JOINT SYMBOL DETECTION AND TIMING ESTIMATION USING PARTICLE FILTERING

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ABSTRACT

This paper addresses joint estimation of the timing epoch and detection of the transmitted symbols in a digital communication system. Most timing recovery techniques found in the literature are either approximately or heuristically derived, since optimal estimators are analytically intractable. Our approach to the problem relies on modeling the symbol timing as an autoregressive process. In this way, the digital communication system can be mathematically represented by a dynamic system in state-space form and the sequential Monte Carlo (SMC) methodology can be applied. SMC algorithms are powerful tools for Bayesian estimation that are based on representing the posterior distribution of the system state by a discrete measure with random support. This representation can be updated recursively, as new information becomes available, allowing for optimal estimation of both, the transmitted symbols and their timing.

1. INTRODUCTION

The fundamental goal of a digital receiver is the detection of transmitted symbols with maximum reliability. However, the signal observed at the receiver is distorted due to the effect of the transmission channel. For accurate symbol detection, several physical parameters must be estimated and compensated for prior to the detection, and they include the symbol timing, the carrier frequency, and the carrier phase. The generalized synchronization problem deals with the estimation of these parameters from the signals collected at the receiver front end.

Different techniques have been proposed for solving the synchronization problem, but most of them are based on approximate and heuristic methods because optimal estimation of the parameters of interest is analytically intractable (see [1] for a review of the subject). Broadly speaking, the synchronization techniques found in the literature can be categorized as decision directed (or data-aided) and non-decision directed (or non-data-aided) [1]. Decision directed schemes depend on the availability of reliable symbol estimates for obtaining parameter estimates and, therefore, they usually require training signals. The most common decision-directed schemes are derived from (approximate) maximum-likelihood (ML) estimation theory. Unlike data-aided techniques, non-data-aided methods do not require knowledge of the transmitted symbols and, instead, they exploit the statistics of the digital waveforms, such as the second order cyclostationarity, which is exhibited by digital modulations [2]. Approximate ML estimation techniques can also be used in non-decision directed methods if the symbols are treated as random variables with known statistics [1].

Sequential Monte Carlo (SMC) techniques [3] (also referred to as *particle filtering* methods) are powerful tools for Bayesian estimation that employ discrete measures with random supports for representing posterior distributions of unknowns of interests. Recently, SMC has been successfully applied in communications including to joint estimation and decoding of space-time trellis codes [4] and equalization [5, 6]. The SMC approach is also potentially useful for joint symbol detection and synchronization because it provides a way to numerically compute optimal estimators when exact solutions cannot be derived analytically [7]. In order to apply common SMC algorithms, e.g., sequential importance sampling (SIS), the observed signal needs to be written as a dynamic system in state-space form. Several authors have addressed the problem of symbol detection with SMC methods, but under the assumption of perfect knowledge of the synchronization parameters [8]. However, as we have already discussed, the actual values of some of the system parameters (propagation delay, phase and frequency offsets) are completely or partially unknown, and they must be estimated.

In this paper we propose a method based on particle filtering that jointly detects the transmitted symbols and measures their timing. The algorithm is derived by considering an extended dynamic system where the symbol delay and the transmitted symbols are state variables. Specifically, the delay is modeled as a first-order autoregressive (AR) stochastic process, while the transmitted symbols are independent and identically distributed (i.i.d.) random variables from a discrete uniform distribution. In this way, both symbols and their delays can be optimally estimated using a particle filter.

The remaining of the paper is organized as follows. Section 2 describes the system model. The proposed algorithm for joint symbol detection and timing estimation is presented in Section 3. Illustrative computer simulations are shown in Section 4. Finally, Section 5 contains our conclusions.

2. SIGNAL MODEL

Consider a digital communication system where symbols, $\{s_m\}$, from an arbitrary alphabet are transmitted in frames of length M. The noisy received complex envelope for any linearly modulated signal has the form

$$y(t) = \sum_{m=0}^{M-1} s_m g\left(t - mT + \tau(t)\right) e^{j(\theta + \omega t)} + v(t)$$

This work was supported by the National Science Foundation under Award CCR-0082607, *Ministerio de Ciencia y Tecnología* of Spain and *FEDER* funds of the European Union (TIC2001-0751-C04-01).

where $g(t)$ is the modulation pulse waveform, T is the symbol period, $0 < \tau(t) < T$ is the time-varying symbol delay, θ and ω are the carrier phase and the carrier frequency offsets, respectively, and $v(t)$ is complex additive white Gaussian noise (AWGN).

The equivalent discrete-time signal model after sampling is given by

$$y_k = \sum_{m=0}^{M-1} s_m g(kT_s - mT + \tau_k) e^{j\theta} e^{j\frac{2\pi}{N_s}k\nu} + v_k$$

where $y_k = y(kT_s)$, $v_k = v(kT_s)$, $k = 0, ..., K-1$ denotes the sample discrete-time index, T_s is the sampling period, $N_s = \frac{T}{T_s}$ is the number of samples per symbol, $\nu = \frac{\omega}{2\pi}T$ is the normalized frequency offset, and $\tau_k = \tau(kT_s)$. Note that, after sampling, the noise term $v(kT_s)$ remains white with variance σ^2.

Without loss of generality, let us assume that $g(t)$ is a causal pulse with finite duration. This happens in practical situations due to the use of truncated Nyquist pulses (i.e., time-shifted raised-cosines with limited duration). Therefore, we can express the received signal as

$$y_k = \sum_{m=k-L}^{k} s_m g(kT_s - mT + \tau_k) e^{j\theta} e^{j\frac{2\pi}{N_s}k\nu} + v_k$$

where $L+1$ is the Inter-Symbol Interference (ISI) span with the assumption that $L < M$. Using vector notation, we arrive at the convenient representation

$$y_k = e^{j\theta} \mathbf{s}_k^\top \mathbf{g}_k(\tau_k, \nu) + v_k, \qquad (1)$$

where $\mathbf{s}_k = [s_{k-L}, ..., s_k]^\top$ is an $(L+1) \times 1$ vector, and

$$\mathbf{g}_k(\tau_k, \nu) = \begin{bmatrix} g(kT_s - (k-L)T + \tau_k) e^{j\frac{2\pi}{N_s}k\nu} \\ g(kT_s - (k-L+1)T + \tau_k) e^{j\frac{2\pi}{N_s}k\nu} \\ \vdots \\ g(kT_s - kT + \tau_k) e^{j\frac{2\pi}{N_s}k\nu} \end{bmatrix}$$

is an $(L+1) \times 1$ vector that represents the channel.

In general, the objective is to jointly estimate the transmitted symbols, s_m, $m = 0 : M-1$, the signal timing, τ_k, the phase rotation θ, and the frequency offset, ν, using the received signal, $y_{0:K-1}$. For clarity of presentation, however, in this paper we restrict ourselves to the problem where only the symbols and the delays are unknown.

3. PARAMETER ESTIMATION USING PARTICLE FILTERING

Following [9], we can model the symbol timing as a first order AR process,

$$\tau_k = a\tau_{k-1} + u_k \qquad (2)$$

where the perturbation variable, u_k, is assumed to be a zero-mean Gaussian with variance σ_u^2. The values of a and σ_u^2 depend on the transmitter and receiver timing jitter. For negligible Doppler shifts and stable local oscillators at both ends, the value of a should be set close to one, and σ_u^2 should be chosen very small [9].

In the sequel, we assume that the carrier phase and frequency offsets, θ and ν respectively, are correctly compensated for and that the received signal is sampled at the symbol rate, i.e., $T = T_s$. Under these assumptions and taking into account the structure of the symbol vectors due to ISI, we can combine (2) and (1) to obtain the following state-space representation of the communication system:

$$\left.\begin{aligned} \tau_k &= a\tau_{k-1} + u_k \\ \mathbf{s}_k &= \mathbf{S}\mathbf{s}_{k-1} + \mathbf{d}_k \end{aligned}\right\} \quad \text{state equation}$$

$$y_k = \mathbf{s}_k^\top \mathbf{g}(\tau_k) + v_k \quad \text{observation equation}$$

where $\mathbf{g}(\tau_k) = [g(LT + \tau_k), g((L-1)T + \tau_k, \ldots, g(\tau_k)]^\top$,

$$\mathbf{S} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

is an $(L+1) \times (L+1)$ shifting matrix and $\mathbf{d}_k = [0, \ldots, 0, s_k]^\top$ is the $(L+1) \times 1$ perturbation vector that contains the new symbol, s_k. Note that the system state at time k is given by $(\mathbf{s}_k, \tau_k)$. The model parameters, a, σ_u^2, σ_v^2 and L, are assumed fixed and known, and we focus on the joint estimation of the symbols, $s_{0:M-1} = \{s_0, \ldots, s_{M-1}\}$, and the delay process, $\tau_{0:M-1} = \{\tau_0, \ldots, \tau_{M-1}\}$, from the available observations $y_{0:M-1} = \{y_0, \ldots, y_{M-1}\}$.

From a Bayesian perspective, all information relevant for the estimation of $\{s_{0:k}, \tau_{0:k}\}$ is contained in the joint posterior probability distribution of the system state[1] $p(\mathbf{s}_{0:k}, \tau_{0:k}|y_{0:k})$. Unfortunately, the estimation of the latter density is analytically intractable and, thus, it is not possible to obtain estimates (e.g., minimum mean squared error or maximum *a posteriori* probability) of the state sequence in closed-form. Therefore, we resort to the sequential importance sampling (SIS) methodology [10]. The basic idea behind SIS is to approximate the posterior distribution by means of a discrete measure with random support that can be recursively updated as new observations become available. More specifically, $p(\mathbf{s}_{0:k}, \tau_{0:k}|y_{0:k})$ is approximated using a set of N particles, $\{(\mathbf{s}_{0:k}, \tau_{0:k})^{(n)}\}_{n=1}^N$, with associated importance weights, $w_k^{(n)}$. The particles are samples from an importance function, $\pi(\mathbf{s}_{0:k}, \tau_{0:k})$, with the same support as the true posterior distribution, and the weights are computed as

$$w_k^{(n)} \propto \frac{p(\mathbf{s}_{0:k}, \tau_{0:k}|y_{0:k})}{\pi(\mathbf{s}_{0:k}, \tau_{0:k})}.$$

It can be shown [7] that the estimate

$$\hat{p}(\mathbf{s}_{0:k}, \tau_{0:k}|y_{0:k}) = \sum_{n=1}^{N} w_k^{(n)} \delta((\mathbf{s}_{0:k}, \tau_{0:k}) - (\mathbf{s}_{0:k}, \tau_{0:k})^{(n)})$$

where $\delta(\cdot)$ is Dirac's delta function, converges in mean squared error to $p(\mathbf{s}_{0:k}, \tau_{0:k}|y_{0:k})$ as $N \to \infty$.

The most salient feature of SIS methods is the possibility to recursively update the particles and weights at time k when a new observation, y_{k+1}, is available. Indeed, if the importance density is factorized as

$$\pi(\mathbf{s}_{0:k}, \tau_{0:k}) = \prod_{i=0}^{k} \pi_i(\mathbf{s}_i, \tau_i)$$

[1] Notice that estimating $s_{0:k}$ given $y_{0:k}$ is completely equivalent to estimating $\mathbf{s}_{0:k}$ given the same observations.

then the particles and weights are updated as

$$
\begin{aligned}
(\mathbf{s}_{k+1}, \tau_{k+1})^{(n)} &\sim \pi_{k+1}(\mathbf{s}_{k+1}, \tau_{k+1}) \\
w_{k+1}^{(n)} \propto w_k^{(n)} & \frac{p(y_{k+1}|\mathbf{s}_{k+1}^{(n)}, \tau_{k+1}^{(n)}) p(\mathbf{s}_{k+1}^{(n)}, \tau_{k+1}^{(n)}|\mathbf{s}_k^{(n)}, \tau_k^{(n)})}{\pi_{k+1}(\mathbf{s}_{k+1}^{(n)}, \tau_{k+1}^{(n)})}, \quad (3)
\end{aligned}
$$

hence the sequentiality of the algorithm.

The efficiency of SIS algorithms largely depends on the choice of. the importance function. The optimal importance function is

$$
\begin{aligned}
\pi_{k+1}(\mathbf{s}_{k+1}, \tau_{k+1}) &= p(\mathbf{s}_{k+1}, \tau_{k+1}|\mathbf{s}_k^{(n)}, \tau_k^{(n)}, y_{k+1}) \\
&\propto p(\mathbf{s}_{k+1}|\mathbf{s}_k^{(n)}, \tau_{k+1}, y_{k+1}) \\
&\quad \times p(\tau_{k+1}|\tau_k^{(n)}, y_{k+1}), \qquad (4)
\end{aligned}
$$

which can be shown to minimize the variance of the importance weights [7]. Unfortunately, the last factor in (4) is difficult to deal with, hence we propose to use

$$
\pi_{k+1}(\mathbf{s}_{k+1}, \tau_{k+1}) \propto p(\mathbf{s}_{k+1}|\mathbf{s}_k^{(n)}, \tau_{k+1}, y_{k+1}) p(\tau_{k+1}|\tau_k^{(n)}) \qquad (5)
$$

instead. The SIS algorithm for joint timing estimation and symbol detection using the importance function (5) is described below:

1. **Initialization.** We assume knowledge of the prior distribution of the state, i.e., $p(\mathbf{s}_{-1}, \tau_{-1})$. This is reasonable in practice. The *a priori* density of the symbol delay is uniform in $(-T/2, T/2)$ or, equivalently, in $(0, T)$ [1]. Also, in digital communication systems where symbols are transmitted in frames, the waveform preceding the first information symbol is a system design parameter and, therefore, is known by the receiver (e.g., the *tail bits* in normal GSM bursts). Therefore, the vector $\mathbf{s}_{-1}$ is known in practice.

As a consequence, the SIS algorithm is initialized at $k = -1$ as $\mathbf{s}_{-1}^{(n)} = \mathbf{s}_{-1}$ and $\tau_{-1}^{(n)} \sim \mathcal{U}(0, T)$, $n = 1, 2, \ldots, N$. All the particles are equally weighted, i.e., $w_{-1}^{(n)} = 1/N$.

2. **Importance sampling.** At time k, the discrete measure of the particle filter computed via the SIS algorithm is $\{\mathbf{s}_k^{(n)}, \tau_k^{(n)}, w_k^{(n)}\}_{n=1}^N$. When y_{k+1} is observed, the state is propagated one time step using the importance function (5). Sampling from this function is practically achieved in two steps. First, the delay is sampled according to

$$
\tau_{k+1}^{(n)} \sim \mathcal{N}(a\tau_k^{(n)}, \sigma_u^2)
$$

where $\mathcal{N}(\mu, \sigma^2)$ is the Gaussian distribution with mean μ and variance σ^2. Then, the vector $\mathbf{s}_{k+1}^{(n)}$ is sampled from $p(\mathbf{s}_{k+1}|\mathbf{s}_k^{(n)}, \tau_{k+1}^{(n)}, y_{k+1})$. Since the transmitted symbols are i.i.d. discrete uniform random variables, the latter density can be decomposed as

$$
\begin{aligned}
p(\mathbf{s}_{k+1}|\mathbf{s}_k^{(n)}, \tau_{k+1}^{(n)}, y_{k+1}) &\propto p(y_{k+1}|s_{k+1}, \mathbf{s}_k^{(n)}, \tau_{k+1}^{(n)}) \\
&= \mathcal{N}(\mu_{k+1}^{(n)}(s_{k+1}), \sigma_v^2) \qquad (6)
\end{aligned}
$$

where $\mu_{k+1}^{(n)}(s_{k+1}) = [s_{k-L+1}^{(n)}, \ldots, s_k^{(n)}, s_{k+1}]^\top \mathbf{g}(\tau_{k+1}^{(n)})$. Notice that, given $\mathbf{s}_k^{(n)}$, we only need to draw the new symbol, $s_{k+1}^{(n)}$, in order to build $\mathbf{s}_{k+1}^{(n)}$. Let $\mathcal{S} = \{S_1, ..., S_J\}$ be the modulation alphabet. According to (6), we can assign posterior probabilities to the symbols in $\mathcal{S}$ and derive the probability mass function

$$
\begin{aligned}
\rho^{(n)}(s_{k+1}) &= p(s_{k+1} = S_j|\mathbf{s}_k^{(n)}, \tau_{k+1}^{(n)}, y_{k+1}) \\
&= \frac{\mathcal{N}\left(\mu_{k+1}^{(n)}(S_j), \sigma_v^2\right)}{\sum_{S\in\mathcal{S}} \mathcal{N}\left(\mu_{k+1}^{(n)}(S), \sigma_v^2\right)} \qquad (7)
\end{aligned}
$$

for $n = 1, ..., N$. Therefore, we draw $s_{k+1}^{(n)} \sim \rho^{(n)}(s_{k+1})$ and build $\mathbf{s}_{k+1}^{(n)} = [s_{k-L+1}^{(n)}, \ldots, s_{k+1}^{(n)}]^\top$.

3. **Weight update.** Once the new particles have been drawn, the importance weights are updated. Substituting (6), (7) and (5) into (3) yields

$$
\begin{aligned}
w_{k+1}^{(n)} &\propto w_k^{(n)} \frac{p(y_{k+1}|\mathbf{s}_{k+1}^{(n)}, \tau_{k+1}^{(n)}) p(\mathbf{s}_{k+1}^{(n)}, \tau_{k+1}^{(n)}|\mathbf{s}_k^{(n)}, \tau_k^{(n)})}{\rho^{(n)}(s_{k+1}^{(n)}) p(\tau_{k+1}^{(n)}|\tau_k^{(n)})} \\
&= w_k^{(n)} \sum_{S\in\mathcal{S}} \mathcal{N}\left(\mu_{k+1}^{(n)}(S), \sigma_v^2\right) p(\mathbf{s}_{k+1}^{(n)}|\mathbf{s}_k^{(n)}) \\
&\propto w_k^{(n)} \sum_{S\in\mathcal{S}} \mathcal{N}\left(\mu_{k+1}^{(n)}(S), \sigma_v^2\right)
\end{aligned}
$$

where we have used that $\mathbf{s}_{k+1}^{(n)}$ and $\tau_{k+1}^{(n)}$ are independent given $\mathbf{s}_k^{(n)}$ and $\tau_k^{(n)}$. In practice, the weights are computed as

$$
\tilde{w}_{k+1}^{(n)} = w_k^{(n)} \sum_{S\in\mathcal{S}} \mathcal{N}\left(\mu_{k+1}^{(n)}(S), \sigma_v^2\right)
$$

and then normalized, $w_{k+1}^{(n)} = \left(\sum_{i=1}^N \tilde{w}_{k+1}^{(i)}\right)^{-1} \tilde{w}_{k+1}^{(n)}$.

4. **Estimation.** The particle filter can be used to approximate any kind of estimator of the state variables at time $k+1$, or the full state trajectory at time M. Here, we consider the minimum mean square error (MMSE) estimate of the delays,

$$
\hat{\tau}_{0:k+1} = \sum_{n=1}^N \tau_{0:k+1}^{(n)} w_{k+1}^{(n)}
$$

and maximum *a posteriori* (MAP) of the symbol estimates,

$$
\hat{\mathbf{s}}_{0:k+1} = \arg\max_{\mathbf{s}_{0:k+1}} \left\{ \sum_{n=1}^N \delta(\mathbf{s}_{0:k+1}^{(n)} - \mathbf{s}_{0:k+1}) w_{k+1}^{(n)} \right\}.
$$

5. **Resampling.** It can be shown that the variance of the importance weights, $w_{k+1}^{(n)}$, can only increase stochastically over time [7]. This means that, after a few time steps of the standard SIS algorithm as described so far, the majority of the normalized importance weights have negligible values and only a few of the particles in the filter have significant weights, i.e., only a few particles are really *useful*. The usual solution to this problem is to resample the existing particles [7]. Intuitively, resampling consists of discarding those particles with negligible weights and replicating those with higher weights. In multinomial resampling, for instance, N new trajectories are created by sampling the discrete set $\{(\mathbf{s}_{k+1}, \tau_{k+1})^{(n)}\}_{n=1}^N$ with probabilities $w_{k+1}^{(n)}$. The resampled trajectories are all equally weighted (i.e., all importance weights are reset to $1/N$).

The recursive steps of the proposed algorithm are summarized in Table 1.

For $k = 0$ to M *(total number of symbols)*
For $n = 1$ to N *(total number of particles)*
Draw $\tau_k^{(n)} \sim \mathcal{N}(a\tau_{k-1}^{(n)}, \sigma_u)$
Draw $s_k^{(n)} \sim \rho^{(n)}(s_k) \propto \mathcal{N}(\mu_k^{(n)}(s_k), \sigma_y^2)$
Build $\mathbf{s}_k^{(n)} = [s_{k-L}^{(n)}, \ldots, s_k^{(n)}]^T$
Update weights $\tilde{w}_k^{(n)} = w_{k-1}^{(n)} \sum_{S \in \mathcal{S}} \mathcal{N}(\mu_k^{(n)}(S), \sigma_y^2)$
Normalize weights $w_k^{(n)} = \left(\sum_{i=1}^N \tilde{w}_k^{(i)}\right)^{-1} \tilde{w}_k^{(n)}$
Resample if $N_{eff} = \frac{1}{\sum_{n=1}^N \left(w_k^{(n)}\right)^2} < N/2$
Timing recovery and symbol detection
$\hat{\tau}_{0:M-1} = \sum_{n=1}^N \tau_{0:M-1}^{(n)} w_{M-1}^{(n)}$
$\hat{\mathbf{s}}_{0:M-1} = \arg\max_{\mathbf{s}_{0:M-1}} \left\{\sum_{n=1}^N \delta(\mathbf{s}_{0:M-1}^{(n)} - \mathbf{s}_{0:M-1}) w_{M-1}^{(n)}\right\}$

Table 1. SIS with resampling algorithm.

4. COMPUTER SIMULATIONS

We have verified the performance of the proposed algorithm by computer simulations of a system with BPSK modulation, ISI span $L + 1 = 3$ and time-limited causal raised-cosine pulses with a roll-off factor $\alpha = 0.7$. The coefficient of the AR process, a, used to model the dynamics of the symbol timing is 0.999, and the variance of the additive noise u_t is $\sigma_u^2 = 0.0001$.

Figure 1 depicts the Bit Error Rate (BER) attained by the proposed algorithm for different values of the Signal-to-Noise Ratio (SNR) when the number of particles used to obtain the estimates is $N = 50$. It is apparent that the achieved BER with unknown symbol timing is very close to the BER obtained considering the same particle filtering method but with known symbol timing. We have also compared the proposed algorithm with the optimal detector given $y_{0:M}$ and known τ. It is clear that the performance of the proposed method is very close to this lower bound.

Figure 2 shows one realization of the actual variation of the normalized symbol timing error and the corresponding estimates for a 2 dB and 12 dB SNRs. As seen from the figure, the proposed algorithm tracks the variation of the symbol timing quite accurately.

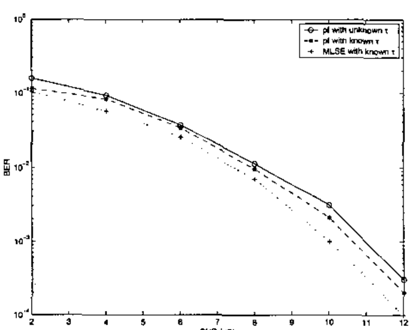

Fig. 1. BER as a function of SNR for known and unknown symbol timing.

5. CONCLUSIONS

A new algorithm for joint symbol detection and timing estimation based on particle filtering is proposed. Our computer simulation experiments show an adequate performance both in terms of BER and tracking of the symbol timing. The method reported in this paper is limited to problems where only the symbol delay is unknown. A logical continuation of this work will include research on symbol detection when additional parameters in synchronization problems are unknown. The proposed algorithm is computationally intensive. However, SMC methods, and specifically SIS, are highly parallelizable and suitable for VLSI implementation.

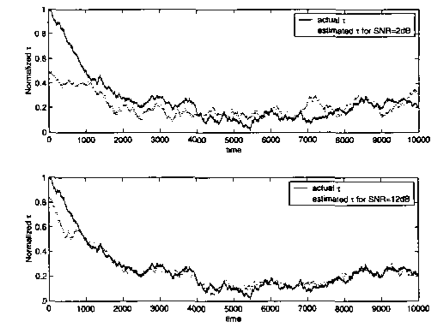

Fig. 2. Actual and estimated τ for two different SNRs.

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