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Author(s)	Location	Extension
R. F. Kazarinov	**MH 1C-304**	**2104**
Serge Luryi	**MH 2C-144**	**6614**

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ABSTRACT

Theory of the quantized Hall resistance R_{xy} in 2-dimensional electron gas is presented. Exactness of the quantization is explained as a purely topological effect. It is shown that lines of constant electrostatic potential represent an effective wave-guide for electron waves. In the Corbino ring geometry the condition for quantization of R_{xy} is the existence of equipotentials encircling the central electrode. The quantum of $R_{xy} = h/e^2$ is shown to be unaffected by a random scattering potential. Collapse of the Hall current on increasing disorder is interpreted as a percolation transition.

Novel quantum device - a voltage amplifier - is proposed whose principle of operation is based on controlled percolation. The device is shown to have ideal transfer characteristic and speed limited only by the cyclotron frequency.

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date: **November 3, 1981**

from: **R. F. Kazarinov**
MH 11152
1C-304 x2104

Serge Luryi
MH 52111
2C-144 x6614

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MEMORANDUM FOR FILE

Recent experiments [1,2] with Hall resistance in semiconductor inversion layers in strong transverse magnetic fields established to the accuracy of better than 10^{-5} that Hall conductivity is quantized in multiples of e^2/h. The exactness of this result, which Laughlin [3] attempted to derive from gauge invariance, has led to the proposal [1] of using the quantum h/e^2 as a standard of resistance. The most fundamental consequence of these experiments is the existence of a long-range order in a fermionic system. This order was interpreted [3] as a long-range phase rigidity analogous to that in a superconductor. As will be shown below this picture is incorrect and the meaning of the long-range order consists here in a self-interference of electrons over macroscopic distances of the sample. A good analogy would be to the Bohm-Aharonov experiment [4], or to the double-slit interference of electrons propagating in vacuum a macroscopic distance from the slits to a screen. Hall quantization represents the first effect of this nature discovered for solid-state electrons.

Consider a 2-dimensional electron gas (2-d EG) of geometry shown in Fig.1c in a strong magnetic field H perpendicular to the plane of the figure. First, let us dicuss an ideal situation with no lateral fluctuation of the electrostatic potential due to a random distribution of the fixed charge which compensates the charge of electrons in the inversion layer. In the absence of an applied electric field the electron energy is characterized by quantum numbers i and n, viz.

$$E_{in} = (n + \tfrac{1}{2})\, \hbar\omega_c + E_i \qquad (1)$$

where the cyclotron frequency $\omega_c = eH/m^* c$. Equation (1) differs from the usual expression [5] for a 3-dimensional electron gas in that the continuous energy spectrum $p_z^2/2m^*$ characterizing the free motion in the direction of H is replaced by discrete energies E_i appropriate for a finite motion in the quantum well confining electrons in the inversion layer. The level splitting in this quantum well is assumed to be so large ($\gg \hbar\omega_c$) that all electrons remain frozen in states with $i = 0$ for all H considered. For the sake of clarity we shall neglect spin effects. The levels E_{in} are degenerate with the number of states in each level given by $N = eHS/hc$ where S is the area of the 2-d EG sample. Thus each filled level contributes $\sigma = e^2 H/hc$ to the surface charge density in the inversion layer ($\sigma/e = 1.3\times10^{11} cm^{-2}$ for $H = 5T$). We shall consider only the case when $\hbar\omega_c >> kT$ so that all states below the Fermi level E_F are completely filled and those above E_F empty, as indicated in Fig.1a. States belonging to the same degenerate level can be labelled by an additional quantum number, x_o, which corresponds to one of the coordinates of a classical cyclotron orbit. Along the x-axis the electron wave-function varies like the

nth eigenfunction of a linear oscillator centered at x_o, and is localized with a dispersion $a_n^2 \equiv <n \left| (x-x_o)^2 \right| n> = (n+½)\, a_L^2$ where the Landau length $a_L \equiv (\hbar c/eH)^{½}$ (for H=5T, $a_L \approx$100Å). In the perpendicular direction the wave function is $exp(ip_y y)$ where $p_y = eHx_o/c$. These results of Landau are strictly valid in a Cartesian system while our coordinates are only locally Cartesian which produces a negligible error of order a_L^2/R_1^2. The choice of direction in which orbits are localized is determined by the gauge for the vector potential **A** of the magnetic field and is quite arbitrary. The above choice of the gauge (in which the locus of x_o is a concentric circle for each state, cf. Fig.1c), is particularly convenient when $V_o \neq 0$, i.e. when there is a radial electric field F. It is easily seen that in this case the Schrödinger equation is satisfied by wave-functions of the form similar to those obtained in the absence of the electric field but with a different relation between x_o and the momentum p_y of the electron wave, viz.

$$p_y = \frac{eH}{c} x_o + m^* c \frac{F}{H} \tag{2}$$

and a different energy spectrum,

$$E_{inx_o} = E_{in} + eFx_o + ½ m^* (cF/H)^2 \tag{3}$$

As seen from Eq.(3) the Landau levels are split by electric field. Each quantum number x_o determines an equipotential. The electronic waves are localized in z-direction by the quantum well and in x-direction by the length a_n, while in the y-direction they propagate along the equipotential lines like light in an optical fiber. Each closed equipotential represents a ring resonator which imposes a cyclic boundary condition on the electronic wave. This results in a discrete spectrum of values for x_o, with a step $\delta x_o = a_L^2/L$, where $L \gg a_L$ is the length of the fiber. Two successive orbits separated by the infinitesimal distance δx_o also differ in the total variation of the phase of the corresponding wave-functions on going around the loop. For the wave-function to be single-valued this phase variation must equal $2\pi l$ (with l integer) for any state. The absolute value of l is gauge-dependent but for two successive orbits the difference $\delta l = 1$ (this makes the two states orthogonal). It can be easily shown that the magnetic flux through the area bounded by two corresponding equipotentials equals hc/e. The first term in Eq.(2) gives no contribution to the total current due to a single electron, because it is exactly compensated by the diamagnetic term in the expression for the current density in a magnetic field [5]. Each electron contributes to the Hall current only in virtue of the additional velocity $v_H = cF/H$ it acquires in the electric field. This velocity also gives rise to the kinetic energy term in Eq.(3). Each filled Landau level contributes a linear current density $J = \sigma v_H$ and the total current in y direction is thus

$$I_o = \bar{n}(e^2/h)\, V_o \equiv G_{xy} V_o \tag{4}$$

where $\bar{n}$ is the number of filled Landau levels and G_{xy} the transverse conductivity. We see that for a given V_o the Hall current does not depend on the width of the ring. Although the number of quantized equipotentials (single-electron fibers) is smaller for a narrower ring, the field F and therefore the Hall velocity v_H of each electron are proportionally increased so that the total current remains the same. Even more remarkable is the fact that the Hall current is unaffected by a random electrostatic potential in the inversion layer. This is a purely topological effect as is demonstrated below.

To be specific we consider the experimental situation of ref.6 in which the vanishing longitudinal resistance effect was first discovered. In this work the inversion layer was formed on the *p-GaAs/n-GaAlAs* interface due to a work function difference, cf. Fig.2a. Due to random variations in the surface density of the fixed charge the shape and the depth of the potential well confining the inversion layer also fluctuate and the energies E_i become ill-defined. However, an important feature of this structure is an undoped *GaAlAs* layer of thickness d separating the fixed donor charge from the inversion layer which is formed on the lightly doped *p-GaAs* side of the heterojunction. If $d \gg a_L$ then the energies E_i become smooth functions of the lateral position in the inversion layer and the quantitiy E_0/e becomes equivalent to a random electrostatic potential for the 2-d electrons. In this case the quantum mechanics of the problem can still be described in local Cartesian coordinates formed by the orthogonal grid of equipotential and field lines, but the equipotentials are no longer concentric circles, cf. Fig.2c. Each equipotential represents an effective wave-guide where an electron is localized to within the Landau length a_L. As before, periodic boundary conditions determine the quantization of equipotentials and turn the latter into fiber ring resonators. Topologically, there are two distinct classes of fibers in the geometry of a Corbino ring: *global* which encircle the central electrode, and *local* fibers, which can be contracted to a point by a continuous deformation. It is the existence of global fibers which embodies the long-range order in the 2-d EG. Because of the potential fluctuation the total number of electrons contributing to the Hall current is reduced, since local fibers obviously do not contribute. Nevertheless the current remains the same as in the ideal situation, Eq.(4). Indeed, consider a radial section of the sample which crosses one or more isolated closed loops, e.g., section S1 in Fig.2c. Because points 2 and 3 lie on an equipotential the sum of voltages dropping in regions 1→2 and 3→4 equals the applied voltage V_o. The effective width of the Corbino ring in section S1 is therefore reduced by the distance 2→3. However, the Hall current for a given V_o does not depend on the width of the ring, as discussed above. It may appear that the accuracy of this argument is influenced by the curvature of a fiber which limits the applicability of local Cartesian coordinates. Indeed, the state of the motion transverse to the fiber (local x-direction) is represented by a linear oscillator wave-function only to the accuracy of a_L^2/R^2 where R is a local curvature radius. Nevertheless, the accuracy of the Hall resistance quantization is far greater, as seen from the following rigorous argument.

Consider a strip of thickness a_L along a global fiber of length L. To the accuracy a_L/L this strip can be regarded as a linear conductor, for which the current I_s and the associated flux Φ_s of magnetic field through the contour I_s are complementary thermodynamic variables. Therefore,

$$I_s = \frac{c\,\partial G_s}{\partial \Phi_s} \tag{5}$$

where G_s is the free energy of electrons in the given strip. The single-electron contribution to the total current is given by $I_1 \equiv \delta I(x_o) = \partial I_s/\partial N_s$, with N_s being the number of electrons in the strip. On the other hand, $\partial G_s/\partial N_s \equiv \mu(x_o)$ where μ is the chemical potential of electrons in the strip. This is a thermodynamic relation valid to within $1/N_s$ which is again a quantity of order a_L/L. Differentiating Eq.(5) we have $\delta I(x_o) = c\,\partial\mu/\partial\Phi_s$.

It should be emphasized that the flux of the magnetic field through any fixed area of the ring is *not* quantized and in contrast to the situation familiar in superconductivity it can vary continuously. The fundamental difference is in the nature of the diamagnetism which in the present case is the Landau diamagnetism of the electron gas. The magnetic flux through a hole in a superconducting ring can vary only discontinuously because of its screening by a macroscopic diamagnetic current on the inner surface of the ring. Such a coherent macroscopic current can exist in a superconductor only in virtue of the bosonic nature of the carriers (Cooper pairs) which can multiply occupy the same quantum state. The Landau diamagnetism, on the other hand, is not due to any macroscopic currents† but to the spatial correlations of current densities. The fact that the flux of magnetic field through a fixed surface is not quantized for any fermionic system was overlooked by Laughlin [3] who based his argument on a supposed quantization of the magnetic flux through a loop formed by a ribbon of 2-dimensional metal.

What is quantized in the present case is the magnetic flux through a variable area bounded by two global orbits on the chosen strip. The minimum flux variation $\delta\Phi_s$ corresponds to adding one extra electron to the strip and equals $\delta\Phi_s = hc/e$. As discussed above this magnitude of the flux "quantum" is an exact consequence of the gauge invariance and cyclic boundary conditions on the wave-functions of the current-carrying states. The corresponding quantum of chemical potential at zero temperature represents the variation of the Fermi energy on filling one successive quantized orbit, $\delta\mu = eF\,\delta x_o$. We thus find

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† The surface (or in our case *edge*) currents have long been recognized to be of no statistical significance for the Landau diamagnetism, which is a volume (or in our case *surface*) effect. For an elegant discussion of the Landau diamagnetism in the 2-dimensional electron gas see Ch.4 in Rudolf Peierls, *Surprises in Theoretical Physics*, Princeton University Press, 1979.

$$\delta I(x_o) = \frac{e}{h}\,\delta\mu = \frac{e^2}{h}\,F\,\delta x_o \tag{6}$$

Summing over all filled global fibers, e.g. between points 1→2 and 3→4, we again arrive at Eq.(4). This proves the exactness of Hall quantization at least to the accuracy of a_L/L, with L being a macroscopic distance of the order of the length of the Corbino ring.

At a finite temperature T in addition to the Hall current $I_y = I_o$ there is a longitudinal current I_x due to generation of mobile carriers, i.e. thermal excitation of "electrons and holes" across the Landau gap $\hbar\omega_c$. With decreasing temperature this current goes to zero as $exp(-\hbar\omega_c/kT)$ and so does all dissipation. The vanishing longitudinal conductivity G_{xx} implies that the longitudinal resistance $R_{xx} = G_{xx}/(G_{xx}^2 + G_{xy}^2)$ also vanishes. However, this phenomenon is different in principle from superconductivity and not only in that the latter occurs discontinuously at a finite temperature. An important difference is in the nature of the long-range coherence which in the case of superconductivity consists in the rigidity of the phase of the Cooper-pair system wave-function. In the present case because of the energy splitting the electron waves oscillate at different frequencies and one cannot speak of a common phase of oscillation. On the other hand for a single fermion the phase is never a quantum number because of the Pauli principle and the uncertainty relation between the phase and the particle number.

Variable-range hopping between localized states (in our case - the local fibers) also contributes to a dissipative current along the electric field. Temperature dependence of this current is described by Mott's law $G_{xx} \propto exp[-(T_o/T)^{1/3}]$ for a two-dimensional system [7]. At a sufficiently low temperature or high degree of disorder this path of current dominates over the generation current. Thus, the temperature dependence of R_{xx} obtained in ref.6 can be explained by the Mott conductivity between local fibers.

An interesting phenomenon will occur if we further increase the disorder. When the effective width of the Corbino ring goes to zero (becomes less than a_L) at least in one cross-section then *all* global fibers are squeezed out and the macroscopic Hall current ceases. This phenomenon can be interpreted as a 1st order phase transition of percolation type with the Hall current playing the role of an order parameter. It should be noted, however, that no real percolation of particles is involved and the term "percolation" is used in a new, quantum, sense to describe the penetration of extended electronic orbits over the entire sample. The exact percolation point may depend on the applied voltage V_o. For a given amount of disorder there exists a critical voltage V_o^{cr} below which the Hall current will not be observed. We can minimize V_o^{cr} by reducing the influence of the random potential variation in the inversion layer, e.g. by increasing the thickness of the undoped buffer layer. In the experiment of ref.6 the Hall current was observed for V_o as low as $50\,mV$.

A remarkable property of the described percolation transition is that it can be brought about in a controlled fashion by simulating "disorder" with the help of a voltage applied to a gate. An entirely new quantum device can be based on this principle. The proposed device, called the PHASER‡ is shown in Fig.3a. It represents a Corbino disk with two pairs of contacts to the inversion layer and two parallel insulated gate electrodes deposited on the surface of the disk. We shall analyze the PHASER operation for an ideal case with no random potential. As discussed above, a moderate amount of disorder will not affect the device performance. In the absence of the gate voltage the electron wave-guides (quantized equipotential lines) represent global fibers (Fig.1c) and the Hall current is flowing. In this case the output voltage $V_{out} = V_o$ to the accuracy of the ratio R_{xx}/R_{xy}. The minimum value of this ratio obtained in ref.6 was less than 10^{-10} and it can be made even smaller but this would be an unnecessary luxury. When an input voltage V_{in} is applied between the gate electrodes the shape of the equipotential lines changes as shown in Fig.3a. The blank area is bounded by a local (though long!) equipotential fiber. Electrons within this area have insufficient energy to pass over the potential barrier created by the gate and suffer a quantum mechanical reflection. Two kinks shown on the global fibers correspond to a *refraction* of electron waves under the gate electrode. The number of global fibers is reduced but the

‡ The name stems from two sources: the controlled *phase* transition and the modulation of electronic *phase.*

Hall current and therefore V_{out} remain constant. This is equivalent to an effective reduction of the Corbino disk width $R_2 - R_1$. When the gate voltage $V_{in} = V_o$ the effective disk width vanishes which is equivalent to the phase transition described above. The ring Hall current ceases abruptly and so does the Hall e.m.f. in the output circuit. In this state $V_{out} = 0$. The resulting transfer characteristic of a PHASER is shown in Fig.3b.

Even though an insulated gate is involved the principle of operation of the PHASER is different from any field-effect device in that the surface density of electrons in the channel remains constant during switching. To illustrate this point we consider a capacitor formed by the 2-d EG and an insulated gate. In a strong magnetic field the differential capacitance, $C \equiv \partial\sigma/\partial V_G$, is "quantized" as shown in Fig.3c. Although this effect has not yet been observed, its existence follows straightforwardly from the existence of gaps in the density of states spectrum shown in Fig.1a. In a $C = 0$ plateau the inversion layer behaves like an insulator with respect to the transverse field so long as the applied voltage is insufficient to transfer an electron from the gate (Fermi level) to the next unfilled Landau level.

A question of principle thus arises: what is the limiting speed of the proposed device? Inasmuch as the input capacitance between the gate electrodes is charged through a vanishing resistance R_{xx} the only fundamental limitation is associated with electron inertia, i.e. the finite equilibration time between the Hall current and Hall e.m.f. This time can be evaluated by considering the effective *proper* inductance L_o and capacitance C_o of the 2-d EG in a strong magnetic field. The kinetic energy W of the Hall electrons (cf. the last term in Eq.3) can be expressed either as $W = \frac{1}{2} L_o I_o^2$ with

$$L_o = \frac{R_2 - R_1}{R_1 + R_2} \frac{\hbar}{\bar{n} e^2 \omega_c} \tag{7}$$

or as $W = \frac{1}{2} C_o V_o^2$ with

$$C_o = \frac{R_1 + R_2}{R_2 - R_1} \frac{\bar{n} e^2}{\hbar \omega_c} \tag{8}$$

Because of the vanishing channel to gate capacitance discussed above, it is the intrinsic LC circuit whose characteristic frequency ν_o determines the transition time τ_o, viz.

$$\tau_o \equiv \nu_o^{-1} = \frac{2\pi}{\sqrt{L_o C_o}} \tag{9}$$

where $\nu_o = \omega_c / 2\pi = 2 \times 10^{12} Hz$ (here and below we use in our estimates the m^* of *GaAs* and $H = 5T$). It may be worthwhile to point out that in addition to the effective contour $L_o C_o$ the Corbino ring along with the contacts represents an ordinary "geometric" LC circuit whose resonant wavelength is of the order of the ring size, R. The delay $\tau_c \approx R/c$ becomes comparable to τ_o only for $R \gtrsim 100\ \mu m$. In terms of ν_o it appears especially attractive to use materials with low effective mass, e.g. *HgTe* ($m^* \sim 10^{-3} m_{el}$) which forms a lattice-matched heterojunction with *CdTe.*

To estimate the power-delay product (energy required to switch *on* or *off* the Hall current) we take $V_o = \hbar\omega_c / e$ ($\sim 10\ mV$, although in principle it can be made smaller depending on the minimum V_o^{cr} achieved) and find

$$W = \frac{\bar{n}}{2} \hbar\omega_c \frac{R_1 + R_2}{R_2 - R_1} \tag{10}$$

The number $\bar{n}$ of filled Landau levels is typically of order unity and the geometrical ratio in Eq.(10) is of order 10, which gives $W \approx 10^{-20} J$. The PHASER as described represents an ideal logic element for integrated circuits. In contrast to all existing logic devices, like field-effect or Josephson-junction transistors, operation of this device requires nearly vanishing dissipation. Take, for example, the case of a Josephson device. During switching the energy of the electric field stored in the tunnel-barrier capacitance dissipates by discharging through the tunnel resistance in its non-superconducting state. In the PHASER, the linear current density in the inversion layer is perpendicular at every point to the local electric field to the accuracy of the ratio R_{xx}/R_{xy}. As discussed above this ratio can be made less than 10^{-10} radians and to this accuracy there is no dissipation. The energy W considered above can disipate only through a finite resistance of the contacts. Consider a (for now Gedanken) situation when the latter are also made superconducting. In this case the logic operation will consist in relocalizing the energy in different parts of the integrated circuit. At a constant temperature T the work done by a power source will be directly related to a change in the entropy content of the information processed by the circuit. Measuring the T dependence of the energy drawn from the battery (which in principle can be negative for certain input routines) we can determine the entropy drop between the input and output terminals.

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R. F. Kazarinov

Serge Luryi

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Figure Captions

Figure 1

Ideal 2-d EG in quantizing magnetic field. a) Density of states in the absense of electric field; b) Energy spectrum in crossed electric and magnetic fields; c) Electron wave-guides in a sample of Corbino geometry.

Figure 2

Quantized inversion layer in a strong transverse magnetic field. a) Cross-section of the structure studied in ref.6; b) Quantized energies of the transverse motion in the potential well confining the inversion layer; c) Schematic diagram of the local and global fibers.

Figure 3

The PHASER. a) Schematic lay-out of the device showing global fibers at a finite input voltage; b) Transfer characteristic; c) Differential capacitance between the gate and the 2-d EG in quantizing magnetic field.

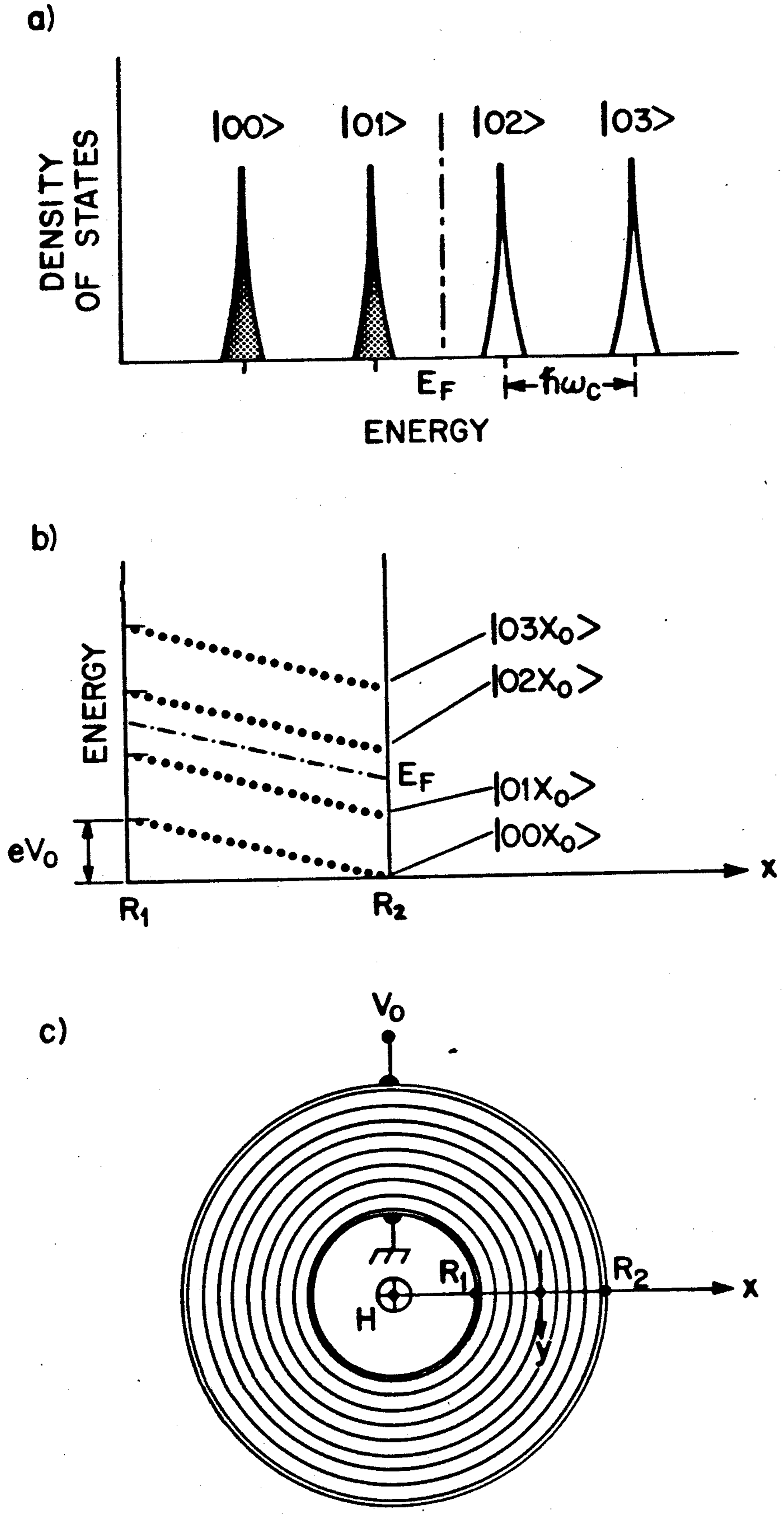
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FIGURE 1

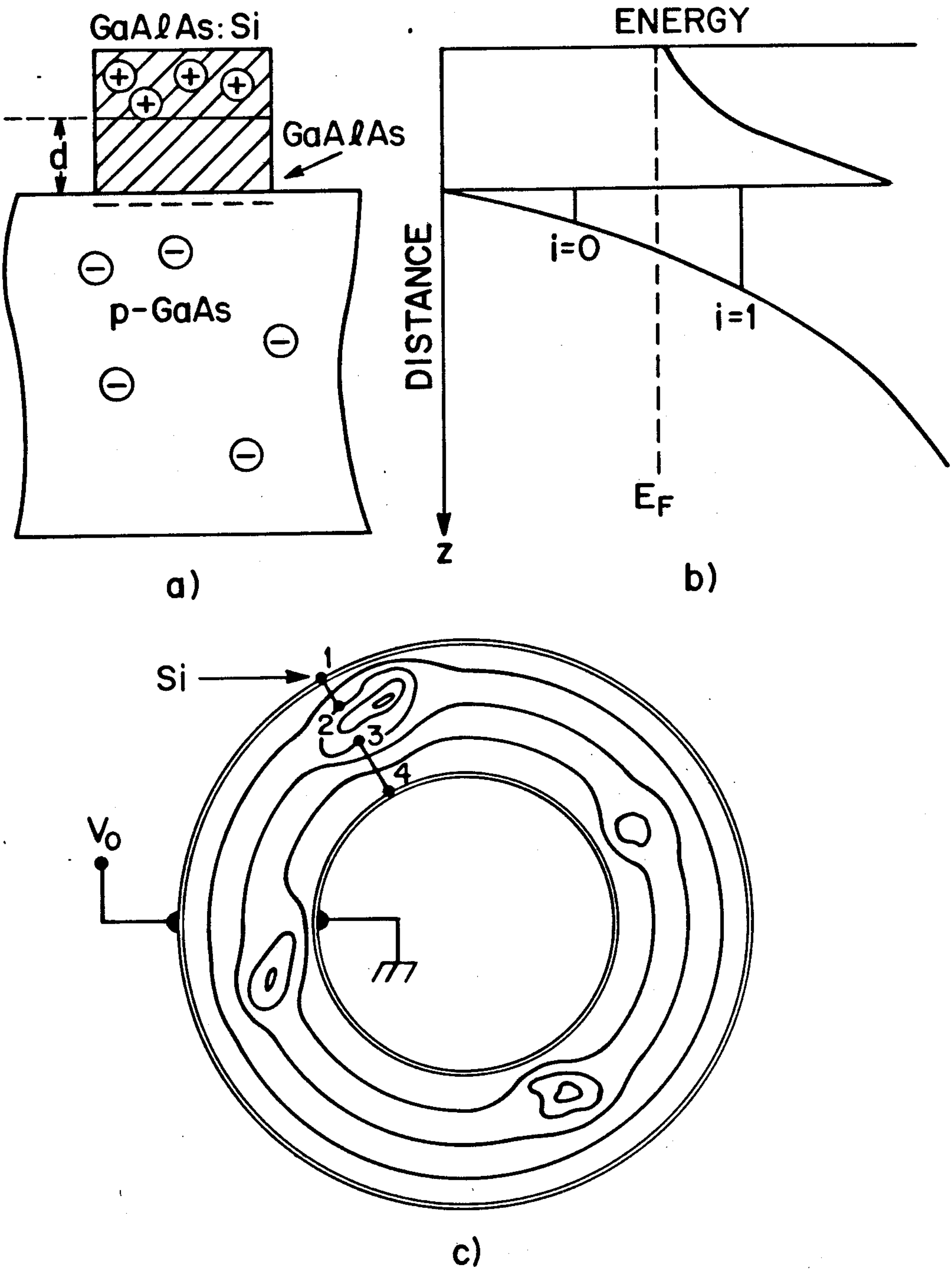
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FIGURE 2

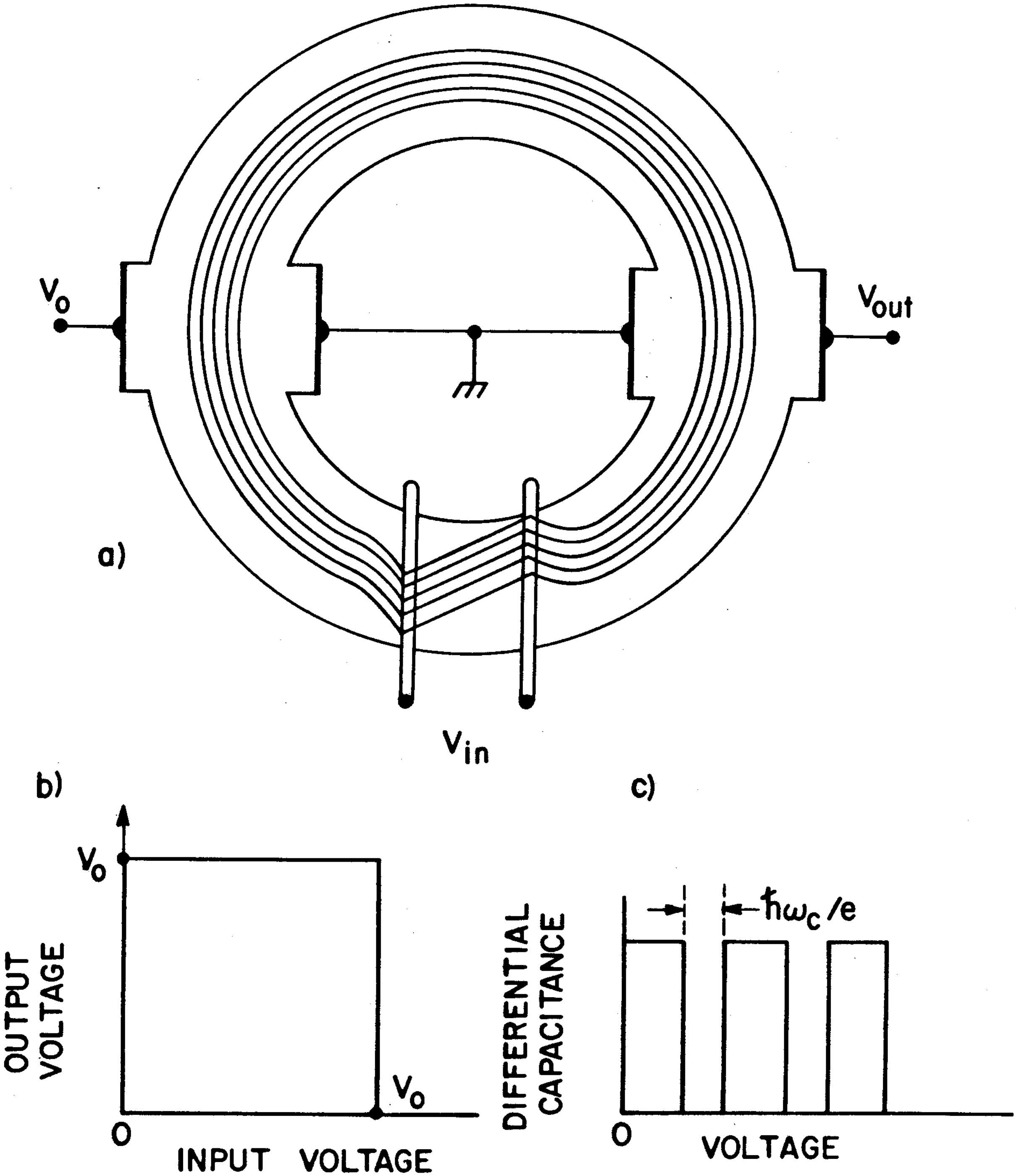
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FIGURE 3