Markov Chain Flow Decomposition for a Two Class Priority Queue

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Abstract —

A simple circulatory structure of probability flux for a two class Markovian priority queueing system is found. This is the second instance of such a finding for a non-product form network, leading to the possibility that many non-product networks have a simple decomposed flow structure. Results of this type may lead to the development of new computational algorithms for Markov chains.

I. Introduction

Markov chains have been a mathematically elegant means of stochastic modeling for almost a century. It is well known that for any Markov chain, the equilibrium state probabilities can be computed by solving the associated set of linear global balance equations. Unfortunately numerical solution techniques for N state arbitrary Markov chains can involve a computational complexity of $O(N^3)$, allowing arbitrary Markov chains of only modest size to be exactly solved. However since the original work of Jackson in 1957 [1] it has been known that certain classes of queueing networks have a tractable analytical solution of the product form type. That is, for this product form class of queueing networks any state equilibrium probability is a product of system parameters and a reference probability.

In 1984 Lazar and Robertazzi [3] showed that the circulatory structure of product form network Markov chains can be decomposed into an aggregation of simpler (usually cyclic) circulations. However product form networks are a limited form of queueing networks. Other types of queueing networks, such as that in [2] and here, have been known as non-product form queueing networks.

II. Main Result

Software has been developed that verifies a simple and exact flow structure for a two class priority queueing system. The flow structure consists of smaller cycles of flow transiting two transitions (as from (n_1,n_2) to (n_1+1,n_2) and back to (n_1,n_2) in the figure) and larger rectangular flows (from (0,0) to (n_1,0) to (n_1,n_2) to (0,n_2) and back to (0,0)). Flows may be negative. We believe that this exact flow structure is not unique for this queueing system.

This type of decomposition appears to be analogous to the mesh equations of circuit theory. Circuit theorists tend to prefer the use of nodal equations as their use often leads to a smaller total number of equations. However queueing network Markov chains have more structure than arbitrtrary circuits and so one can hope that this additional structure will lead to the development of new and efficient computational algorithms.

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III. Conclusion

The fact that simple and exact decompositions have been found for tandem queues with blocking [2] and a priority queueing system makes it clear that such decompositions are possible for a variety of non-product form queueing networks. The open research question is whether this will lead to new computational algorithms for Markov chain state equilibrium probabilities (and hence performance measures).

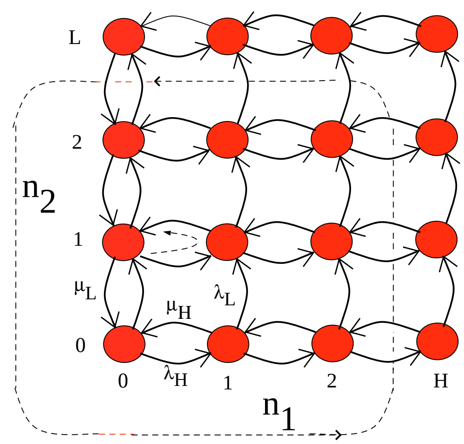

Figure 1: Markov chain flow

References

[1] J.R. Jackson, "Networks of Waiting Lines," *Operations Research*, vol. 5, 1957, pp. 518-521.

[2] T.G. Robertazzi, "Twinflows: Markov Chain Flow Decomposition for Tandem Queues with Blocking, " *Proc. of the 2002 Conference on Information Sciences and Systems*, Princeton University, March 2002.

[3] T.G. Robertazzi, *Computer Networks and Systems: Queueing Theory and Performance Evaluation*, 3rd ed., Springer-Verlag, NY, 2000.