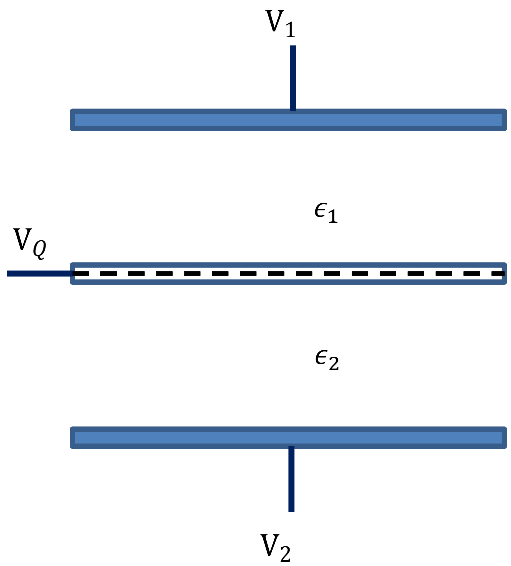

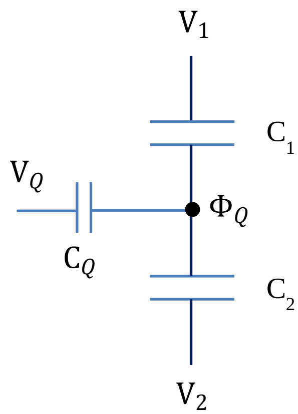

Notes

Geometric capacitances C_1 and C_2 enter symmetrically as they should.

Letting $V_2 = V_Q$ = GND we recover the equivalent circuit of my 1988 paper.

It is, perhaps, instructive to ponder the physical meaning of the two capacitor plates of C_Q in the above equivalent circuit. The left "Fermi-level" plate is connected to the voltage terminal and represents the chemical potential level. The right "electrostatic" plate represents the electrostatic potential at the quantum well so that the voltage drop across C_Q equals the Fermi level of carriers in the QW.

Examples of more complicate equivalent circuits involving quantum capacitance can be found in the paper by Y. Katayama and D. C. Tsui, "Lumped circuit model of two-dimensional to two-dimensional tunneling transistors", Appl. Phys. Lett. **62**, pp. 2563-2565 (1993).

In my 1988 derivation, I had cut corners in Eqs. (3 a,b). Indeed, these equations *assumed* that the charge on the first electrode is the biggest of the three, so that the other two can be parameterized by a single angle φ. Obviously, this is not the case in a general three-terminal structure. But this is a minor point.

The more important point is that the three voltages, V_1, V_2, and V_Q, are fixed as "boundary conditions" and the quantities to be determined are the three charge densities σ_1, σ_2, and σ_Q, as well as the electrostatic potential Φ_Q of the quantum well.

The electric fields F_1 and F_2, are given in terms of the charge densities as $F_i = (4\pi/\epsilon_i)\,\sigma_i$, and in terms of the potential Φ_Q as follows: $F_i d_i = \left(\frac{4\pi}{\epsilon_i}\right) d_i \sigma_i = V_i - \Phi_Q$. Whence we have two "electrostatic" equations,

$$\sigma_i = \frac{\epsilon_i}{4\pi d_i}\left(V_i - \Phi_Q\right) \qquad (i = 1,2), \tag{1}$$

and a third, "chemical" relation,

$$\sigma_Q = \frac{me^2}{\pi\hbar^2}\left(V_Q - \Phi_Q\right) \tag{2}$$

Equations (1) and (2) express all unknown quantities in terms of the electrostatic potential Φ_Q of the quantum well, which is then found from the neutrality condition, $\sigma_1 + \sigma_2 + \sigma_Q = 0$, and expressed in terms of the three voltages, V_1, V_2, and V_Q. The three equations (1) and (2) are embodied in the equivalent circuit above.

The problem is solved, no need for minimization of the total energy! We have gotten away so cheaply, because now we *assumed known* the "chemical" relation (2) that defines the quantum capacitance.

If we wanted to follow in the footsteps of Luryi (88), we could *derive* Eq. (2) by minimization of the total energy, $E_{tot} = E_1 + E_2 + E_Q$, where $E_i = \left(\frac{2\pi d_i}{\epsilon_i}\right)\sigma_i^2 \;\; (i = 1,2)$ represent the field energies and E_Q is the chemical energy of 2D electrons, $E_Q = \left(\frac{\pi\hbar^2}{2me^2}\right)\sigma_Q^2$. Of course, the minimization is subject to the neutrality condition, $\sigma_1 + \sigma_2 + \sigma_Q = 0$. If you minimize E_{tot} with respect to a single parameter, say σ_Q , then Eq. (2) will emerge! The calculation is somewhat cumbersome and quite unnecessary, given the fact that the quantum capacitance (2) had been already established in a simpler model.