Erratum to "A New Class of Particle Filters for Random Dynamic Systems with Unknown Statistics"

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We have found an error in the proof of Lemma 1 presented in our paper "A New Class of Particle Filters for Random Dynamic Systems with Unknown Statistics" (EURASIP Journal on Applied Signal Processing, 2004). In the sequel, we provide a restatement of the lemma and a corrected (and simpler) proof. We emphasize that the original result in the said paper still holds true. The only difference with the new statement is the relaxation of condition (3), which becomes less restrictive.

Lemma 1 in [1] should be as follows.

Lemma 1. *Let $\{\mathbf{x}_t^{(i)}\}_{i=1}^M$ be a set of particles drawn at time t using the propagation pdf $p_t^{M'}(\mathbf{x})$, let $\mathbf{y}_{1:t}$ be a fixed bounded sequence of observations, let $\Delta\mathcal{C}(\mathbf{x} \mid \mathbf{y}_t) \geq 0$ be a continuous cost function, bounded in $S\{\mathbf{x}_t^{\mathrm{opt}}, \varepsilon\}$, with a minimum at $\mathbf{x} = \mathbf{x}_t^{\mathrm{opt}}$, and let $\mu_t : A \subseteq \{\mathbf{x}_t^{(i)}\}_{i=1}^M \to [0, \infty)$ be a set function defined as*

$$\mu_t\left(A \subseteq \{\mathbf{x}_t^{(i)}\}_{i=1}^M\right) = \sum_{\mathbf{x} \in A} \mu(\Delta\mathcal{C}(\mathbf{x} \mid \mathbf{y}_t)). \quad (1)$$

If the following three conditions are met:

(1) *Any ball with center at $\mathbf{x}_t^{\mathrm{opt}}$ has a nonzero probability under the propagation density, that is,*

$$\int_{S\{\mathbf{x}_t^{\mathrm{opt}}, \varepsilon\}} p_t^{M'}(\mathbf{x})d\mathbf{x} = \gamma > 0, \quad \forall \varepsilon > 0, \quad (2)$$

(2) *the supremum of the function $\mu(\Delta\mathcal{C}(\cdot \mid \cdot))$ for points outside $S(\mathbf{x}_t^{\mathrm{opt}}, \varepsilon)$ is a finite constant, that is,*

$$\mathbf{S}_{\mathrm{out}} = \sup_{\mathbf{x}_t \in \mathbb{R}^{L_x} \setminus S(\mathbf{x}_t^{\mathrm{opt}}, \varepsilon)} \{\mu(\Delta\mathcal{C}(\mathbf{x}_t \mid \mathbf{y}_t))\} < \infty, \quad (3)$$

(3) *the expected value of $1/\mu_t(\{\mathbf{x}_t^{(i)}\}_{i=1}^M)$ satisfies*

$$\lim_{M \to \infty} E\left(\frac{1}{\mu_t(\{\mathbf{x}_t^{(i)}\}_{i=1}^M)/M}\right) = 0, \quad (4)$$

then

$$\lim_{M \to \infty} \Pr\left[1 - \frac{\mu_t(S^M(\mathbf{x}_t^{\mathrm{opt}}, \varepsilon))}{\mu_t(\{\mathbf{x}_t^{(i)}\}_{i=1}^M)} \geq \delta\right] = 0, \quad \forall \delta > 0, \quad (5)$$

where $\Pr[\cdot]$ denotes probability, that is,

$$\lim_{M \to \infty} \frac{\mu_t(S^M(\mathbf{x}_t^{\mathrm{opt}}, \varepsilon))}{\mu_t(\{\mathbf{x}_t^{(i)}\}_{i=1}^M)} = 1 \quad (i.p.), \quad (6)$$

where i.p. stands for "in probability."

Proof. The proof is based on Markov inequality. We write

$$\lim_{M \to \infty} \Pr\left[1 - \frac{\mu_t(S^M(\mathbf{x}_t^{\mathrm{opt}}, \varepsilon))}{\mu_t(\{\mathbf{x}_t^{(i)}\}_{i=1}^M)} \geq \delta\right]$$

$$= \lim_{M \to \infty} \Pr\left[\frac{\mu_t(\{\mathbf{x}_t^{(i)}\}_{i=1}^M) - \mu_t(S^M(\mathbf{x}_t^{\mathrm{opt}}, \varepsilon))}{\mu_t(\{\mathbf{x}_t^{(i)}\}_{i=1}^M)} \geq \delta\right] \quad (7)$$

$$= \lim_{M \to \infty} \Pr\left[\frac{\mu_t(\{\mathbf{x}_t^{(i)}\}_{i=1}^M \setminus S^M(\mathbf{x}_t^{\mathrm{opt}}, \varepsilon))}{\mu_t(\{\mathbf{x}_t^{(i)}\}_{i=1}^M)} \geq \delta\right].$$

Using the second condition, we infer that

$$\lim_{M \to \infty} \Pr\left[\frac{\mu_t(\{\mathbf{x}_t^{(i)}\}_{i=1}^M \setminus S^M(\mathbf{x}_t^{\mathrm{opt}}, \varepsilon))}{\mu_t(\{\mathbf{x}_t^{(i)}\}_{i=1}^M)} \geq \delta\right]$$

$$\leq \lim_{M \to \infty} \Pr\left[\frac{M S_{\mathrm{out}}}{\mu_t(\{\mathbf{x}_t^{(i)}\}_{i=1}^M)} \geq \delta\right].$$

$$(8)$$

Finally, we apply Markov inequality to the last expression on the right and obtain

$$
\lim_{M \to \infty} \Pr \left[\frac{\mu_t \left(\{ \mathbf{x}_t^{(i)} \}_{i=1}^{M} \setminus S^M(\mathbf{x}_t^{\mathrm{opt}}, \varepsilon) \right)}{\mu_t \left(\{ \mathbf{x}_t^{(i)} \}_{i=1}^{M} \right)} \geq \delta \right]
$$
$$
\leq \frac{S_{\mathrm{out}}}{\delta} \lim_{M \to \infty} \left[E \left(\frac{1}{\mu_t \left(\{ \mathbf{x}_t^{(i)} \}_{i=1}^{M} \right)/M} \right) \right]. \tag{9}
$$

Clearly, if

$$
\lim_{M \to \infty} \left[E \left(\frac{1}{\mu_t \left(\{ \mathbf{x}_t^{(i)} \}_{i=1}^{M} \right)/M} \right) \right] = 0, \tag{10}
$$

we can claim that

$$
\lim_{M \to \infty} \frac{\mu_t \left(S^M(\mathbf{x}_t^{\mathrm{opt}}, \varepsilon) \right)}{\mu_t \left(\{ \mathbf{x}_t^{(i)} \}_{i=1}^{M} \right)} = 1 \quad (\mathrm{i.p.}). \tag{11}
$$

□

REFERENCES

[1] J. Míguez, M. F. Bugallo, and P. M. Djurić, "A new class of particle filters for random dynamic systems with unknown statistics," *EURASIP Journal on Applied Signal Processing*, vol. 2004, no. 15, pp. 2278–2294, 2004.

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