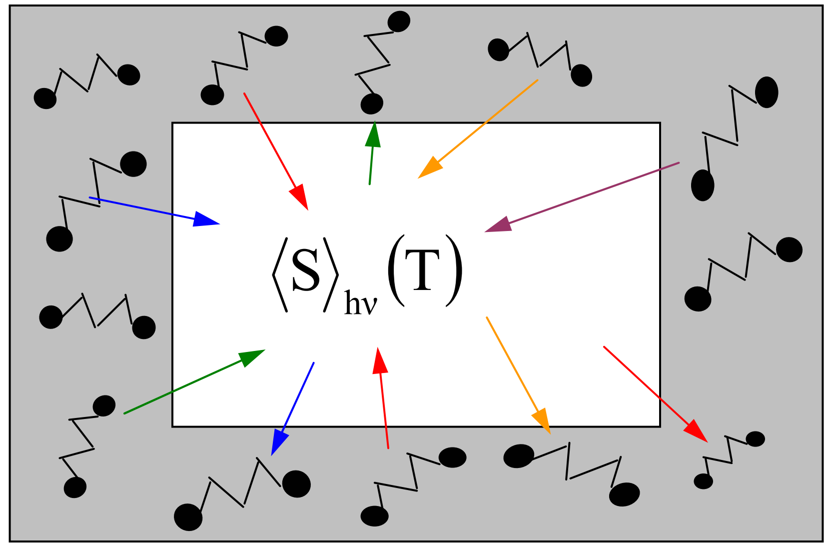
Let us use Einstein's approach to relate gain and spontaneous emission.

According to Plank:

1.The probability of radiation taking place from a black body decreased as the frequency of the radiation increased.

2. The black body radiation flow out in discrete, lumpy quantities $h\nu$

MODEL



Planck postulated:

$$\langle S \rangle_{h\nu} \propto \frac{(h\nu)^2}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

$$h = 6.626 \cdot 10^{-34}\ J \cdot s$$

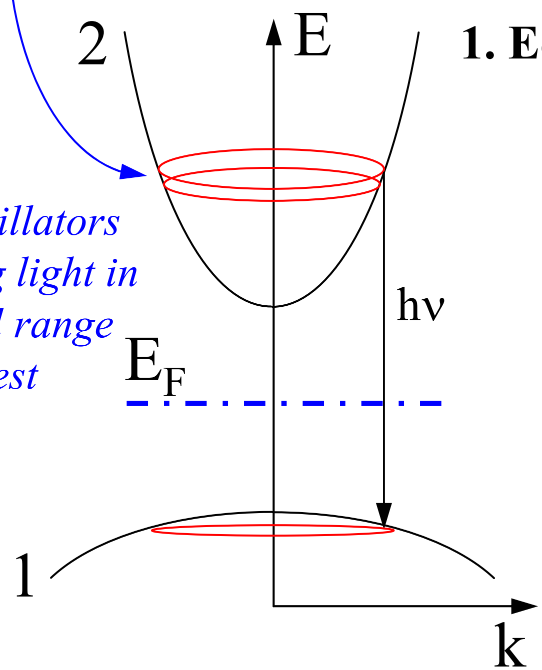
Optical gain spectrum measurements: From TSE

Einstein's approach to gain and spontaneous emission

Change of the photon numbers S:

$$\Delta S = -B_{STIM}^{ABS} \cdot S \cdot f_1 \cdot (1 - f_2) + B_{STIM}^{EM} \cdot S \cdot f_2 \cdot (1 - f_1) + A_{SPON}^{EM} \cdot f_2 \cdot (1 - f_1)$$

Possible contribution of the various electron transitions to emission/absorption of photons in energy interval [hν,hν+dhν] *is accounted for in A and B constants.*



The oscillators emitting light in spectral range of interest

1. Equilibrium: ΔS=0 $$\langle S \rangle_{h\nu} = \frac{A_{SPON}^{EM} / B_{STIM}^{EM}}{B_{STIM}^{ABS} / B_{STIM}^{EM} \cdot \exp\left(\frac{h\nu}{kT}\right) - 1}$$

Blackbody radiation: $$\langle S \rangle_{h\nu} \propto \frac{(h\nu)^2}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

$$\boxed{\begin{array}{l} B_{STIM}^{EM} = B_{STIM}^{ABS} = B \\ A_{SPON}^{EM} = A = \propto (h\nu)^2 \cdot B \end{array}}$$

Comments

Equilibrium $\Delta S=0$:

$$-B_{STIM}^{ABS}\cdot f_1\cdot(1-f_2)\cdot S+B_{STIM}^{EM}\cdot f_2\cdot(1-f_1)\cdot S+A_{SPON}^{EM}\cdot f_2\cdot(1-f_1)=0$$

$$S\cdot\left(-B_{STIM}^{ABS}\cdot f_1\cdot(1-f_2)+B_{STIM}^{EM}\cdot f_2\cdot(1-f_1)\right)=-A_{SPON}^{EM}\cdot f_2\cdot(1-f_1)$$

$$S=\frac{-A_{SPON}^{EM}\cdot f_2\cdot(1-f_1)}{-B_{STIM}^{ABS}\cdot f_1\cdot(1-f_2)+B_{STIM}^{EM}\cdot f_2\cdot(1-f_1)}=\frac{A_{SPON}^{EM}\cdot f_2\cdot(1-f_1)}{B_{STIM}^{ABS}\cdot f_1\cdot(1-f_2)-B_{STIM}^{EM}\cdot f_2\cdot(1-f_1)}$$

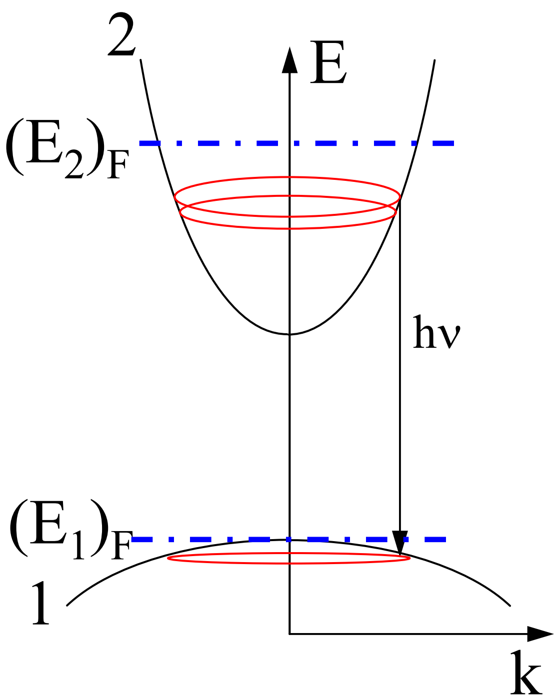
$$S=\frac{A_{SPON}^{EM}/B_{STIM}^{EM}}{B_{STIM}^{ABS}/B_{STIM}^{EM}\cdot\frac{f_1\cdot(1-f_2)}{f_2\cdot(1-f_1)}-1};\quad \frac{f_1\cdot(1-f_2)}{f_2\cdot(1-f_1)}=\frac{\frac{1}{1+\exp\left(\frac{E_1-E_F}{kT}\right)}\cdot\left(1-\frac{1}{1+\exp\left(\frac{E_2-E_F}{kT}\right)}\right)}{\frac{1}{1+\exp\left(\frac{E_2-E_F}{kT}\right)}\cdot\left(1-\frac{1}{1+\exp\left(\frac{E_1-E_F}{kT}\right)}\right)}$$

$$\frac{f_1\cdot(1-f_2)}{f_2\cdot(1-f_1)}=\frac{\exp\left(\frac{E_2-E_F}{kT}\right)}{\exp\left(\frac{E_1-E_F}{kT}\right)}=\exp\left(\frac{E_2-E_1}{kT}\right)=\exp\left(\frac{h\nu}{kT}\right)$$

Optical gain spectrum measurements: From TSE

Gain and spontaneous emission relation

No equilibrium: $E_{F1} \neq E_{F2}$, $\Delta S \neq 0$.



$$G(h\nu) = \frac{\Delta S_{STIM}}{\Delta x} \cdot \frac{1}{S} = \frac{B}{\Delta x} \cdot (f_2 - f_1)$$

$$R_{SPON}(h\nu) = \frac{\Delta S_{SPON}}{\Delta t} = \upsilon_g \cdot \frac{\Delta S_{SPON}}{\Delta x} = \upsilon_g \cdot \frac{A}{\Delta x} \cdot f_2 \cdot (1 - f_1)$$

$$R_{SPON}(h\nu) \propto G(h\nu) \cdot (h\nu)^2 \cdot \frac{f_2 \cdot (1 - f_1)}{(f_2 - f_1)}$$

$$\boxed{I_{SP}(h\nu) \propto \frac{G(h\nu) \cdot (h\nu)^2}{\left(1 - \exp\left(\frac{h\nu - \Delta E_F}{kT}\right)\right)}}$$

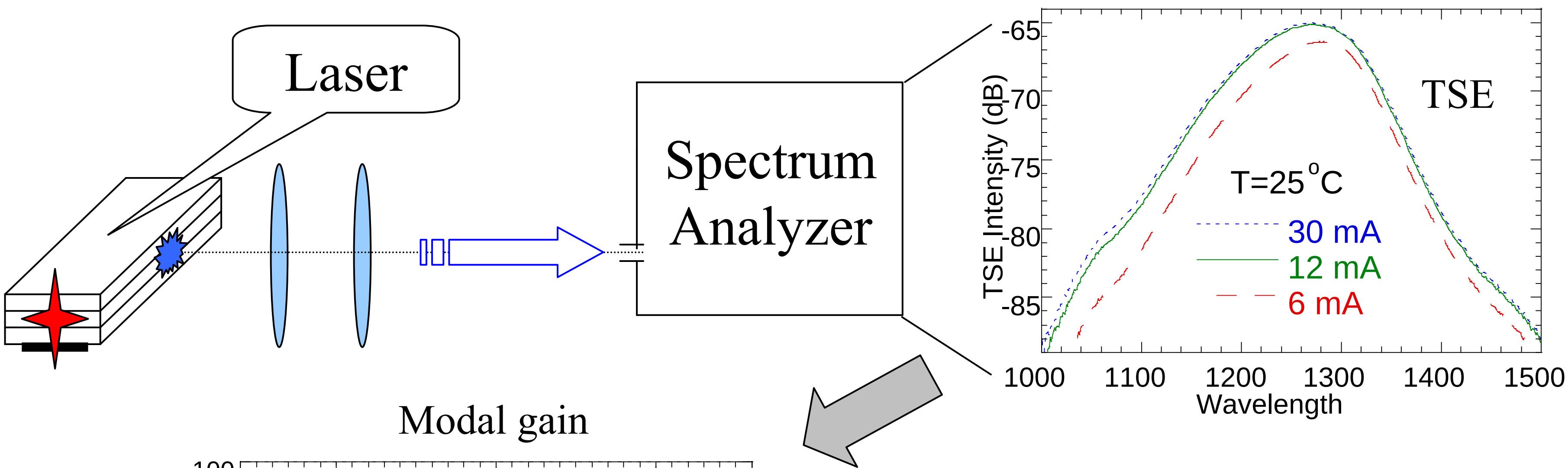
Comments

$$\frac{f_2 \cdot (1-f_1)}{(f_2 - f_1)} = \frac{\dfrac{1}{1+\exp\left(\dfrac{E_2 - E_{F2}}{kT}\right)} \cdot \left(1 - \dfrac{1}{1+\exp\left(\dfrac{E_1 - E_{F1}}{kT}\right)}\right)}{\left(\dfrac{1}{1+\exp\left(\dfrac{E_2 - E_{F2}}{kT}\right)} - \dfrac{1}{1+\exp\left(\dfrac{E_1 - E_{F1}}{kT}\right)}\right)} =$$

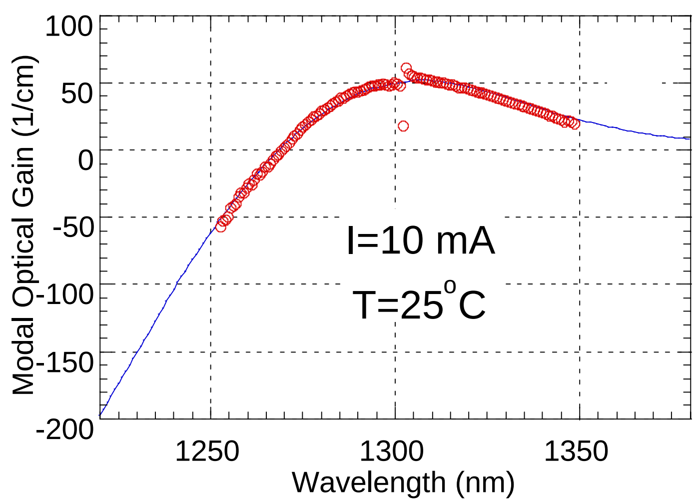
$$= \frac{\exp\left(\dfrac{E_1 - E_{F1}}{kT}\right)}{\exp\left(\dfrac{E_1 - E_{F1}}{kT}\right) - \exp\left(\dfrac{E_2 - E_{F2}}{kT}\right)} = \frac{1}{1-\exp\left(\dfrac{E_2 - E_1 - (E_{F2} - E_{F1})}{kT}\right)} = \frac{1}{1-\exp\left(\dfrac{h\nu - \Delta E_F}{kT}\right)}$$

Optical gain spectrum measurements: From TSE

$$G(h\nu) \propto \frac{I_{SP}(h\nu)}{(h\nu)^2} \cdot \left(1 - \exp\left(\frac{h\nu - \Delta E_F}{kT}\right)\right)$$



Modal gain

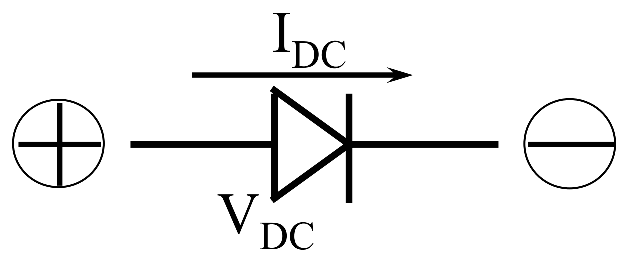


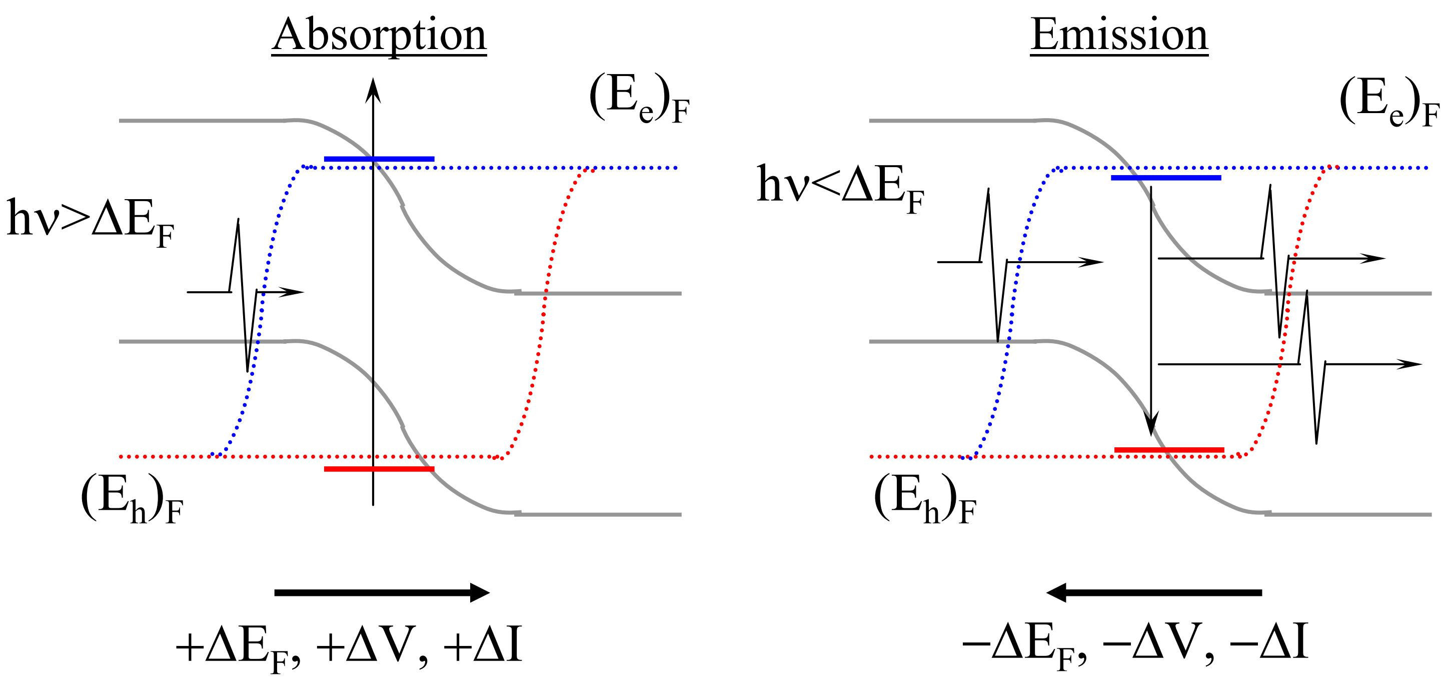
Advantage: measurable in wider spectral range comparing with H-P.

Complication: ΔE_F should be determined from the independent experiment. Needs calibration to find absolute value of the gain.

Determination of transparency energy: Andrekson technique

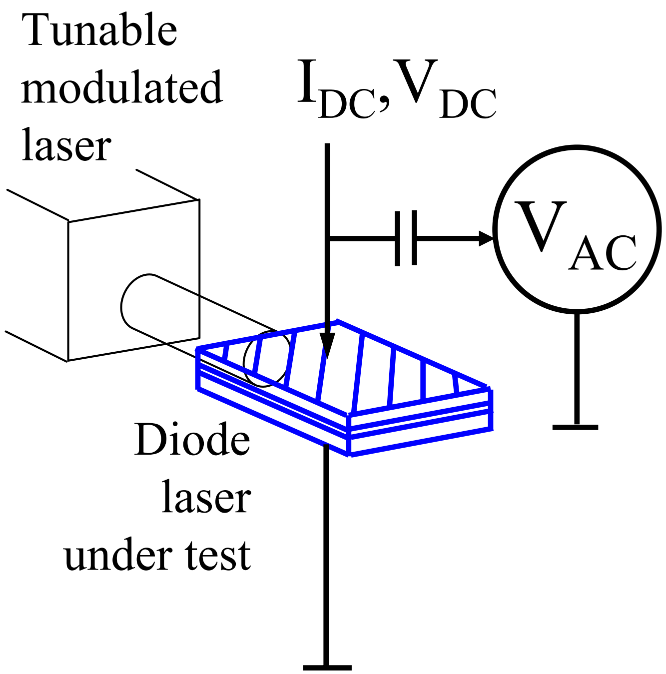
Detection properties of laser diode

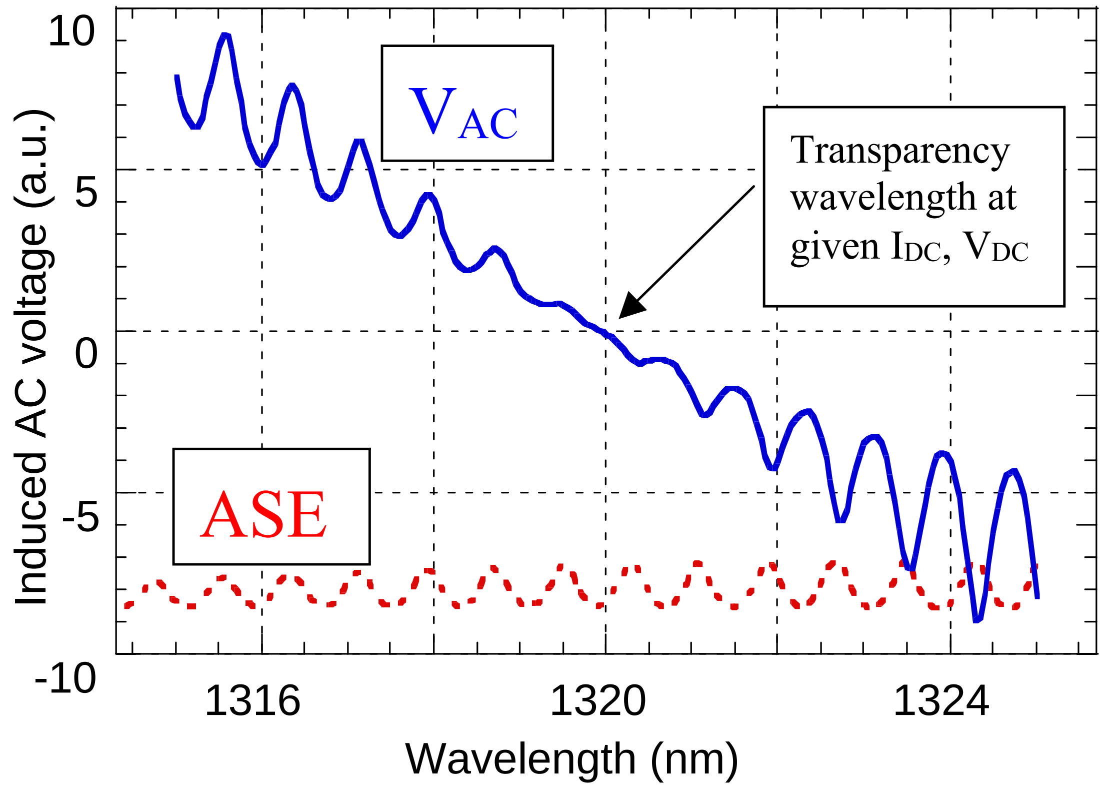




NO VOLTAGE CHANGE ACROSS DIODE WHEN **$h\nu = \Delta E_F$!**

Determination of transparency energy: Andrekson technique

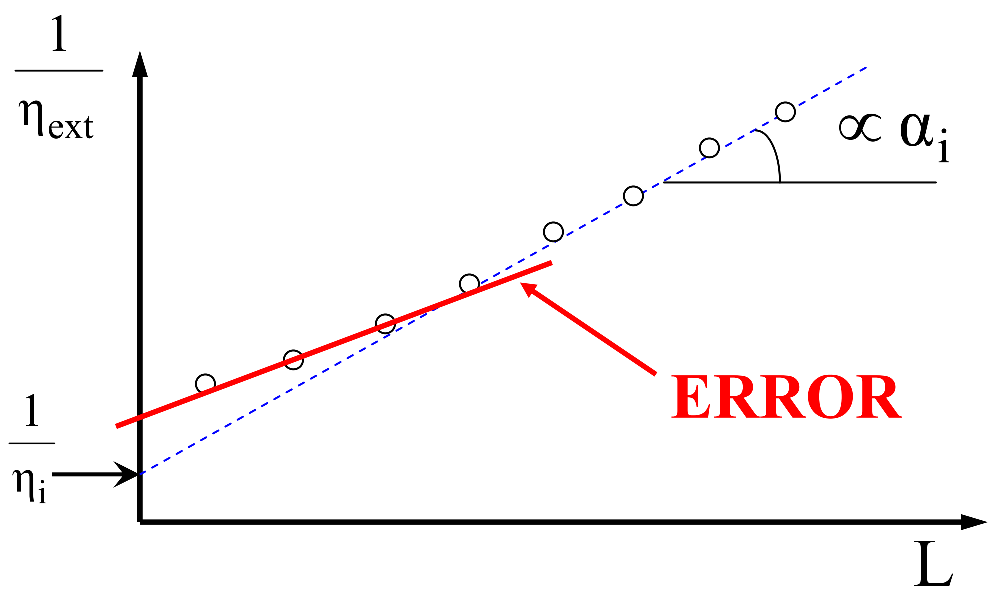




The transparency energy is equal to $\Delta E_F=(E_e)_F-(E_h)_F$.

** Induced voltage is "in phase" with ASE for $h\nu>(E_e)_F-(E_h)_F$ since larger absorption means larger voltage. For $h\nu<(E_e)_F-(E_h)_F$ modes supported by resonator are amplified stronger and maximum of ASE corresponds to the voltage minimum. The energy of voltage oscillation "phase change" corresponds to transparency energy.*

Measurements of optical loss by Variable cavity length method



$$\eta_{ext} = \eta_i \cdot \frac{\alpha_m}{\alpha_m + \alpha_i}$$

$$\frac{1}{\eta_{ext}} = \frac{1}{\eta_i} \cdot \left(1 + \frac{\alpha_i}{\alpha_m}\right)$$

$$\alpha_m = \frac{1}{L} \cdot \ln \frac{1}{\sqrt{R_1 R_2}}$$

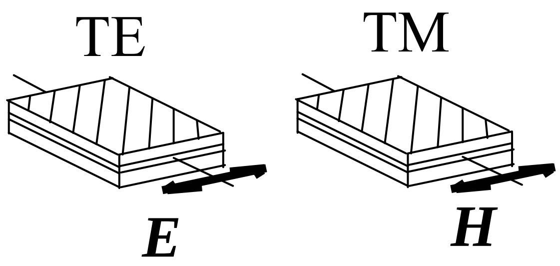
$$\boxed{\frac{1}{\eta_{ext}} = \frac{1}{\eta_i} - \left(\frac{\alpha_i}{\eta_i \ln \sqrt{R_1 R_2}}\right) \cdot L}$$

Assumption: η_i and α_i are cavity length independent

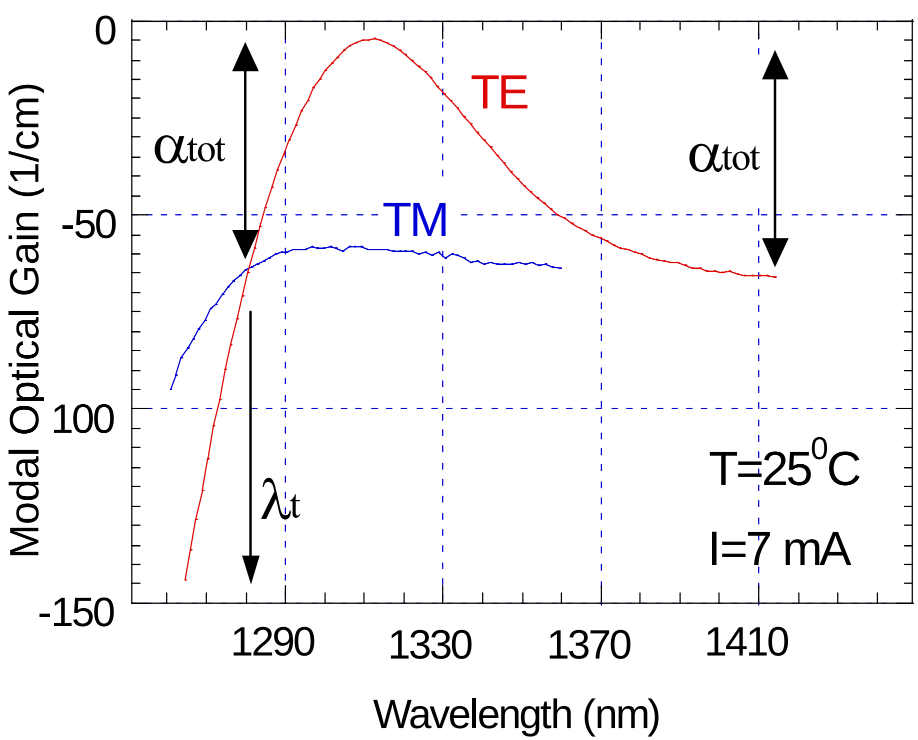
Advantage: Simplicity of measurements

Shortcoming: For short cavities or small internal loss threshold is L-dependent and errors arise. Also a lot of laser material is required.

Measurements of optical loss from modal gain spectrum



$$g^{TE/TM}(h\nu) = \Gamma^{TE/TM} \cdot G^{TE/TM}(h\nu) - \alpha_{tot}^{TE/TM}$$



1. TE and TM modal gains intersection

TE – comes from C-HH transition; TM – comes from C-LH transition. Thus, spectra for TE and TM gains are different and corresponding gains and can be equal only when ***material*** *gain G=0 (transparency point).*

2. Saturation for $h\nu<E_g$

For photon energies below bandgap, ***material*** *gain is equal to zero.* ***Modal*** *(g) gain is equal to total loss within this spectra region. Usually, long wavelength tail of the modal gain spectra gives total loss value. When loss are determined, transparency energy (condition G=0) can be estimated from the gain spectrum.*