Blind sequential detection for Rayleigh fading channels using hybrid Monte Carlo-recursive identification algorithms

Jayesh H. Kotecha\textsuperscript{a}, Petar M. Djurič\textsuperscript{b,}\textsuperscript{*}

\textsuperscript{a}Department of Electrical and Computer Engineering, University of Wisconsin at Madison, Madison, WI 53706, USA
\textsuperscript{b}Department of Electrical and Computer Engineering, State University of New York at Stony Brook, Stony Brook, NY 11794, USA

Received 15 June 2000; received in revised form 25 June 2002

Abstract

Detection of data transmitted over a Rayleigh fading channel, where the channel is unknown, has been a problem of interest for many researchers. In this paper, we present a new algorithm for joint detection and channel estimation for Rayleigh fading channels. Our algorithm combines Monte Carlo sampling with classical recursive identification methods. The channel is modeled as an autoregressive process, which allows for representation of the communication system by a dynamic state space model. A more accurate modeling of the channel, especially in fast fading along with exploitation of time diversity in the received signal, is also considered. Simulation results illustrating the effectiveness of this algorithm are presented.

© 2004 Elsevier B.V. All rights reserved.

Keywords: Rayleigh fading; Blind detection; Channel estimation; Monte Carlo filtering; Recursive identification; Sequential importance sampling

1. Introduction

Transmission of data over mobile communication channels is severely impaired due to fading. Frequency flat fading induces distortion in the form of random amplitude changes in the transmitted signal. In order to detect transmitted symbols optimally, it is necessary to track the amplitude variations. In the absence of a direct line-of-sight component, a Rayleigh fading assumption is often made, i.e., the received amplitude is a random variable with a Rayleigh density. The rate of variation of the received amplitude or the fading rate is governed by the Doppler bandwidth, which in turn is proportional to the transmission frequency and the mobile speed. This variation should be addressed during design of robust receivers. Consequently, in fast fading channels, the variation of the received amplitude over a symbol period cannot be neglected.

In discretizing the received signal and in presence of slow fading channels, the sampling frequency is the Nyquist frequency of the transmitted signal. This discrete time model cannot be used for fast fading channels because the bandwidth of the faded signal is higher than the bandwidth of the transmitted signal. There, the sampling is at a rate \( n \) times that of the
Nyquist frequency of the transmitted signal, resulting in oversampling that can be exploited to achieve improved performance. This approach has been taken in \cite{9,11,13}, and will be used here.

Design of reliable detectors for the Rayleigh fading channel has become a subject of interest for many researchers recently, where the common assumption is that the second-order statistics of the channel are known. Various approaches have been reported in \cite{5,9,11,12}.

It has been shown in \cite{14} that the time varying fading coefficients can be modeled as an autoregressive (AR) process. This allows for a representation of the communication system by a dynamic state space (DSS) model, where the channel coefficients and the transmitted data are dynamic state (hidden) variables and the received signal represents observations. This is the approach in \cite{13}, where Kalman filtering (KF) is performed to estimate the channel coefficients. A Viterbi scheme is used for maximum likelihood symbol detection along with the KF. In \cite{1,10} a similar approach was proposed, but the discrete time model used is inadequate for fast fading channels, as has been indicated in \cite{5,6}. In all the detectors referred above, the AR coefficients of the fading process were assumed known. Equivalently, as in all of the detectors referred above, the second order statistics of the channel are assumed known. However, with the exception of \cite{13}, none of the detectors allow for the case when such statistics are unknown. In continuation of our previous work, here we allow for uncertainty in the AR coefficients, where the uncertainty is represented in the form of a prior density. This implies almost no knowledge of the channel, hence the scheme presented here is a blind detection algorithm with joint channel estimation. In order to tackle the uncertainty in the channel characteristics, we use a novel scheme called hybrid Monte Carlo-recursive identification filter (MCRIF). The MC sampling methods are combined with recursive identification techniques to obtain a hybrid scheme. Notably, the implicit assumption made in most detectors is that the observation noise is Gaussian. The detector proposed here allows for non-Gaussian noise, without additional complexity.

2. System model

We consider transmission of data \(b_k\) from a given discrete complex set \(L = \{l_1, \ldots, l_m\}\), over a frequency flat Rayleigh fast fading channel. The time-varying complex coefficient of the Rayleigh fading channel, denoted as \(c(t)\), is a complex Gaussian random process, whose amplitude \(|c(t)|\) is Rayleigh distributed. Fig. 1 shows the system block diagram, where \(g(t)\) is the pulse shaping filter, and \(s(t)\) is the input to the channel. The channel consists of the fading process \(c(t)\) with an additive noise \(v(t)\). The channel \(c(t)\) can be modeled as the output of a low-pass filter with cut-off frequency \(f_d\), which is the Doppler frequency. Denote the noise-free faded (received) signal as \(\tilde{s}(t) = s(t)c(t)\) whose Fourier transform is given by \(\tilde{S}(jw) = S(jw) \ast C(jw)\), where ‘*’ denotes convolution. Then the one-sided bandwidth of \(\tilde{s}(t)\) is equal to \(f_d + f_s\), where \(f_s = 1/(2T)\) is the one-sided bandwidth of the pulse shaping filter, and \(T\) is the symbol period. At the receiver, the signal is sampled at sampling frequency \(1/T_s\), where \(T_s = T/M\), and \(M\) is an integer \(\geq 2\) and is called the oversampling factor. To prevent inter-symbol interference (ISI) due to oversampling, \(g(t)\) is chosen to be \(M\) time-displaced raised-cosine pulses with centers at \(mT/M, m = 0, \ldots, M - 1\). If \(d_k\) is an upsampled version of \(b_k, d_k = b_{\lfloor k/M \rfloor}\), where \(\lfloor \cdot \rfloor\) indicates the smallest integer greater than or equal to \((\cdot)\). The fading process is modeled as an AR process of order \(r\), denoted as AR\((r)\), i.e., \(c_k\) is the output of an all pole filter of order \(r\) with white noise input. We can represent the above communication system by the following dynamic state space (DSS) model

\[
\begin{align*}
    y_k &= d_k^T c_k + v_k \quad \text{(observation equation)}, \\
    c_k &= A c_{k-1} + u_k \quad \text{(state equation)}, \\
    A &= \begin{bmatrix}
        a_1 & a_2 & \ldots & a_{r-1} & a_r \\
        1 & 0 & \ldots & 0 & 0 \\
        \vdots & \vdots & \ddots & \vdots & \vdots \\
        0 & 0 & \ldots & 1 & 0
    \end{bmatrix},
\end{align*}
\]

where \(d_k^T = [d_k g_k 0 0 \ldots 0]\), \(c_k^T = [c_k c_{k-1} \ldots c_{k-r+1}]\), and \(g_k\) is the discrete impulse response of the transmitting (pulse shaping) filter. \(v_k\) is complex white
Gaussian noise, whose real and imaginary parts are zero mean, independent and identically distributed with variance $\sigma^2_w/2$. The vector $u_k^T = [u_{1k} \ 0 \ \ldots \ \ 0]$, where $u_{1k}$ is a white Gaussian random variable with variance $\sigma^2_u$, and $a_1, a_2, \ldots, a_r$ are unknown. Note that $c_k, d_k$ and $u_k$ are vectors of length $r$ and $A$ is a matrix of size $(r+1) \times r$. Here the unknown (hidden) variables are $x_k^T = [c_k^T \ d_k \ a_1 \ \ldots \ \ a_r]$. Given the dynamic state space model (1), it is our interest to detect the transmitted data and track the channel variation, i.e., estimate $x_k$, given the observations $y_{1:k} = [y_1 \ \ldots \ \ y_k]$. From a Bayesian perspective, all the knowledge of $x_k$ given the observation $y_{1:k}$ can be summarized by the marginalized posterior probability $p(x_k | y_{1:k})$. Equivalently, we are interested in the joint detection of transmitted data and estimation of channel state information (CSI). Inference for symbol detection can be based on the marginalized density $p(d_k | y_{1:k})$, which can be obtained from the joint density $p(x_k | y_{1:k})$. Since model (1) is highly nonlinear, recursive closed-form solutions of the posterior densities do not exist. We propose a new algorithm which uses the well-known recursive least-squares (RLS) algorithm and particle filters to detect the transmitted data.

### 3. Monte Carlo filtering

In this section, we give a brief description of Monte Carlo sampling filters. For details, however, the reader is directed to [2,4,7]. Consider the following DSS model:

$$x_k = f_k(x_{k-1}, u_k) \quad \text{(state equation)},$$

$$y_k = h_k(x_k, v_k) \quad \text{(observation equation)},$$

where $x_k$, $y_k$, $u_k$ and $v_k$ are the hidden state, observation, state noise and observation noise respectively, of given dimensionalities. We would like to estimate $p(x_{1:k} | y_{1:k})$, where $y_{1:k} = (y_1, y_2, \ldots, y_k)$. Often of interest is also the expectation $E_p(q(x_k) | y_{1:k})$. For a linear model with Gaussian noise, $p(x_{1:k} | y_{1:k})$ is Gaussian and the celebrated Kalman filter can be used to obtain a closed-form solution. However, with non-linearity (as in our problem) and non-Gaussianity in the model, there generally exist no such closed-form solutions and analytical computation is infeasible practically. Monte Carlo based filters provide a practical methodology for estimation in such problems.

The basic idea is to represent the distribution as a collection of samples (particles) and weights associated with the particles. $N$ particles, $x_{1:k} = \{x_{1:k}^{(1)}, \ldots, x_{1:k}^{(N)}\}$, from the so-called importance sampling distribution $\pi(x_{1:k} | y_{1:k})$ are generated. Subsequently, the particles are weighted as $w_k^{(n)} = p(y_k | x_{1:k}^{(n)})/\pi(x_{1:k}^{(n)} | y_{1:k})$. A Monte Carlo estimate of $E_p(q(x_k))$ can be written as

$$\hat{E}_p(q(x_k)) = \sum_{n=1}^{N} w_k^{(n)} q(x_k^{(n)}). \quad (3)$$

To minimize the variance of $\hat{E}_p(q(x_k))$, the dispersion of the weights $w_k$ should be minimized, which implies that $\pi(x_{1:k} | y_{1:k})$ must be “similar” to $p(x_{1:k} | y_{1:k})$ [3]. The support of $\pi(x_{1:k} | y_{1:k})$ must include that of $p(x_{1:k} | y_{1:k})$, and $\pi(x_{1:k} | y_{1:k})$ should be easy for sampling. Due to the Markovian nature of the state equation, we can obtain a sequential procedure called SIS, which sequentially yields the weights of the samples that approximate $p(x_{1:k} | y_{1:k})$. More details on the procedure can be found in [2] or [7]. At each time instant, samples $x_k^{(n)}$ are obtained from the importance sampling distribution $\pi(x_k | x_{1:k-1}, y_{1:k})$, and the weight update is done by

$$w_k^{(n)} = \frac{p(y_k | x_k^{(n)}, y_{1:k-1}) p(x_k^{(n)} | x_{1:k-1}, y_{1:k-1})}{\pi(x_k^{(n)} | x_{1:k-1}, y_{1:k-1})}. \quad (4)$$

In SIS, degeneration of particles occurs with $k$. Effectively, the weights of only a few particles remain significant. This results in a poor approximation of the expectation in (3). A procedure called resampling can be used to reduce this degeneration. The basic idea is to duplicate the particles which have significant weights, in proportion to the weights of the particles. For details, see [2,7].
4. Hybrid Monte Carlo-recursive identification filtering (hybrid MC-RIF) algorithms

In [6], we considered the problem of detection over a Rayleigh fast fading channel where the coefficients of the AR process are assumed unknown. The SIS algorithm was applied for channel tracking and data detection. Possible channel fading coefficients are sampled from the so-called importance sampling (IS) density and the particles are then weighted according to the updated posterior distribution. However, when the AR coefficients are assumed unknown, the problem becomes one of blind detection. It is possible to treat the unknown AR parameters similar to the fading coefficients and sample them from an importance sampling density. This leads to an increase in the dimensionality of the hidden variables. Due to the nonlinearity of the problem and the increase in the dimensionality, the Monte Carlo variation increases and a large number of particles are required to obtain satisfactory estimates in Eq. (3).

In order to overcome this difficulty, we propose a hybrid MC-RIF algorithm. We combine the SIS and recursive identification [8] filters to reduce the Monte Carlo variation of the SIS filter and obtain an effective methodology which is applicable in various scenarios. The basic idea is to treat the sets of unknown variables differently. Specifically, in the blind detection problem, the unknown channel coefficients and transmitted data are treated with an SIS strategy, while the AR parameters are estimated using a recursive least-squares method. The channel fading coefficients $c_k$ are sampled recursively from an importance sampling distribution. The obtained coefficient trajectories are used to estimate the AR parameters $\mathbf{a}^T = (a_1, \ldots, a_r)$ recursively using a recursive least squares (RLS) algorithm. The estimated trajectories of $c_k$ are noisy estimates of the AR process, which are then used to estimate $\mathbf{a}$. Since the trajectories are weighted, each of the RLS estimates is also weighted to obtain the final inference. In essence, the distribution of the AR parameters is represented as a mixture model of weighted Gaussian distributions. The mean and variance of each of the individual Gaussian is estimated by the RLS algorithm and the weights of the mixture model are simply the weights of the trajectories obtained in the SIS update procedure. Thus, the SIS methodology is used in a novel manner to update a given mixture distribution.

4.1. Choice of importance sampling density

Let $\mathbf{b}_k = (b_1, \ldots, b_k)$, and assume that the data transmitted from the source are independent, i.e., $P(b_k = l_i | \mathbf{b}_{k-1}) = P(b_k = l_i) = q_i$. In order to simplify the implementation of the SIS filter, we make an assumption that the $d_j$ are independent with $k$. The time diversity in $d_k$, which results due to the upsampling of $b_k$ will be exploited as shown in the next section. The importance sampling density is chosen as

$$
\pi(\mathbf{c}_k | \mathbf{c}_{1:k-1}, y_{1:k-1}) = p(\mathbf{c}_k | \mathbf{c}_{1:k-1}, y_{1:k-1}) \pi(d_k) = p(d_k) \int p(\mathbf{c}_k | \mathbf{c}_{1:k-1}, \mathbf{a}) p(\mathbf{a} | \mathbf{c}_{1:k-1}) d\mathbf{a}. \quad (5)
$$

The second term in the above equation can be written as

$$
p(\mathbf{a} | \mathbf{c}_{1:k-1}) = \prod_{j=2}^{k-2} p(c_j | c_{j-1}, \mathbf{a}) p(\mathbf{a}). \quad (6)
$$

If the prior $p(\mathbf{a})$ is chosen as a Gaussian, then $p(\mathbf{a} | \mathbf{c}_{1:k-1})$ is a Gaussian, whose mean and variance denoted as $\mu_{k-1}$ and $\Sigma_{k-1}$, respectively, can be updated using the RLS algorithm. From (5) and (6), it can be shown that $p(c_k | c_{1:k-1}, y_{1:k-1})$ is a Gaussian with mean $\mu_{k-1} = c_{k-1} + \Sigma_u$, where $\Sigma_u = E(\mathbf{u}\mathbf{u}^T)$. Thus, the resulting integration in (5) yields a Gaussian which is easy for sampling. Effectively, the importance function is chosen by integrating out $\mathbf{a}$, which is a procedure known as Rao-Blackwellization [2,7]. With the above importance sampling density, the weight update is given by

$$
w_k^{(n)} = w_{k-1}^{(n)} p(y_k | c_k^{(n)}, d_k^{(n)}). \quad (7)
$$

Note that $p(y_k | c_k^{(n)}, d_k^{(n)})$ is the density of the channel noise, which need not be only Gaussian, but it should be a density which can be evaluated. Hence, the detection algorithm can accommodate the case of non-Gaussian noise too.

4.1.1. Hybrid filter

The hybrid MC-RLS filter used for blind detection is given below, where the unknown variables are $x_k = (\theta_k, \mathbf{a}_k)$ and $\theta_k^T = (c_k^T, d_k^T)$. The particles of $x_k$ are obtained in two steps. An MC filter is run for $\theta_k$, while
the RLS is used to update the estimate of \( \mathbf{a} \). First, the SIS procedure is used to sample \( \mathbf{\theta}_k \) and subsequently, based on the sampled values \( \mathbf{\theta}_k^{(n)} \), the updating of \( \mathbf{a} \) for each trajectory is carried out using the RLS algorithm. The Hybrid MC-RLS algorithm can be written as follows:

(1) At time \( k=0 \), we start with \( N \) samples from the IS density \( \pi(\mathbf{\theta}_0) \) and denote them \( \mathbf{\theta}_0^{(n)}; n=1,\ldots,N \), with weights

\[
\mathbf{w}_0^{(n)} = p(\mathbf{\theta}_0^{(n)})/\pi(\mathbf{\theta}_0^{(n)}).
\]

Initialize, \( \mathbf{\mu}_0^{(i)} \) and \( \mathbf{\Sigma}_0^{(a)} \) for \( n=1,\ldots,N \).

(2) At times \( k=1,\ldots,K \), let \( \mathbf{\Theta}_k = \{ \mathbf{\theta}_k^{(n)}; n=1,\ldots,N \} \) be particles of \( \mathbf{\theta}_k \) with weights \( \mathbf{W}_k = \{ \mathbf{w}_k^{(n)}; n=1,\ldots,N \} \), and \( \mathbf{A}_k = \{ \mathbf{\mu}_k^{(a)}; n=1,\ldots,N \} \) and \( \mathbf{P}_k = \{ \mathbf{\Sigma}_k^{(a)}; n=1,\ldots,N \} \) be the corresponding mean vectors and covariance matrices. At time \( k-1 \), denote the particle set \( \mathbf{\Phi}_{k-1} = \{ \mathbf{\Theta}_{k-1}, \mathbf{W}_{k-1}, \mathbf{A}_{k-1}, \mathbf{P}_{k-1} \} \). We obtain \( \mathbf{\Phi}_k \) from steps 3, 4, 5 and 6.

(3) For \( n=1,\ldots,N \), sample \( \mathbf{\theta}_k^{(n)} \sim \pi(\mathbf{\theta}_k^{(n)}; y_{1:k-1}, y_{1:k}) \).

(4) For \( n=1,\ldots,N \), update the weights using

\[
\mathbf{w}_k^{(n)} = \tilde{\mathbf{w}}_k^{(n)} = \frac{p(y_k|\mathbf{\theta}_k^{(n)}, d_k^{(n)})}{\sum_{j=1}^{N} \tilde{w}_k^{(j)}}.
\]

(5) Normalize according to

\[
\tilde{\mathbf{w}}_k = \frac{\mathbf{w}_k}{\sum_{j=1}^{N} \mathbf{w}_k^{(j)}}.
\]

(6) For \( n=1,\ldots,N \), calculate \( \mathbf{A}_k \) and \( \mathbf{P}_k \) using the RLS algorithm. The observations used in the RLS update are \( y_k \) and \( \mathbf{\Theta}_k \).

(7) Resample periodically to prevent sample degeneration.

Thus, we see that the SIS filter is applied as usual for \( \mathbf{\theta}_k \), using estimates of \( \mathbf{a} \) obtained by the RLS algorithm.

5. Detection and channel estimation

The symbol \( d_k \) represents an upsampled version of the transmitted data \( b_k \). Therefore, all \( d_k \) obtained from oversampling, for the same symbol period \( T \) are identical. This time diversity is now exploited in the decision process. The posterior probability of \( d_k \) can be written as

\[
P(d_k = l_i|y_{1:k}) = E(I(d_k = l_i)|y_{1:k})
\]

\[
\approx \frac{\sum_{n=1}^{N} \mathbf{w}_k^{(n)} I(d_k^{(n)} = l_i)}{\sum_{n=1}^{N} \mathbf{w}_k^{(n)}} \quad i = 1,\ldots,|\mathbb{I}|,
\]

where \( I(d_k = l_i) = 1 \) if \( d_k = l_i \) and 0 otherwise. Then for \( i = 1,\ldots,|\mathbb{I}| \),

\[
P(b_k = l_i|y_{1:k}) = \frac{1}{M} \sum_{m=0}^{M-1} P(d_{Mk-m} = l_i|y_{1:k})
\]

and choose \( b_k = l_i \) so that

\[
\hat{b}_k = \max_{l_i \in \mathbb{I}} P(b_k = l_i|y_{1:k}).
\]

Inference about the channel fading coefficients and parameters can be obtained in a similar manner. The channel and parameter estimates at time \( k \) are given by

\[
\hat{\mathbf{\phi}}_k = \frac{\sum_{n=1}^{N} \mathbf{w}_k^{(n)} \hat{\mathbf{\mu}}_n^{(a)}}{\sum_{n=1}^{N} \mathbf{w}_k^{(n)}},
\]

\[
\hat{\mathbf{\mu}}_k = \frac{\sum_{n=1}^{N} \mathbf{w}_k^{(n)} \mathbf{\mu}_n^{(a)}}{\sum_{n=1}^{N} \mathbf{w}_k^{(n)}},
\]

respectively, where \( \mathbf{\mu}_n^{(a)} \) is obtained by the RLS algorithm.

The resulting algorithm is computationally very intensive but highly parallelizable, giving an advantage for hardware implementation using VLSI technology. Direct comparison of the computational load of the algorithm with other methods when they are run on sequential machines does not make much sense because in real applications the proposed algorithm should be implemented on a specially designed hardware. The algorithm has four important computations: (1) generation of new particles, (2) updating of the weights, (3) computation of the mean vectors and covariance matrices of the AR parameters, and (4) resampling. For all the particles, the computations can be implemented in parallel, which implies that in terms of speed, the...
6. Simulation results

Many computer simulations were generated and here we show the results for fading rates \( f_d T = 0.01 \) and \( f_d T = 0.001 \) as examples. The method was implemented on a system where the modulation scheme used was differentially encoded BPSK. The sampling period was \( T_s = T/2 \) or \( M = 2 \). The transmitted data, \( b_k \in \{-1, 1\} \) were equally likely. The channel was modeled as an AR(3) process driven by complex white Gaussian noise, with AR coefficients given by \( a_{\text{true}} = (2.9145, -2.8344, 0.9197) \) for \( f_d T = 0.01 \) and \( a_{\text{true}} = (2.9916, -2.9833, 0.9917) \) for \( f_d T = 0.001 \). The number of trajectories used was \( N = 1000 \). In the simulations, we assumed \( e_0 \) as known and the prior for \( a \) as Gaussian with mean \( a_{\text{true}} \) and covariance \( 3I_3 \), where \( I_3 \) is a \( 3 \times 3 \) identity matrix. Figs. 2 and 3 show the bit error rate (BER) as a function of the signal-to-noise ratio (SNR) (in dB) for \( f_d T = 0.001 \) and \( f_d T = 0.01 \), respectively. The SNR is calculated as \( 10 \log E(|c_k|^2)/\sigma_e^2 \). The performance of the SIS detector, with unknown channel coefficients and unknown AR parameters is compared with the clairvoyant matched filter (which knows the channel coefficients). The performance of the clairvoyant matched filter represents a lower bound and is a benchmark for ideal conditions. In general, it was observed that the performance increases relatively as the fading rate increases, since at lower fading rates the channel stays in deep fades for longer durations of time causing the low SNR conditions to adversely affect the performance of the receiver. As shown in Fig. 2, at SNR of 5 dB the algorithm was unable to track the channel and diverged. However, as the SNR increased, the tracking performance improved.

7. Conclusion

A novel method was proposed for channel estimation and detection of data transmitted over a Rayleigh
fading channel. The presented methodology can be extended to many other channels with different characteristics. A simple extension of the above algorithm can be made for Rician fading channels. Another significant extension is that of tracking of channels with time varying characteristics. A change in the characteristics, implies a change in the AR coefficients of the channel model. Then the same algorithm is applicable, without any significant change. Additionally, for channels with non-Gaussian additive noise, the same algorithm can be used as long as the noise probability density can be evaluated.

References


