Instantaneous Phase Tracking in Power Networks by Demodulation

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***Abstract*—In the paper we present results of our study of a technique for instantaneous monitoring of phase angles during transients in power networks. The technique is based on demodulation and subsequent filtering of the measured signal. It allows very accurate tracking of transient phase angles (with estimation errors within 1 degree). We specifically examine the effects of the low-pass filter on the estimation error. Various FIR and IIR filters were tested on simulated scenarios of transient conditions in power networks. The simplicity of the method and the shown remarkable performance make it an excellent candidate for real-time implementation.**

I. Introduction

The power system operating at constant frequency is completely characterized by its state vector (a set of positive sequence voltage phasors at all buses for a balanced system). The utilities maintain the operating frequency almost constant, by adjusting the generation to match the load fluctuations. The minor load fluctuations which escape the automatic generation control dynamics cause a frequency deviation which is normally very small (of the order of mHz). The accurate measurement of state vectors in steady state requires that voltage magnitudes be measured with precision better than 1 percent, which is relatively easy, and that the phase angles be measured with accuracy better than 0.1 degree and precisely time-tagged for coordination purposes, when they are received at a central location. Until recently, the system-wide synchronization of measurements required for direct measurement of phase angles, expected to be below 1 microsecond to accommodate the above requirements, has been a major obstacle to providing a fast state acquisition of the power system. That problem has been solved with the advent of Global Positioning Satellite system (GPS) [16], which is capable of providing the time information with up to 100 nanosecond accuracy to commercial users. The price of real-time phasor measurement systems capable of substituting for today's slow and somewhat inaccurate SCADA systems is still high, but several prototypes are undergoing field tests in various utilities. Inasmuch as the advances in hardware technology and decline of prices of GPS and phasor measurement hardware are making it more attractive for implementation, there is a function which only they could accomplish in tomorrow's modern control center. It is a real-time tracking of the system state for monitoring and enhancement of transient stability, which requires that the measurement information be transferred to the central location in the time frame of a few cycles. Some strategies presently used to fight the transient oscillations include dynamic braking and generator excitation control. These strategies rely upon local information about transient swings, without knowledge about dynamics of the overall power system. They could be greatly enhanced if the systemwide transient state information were available in real time, and a coordinated strategy worked out for the control purposes. This is the ideal application for a phasor measurement system, and the requirements for such measurements are: the phase angle information is needed with accuracy of 1 degree (the transient is typically characterized by the maximum rates of change of phase angles of the order of 1 rad/s in the form of damped oscillations with periods of 0.5 to 2 s). Although a number of algorithms for measurement of instantaneous frequency and phase angle have been proposed, [3]–[6], [8], [9], [12], [14], [16], dynamic accuracy remains a difficult objective to accomplish. The authors are presenting herewith a study of an approach to the measurement of the transient phase angle that is an excellent candidate for monitoring of transient stability in power systems.

In the paper it is assumed that two discrete signals are measured at two different sites. The phases of these signals are unknown, and the primary interest is to estimate accurately the instantaneous difference between them. The signals are sinusoids in noise whose frequencies are identical and equal to $f_0 = 60$ Hz. It should be noted that in real systems the problem is more complex since the number of measured signals is much greater than two, and the task is to provide estimates of the phase differences among all these signals. We believe, however, that once the procedure for accurate phase estimation for the simplified problem as defined below is soundly developed, its extension for use in such large systems will not present a significant difficulty. The method that we investigate exploits the principle of demodulation and is well known in

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the field of communications [1]. The measured signals are multiplied by sine and cosine functions whose frequencies are 60 Hz. The signals so obtained (known as quadrature signals) are filtered by low-pass filters, and complex signals are formed whose instantaneous phases are almost identical to the instantaneous phases of the original signals. The phases of the signals are then estimated by taking the inverse tangent of the ratio of the quadrature components. To achieve high accuracy a very important issue is the filtering of the signals. In the paper we emphasize this, and we provide guidance on how to carry it out. The filtering should minimally affect the phases of the signals. Filters with finite (FIR) and infinite impulse responses (IIR) are examined. The former have the nice property that they can easily be designed to have linear phase responses and therefore constant group delays. The latter may achieve the same attenuations as FIR filters with their orders being much lower than the orders of the FIR filters. This implies less arithmetic involved in estimating the phase functions. The procedure has been tested on simulations of realistic cases of phase variations during transient oscillations. The results show that the method, when properly applied, yields excellent results.

II. Problem Formulation

We start with the assumption that there are two discrete signals measured at different sites whose forms are given by

$$x_1[n] = A_1 \cos(\omega_0 n + \phi_1[n]) + \epsilon_1[n] \quad (1)$$

and

$$x_2[n] = A_2 \cos(\omega_0 n + \phi_2[n]) + \epsilon_2[n] \quad (2)$$

A_1 and A_2 are the amplitudes of the signals, ω_0 is the known fundamental frequency normalized by the sampling frequency, and $\phi_1[n]$ and $\phi_2[n]$ are the instantaneous phases of the signals whose values are unknown. $\epsilon_1[n]$ and $\epsilon_2[n]$ represent noise and/or measurement errors whose probabilistic distributions are unknown. The only assumption regarding the noise is that the signal-to-noise ratios (SNR's) defined by

$$\rho_i = 10 \log \frac{A_i^2}{2\sigma_i^2} \qquad i = 1, 2$$

where σ_i^2, $i = 1, 2$, is the variance of $\epsilon_i[n]$, are high (above 10 dB). This assumption is valid because in practice these SNRs are much higher. Since the information about any changes in the system is encrypted in $\phi_1[n]$ and $\phi_2[n]$, the ultimate objective is to estimate

$$\Delta\phi[n] = \phi_2[n] - \phi_1[n]. \quad (3)$$

The estimator must be accurate, simple, and applicable for on-line implementation. The delay in getting the estimates with the required accuracy should be shorter than a predefined time interval (usually about 50 ms).

III. Instantaneous Phase Tracking

We exploit the principle of demodulation. Consider the signal

$$x[n] = A \cos(\omega_0 n + \phi[n]) + \epsilon[n].$$

Two new signals are formed according to

$$y_1[n] = x[n] \cos(\omega_0 n),$$

and

$$y_2[n] = -x[n] \sin(\omega_0 n).$$

$y_1[n]$ and $y_2[n]$ become

$$y_1[n] = \frac{A}{2}(\cos(\phi[n]) + \cos(2\omega_0 n + \phi[n])) + \epsilon[n] \cos(\omega_0[n]) \quad (4)$$

and

$$y_2[n] = \frac{A}{2}(\sin(\phi[n]) - \sin(2\omega_0 n + \phi[n])) - \epsilon[n] \sin(\omega_0[n]). \quad (5)$$

In $y_1[n]$ and $y_2[n]$ we have two signal components that carry the information of $\phi[n]$, one around DC, and the remaining around 120 Hz. To facilitate the estimation of $\phi[n]$ this redundancy is removed by low-pass filtering $y_1[n]$ and $y_2[n]$. If the cutoff frequency of the filter is high enough, the low-frequency component will pass through the filter undistorted, and the one around 120 Hz will be strongly attenuated. At the output of the filter we will have

$$\tilde{y}_1[n] = \frac{A}{2}\cos(\phi[n]) + \tilde{\epsilon}_1[n] \quad (6)$$

and

$$\tilde{y}_2[n] = \frac{A}{2}\sin(\phi[n]) + \tilde{\epsilon}_2[n] \quad (7)$$

where $\tilde{\epsilon}_1[n]$ and $\tilde{\epsilon}_2[n]$ represent respectively the sum of the filtered noise sequences from (4) and (5) and the attenuated signals at 120 Hz.

Now, using the assumption of high SNR, it can be shown that the phase estimate can be obtained from (6) and (7) according to

$$\tilde{\phi}[n] = \arctan \frac{\tilde{y}_2[n]}{\tilde{y}_1[n]}.$$

For a related derivation see [15]. Moreover, it is found that when the noise is Gaussian and the phase is constant, the estimator derived along these lines has variance identical to the Cramer–Rao bound.

If we want to estimate the instantaneous frequency, an additional step should be added. For example, if the instantaneous frequency is defined via the phase difference $\phi[n] - \phi[n-1]$, a new signal, say $v[n]$, may be formed according to [7]:

$$v[n] = (\tilde{y}_1[n] + j\tilde{y}_2[n])(\tilde{y}_1[n-1] - j\tilde{y}_2[n-1])$$

and the inverse tangent function applied to estimate the instantaneous frequency directly. If the phase function is modeled as a polynomial of higher degree, then one may apply the algorithms from [2].

IV. Discussion

An important role in the procedure is played by the low-pass filters. We want to extract the low-pass signals in (4) and (5), and at the same time attenuate the component at 120 Hz as much as possible. The extraction of the useful signal has to be done in a way that its phase remains undistorted.

Attractive candidates for this job are the filters with finite impulse response. They are easily designed to have linear phase responses. When filters have linear phase responses they have constant group delays. This is quite an important feature since signals processed with filters that have constant group delays will not need additional processing. Recall that our primary interest is to be able to estimate phases of signals during transients. When they occur, the instantaneous frequencies of the sinusoids vary which entails that the signals that are filtered also have variable frequencies. If the filters used do not have constant group delays, the variable delays for the various frequencies will introduce systematic errors in the estimated phases. To remove these errors we need to correct the estimated phases. This can be achieved by finding the instantaneous frequencies of the signal and accordingly add or subtract the phase that corresponds to the difference in the group delay at that particular frequency. This certainly complicates the algorithm. As mentioned above, this is not needed when we use FIR filters. However, to achieve appropriate attenuation at 120 Hz, we may need a filter of relatively high order. But the higher the order, the more computations to get the filtered signal, and the longer the delay in finding the estimate.

Alternatives to the FIR filters are the filters with infinite impulse responses. They can achieve the same attenuations as the FIR filters, and have the advantage in requiring fewer arithmetic operations per filtered sample. This is due to their low order. For example, attenuations of the signal component at 120 Hz of 80 dB may be achieved by elliptic filters of the second order, and that requires only four multiplications, four additions and/or four subtractions per sample. However, the phase responses of these filters are not linear, and that creates the problems that were discussed above. Promising candidates for the low-pass filtering job could be Bessel filters. They are characterized by having maximally flat group delays at very low frequencies [13], which is precisely the range where we need constant group delays.

The choice of the cutoff frequency may also be a crucial issue for the performance of the method. If the phase variations are larger, then a higher cutoff frequency will be required. If it is not high enough, whenever there are quick phase changes, the procedure will produce errors. The filter simply cuts off useful components of $y_1[n]$ and $y_2[n]$ in (4) and (5). If a higher cutoff frequency of the filter is allowed, this will not occur. However, larger noise samples will be output through the filter which will introduce larger errors in the estimates. So, it is quite important to keep the cutoff frequency at the lowest possible value. If the noise in the measurements is insignificant, then the value of the cutoff frequency is not that important.

V. Simulation Results

The procedure was tested using the following models of $\phi_1[n]$ and $\phi_2[n]$ in (1) and (2):

$$\phi_1[n] = \Phi_1 e^{-n/\tau_1} \cos(\omega_1 n + \psi_1) \tag{8}$$

and

$$\phi_2[n] = \Phi_2 e^{-n/\tau_2} \cos(\omega_2 n + \psi_2). \tag{9}$$

(8) and (9) are crude approximations of a phase transient in a power system. The noise sequences $\epsilon_1[n]$ and $\epsilon_2[n]$ were independent and Gaussian. (It should be noted that the procedure does not exploit the probabilistic structure of the noise.) The sampling frequency throughout the simulations was 720 Hz. (The procedure does not require any special value of the sampling frequency. The only restriction is that it must be greater than 240 Hz.) The SNR was 30 dB, which is considered to be a low SNR. The rest of the parameters in (4), (5), (8), and (9) were $A_1 = 1$, $A_2 = 1$, $\Phi_1 = 30°$, $\Phi_2 = 0°$, $\tau_1 = 1$ s, $\omega_1 = 2\pi/720$ rad/s, and $\psi_1 = 0°$. Fig. 1 shows the function $\Delta\phi[n] = \phi_2[n] - \phi_1[n]$.

We performed a set of experiments to test various FIR and IIR filters. In every experiment five different realizations, each 2000 samples long, were processed, and the phase estimation errors were depicted on the same plot. The same five noise sequences were used throughout each experiment. The effects of three filter characteristics on the phase estimation were thoroughly examined. These characteristics are the filter cutoff frequency, its transition bandwidth, and the attenuation of the filter at 120 Hz.

The FIR filters were designed by the method of Parks–McClellan which uses the Remez exchange algorithm [11], [10]. These filters have equiripple response and are optimal in the sense that they minimize the maximum error between the desired and the actual frequency responses. Since the maximum allowed delay was set to be 50 ms, the maximum order of the filter was 72.

In Fig. 2 we show the results obtained when the filter's cutoff frequency was 51 Hz. The attenuation at 120 Hz was 187 dB. The transition bandwidth was 49 Hz. The estimation error was most of the time within the $\pm 1°$ limits. Improved results were obtained when the cutoff frequency was decreased. In Fig. 3 we see the results obtained when the applied filter had a cutoff frequency of 26 Hz. To achieve lower cutoff frequency, we had to give up attenuation in the stopband. At 120 Hz it was 101 dB. The transition bandwidth was 24 Hz. The improvement is due to the fact that more noise had been cut off by the filter, while the attenuation in the stopband was still kept high.

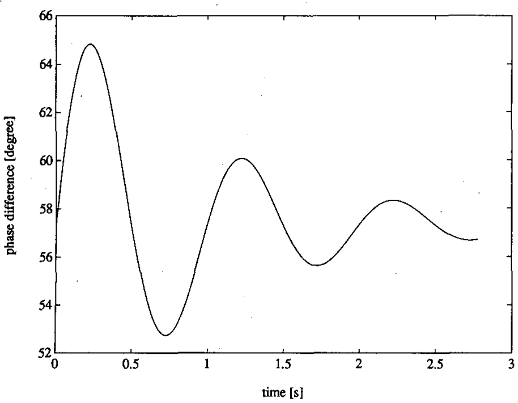

Fig. 1. Instantaneous phase difference as a function of time.

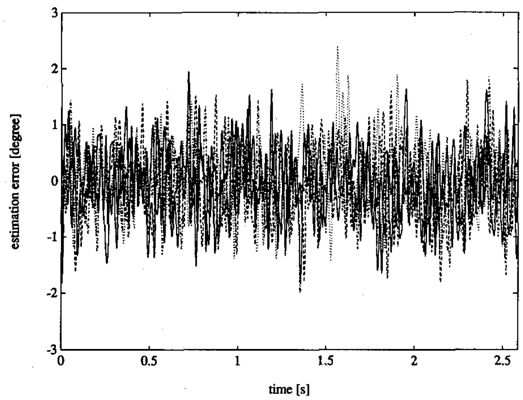

Fig. 2. Estimation error of $\Delta\phi[n]$. The filter used is FIR of order 72. Its cutoff frequency is 51 Hz, and the attenuation is 187 dB.

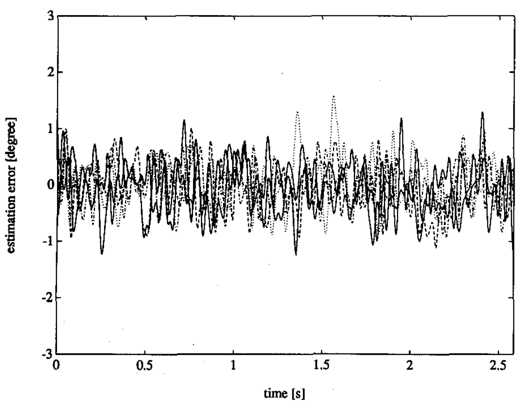

Fig. 3. Estimation error of $\Delta\phi[n]$. The filter used is FIR of order 72. Its cutoff frequency is 26 Hz, and the attenuation is 101 dB.

We continued the process of decreasing the cutoff frequency. The results in Fig. 4 were obtained when the filter's cutoff frequency was 13.5 Hz. The attenuation in the stopband had to drop to 45 dB. The transition bandwidth was 11.5 Hz. It can be noticed that the error remains within the limits of $\pm 1°$. This error, unlike the one from Fig. 3, contains an easily visible 120 Hz component. If

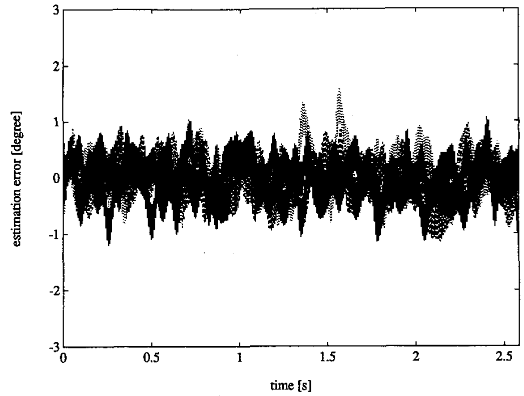

Fig. 4. Estimation error of $\Delta\phi[n]$. The filter used is FIR of order 72. Its cutoff frequency is 13.5 Hz, and the attenuation is 45 dB.

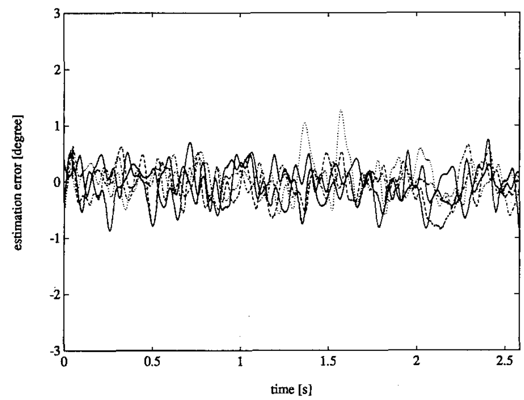

Fig. 5. Estimation error of $\Delta\phi[n]$. The filter used is FIR of order 72. Its cutoff frequency is 13.5 Hz, and the attenuation is 45 dB. In addition, a notch filter with attenuation of 40 dB at 120 Hz is applied.

in addition we apply a notch filter to remove the strong 120 Hz component in the processed signal, we obtain the result in Fig. 5. The attenuation of the notch filter at 120 Hz was 40 dB. The overall results in Fig. 5 are better than those in Fig. 3.

The next filter had a cutoff frequency of 7 Hz. The attenuation in the stopband was 29 dB. The transition bandwidth was 5 Hz. The filter was followed by two identical notch filters with attenuation of 40 dB at 120 Hz. The results are shown in Fig. 6. The error continues to be within the required limits. Besides the strong low-frequency component of the error, there are superimposed high-frequency components due to the noise which have not been removed by the low-pass filter.

The same experiment was repeated with a Bessel filter of fourth order with a cutoff frequency at 6 Hz, attenuation of 88 dB at 120 Hz, and group delay of 50 ms. The estimation errors are shown in Fig. 7. This filter has a satisfactory attenuation at 120 Hz, a low cutoff frequency, and narrow transition bandwidth. In addition, its group delay is constant for low frequencies. The results are comparable to those obtained by the FIR and the notch filter from Fig. 5.

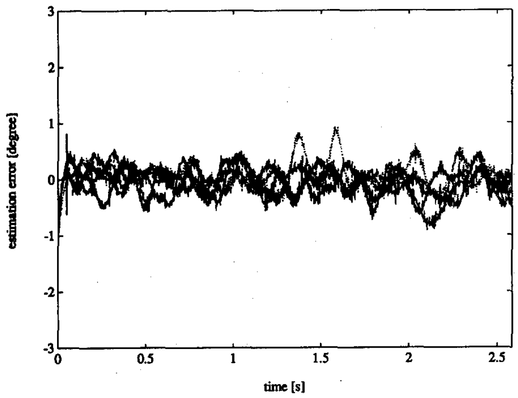

Fig. 6. Estimation error of $\Delta\phi[n]$. The filter used is FIR of order 72. Its cutoff frequency is 7 Hz, and the attenuation is 29 dB. In addition, two notch filters with attenuation of 40 dB at 120 Hz are applied.

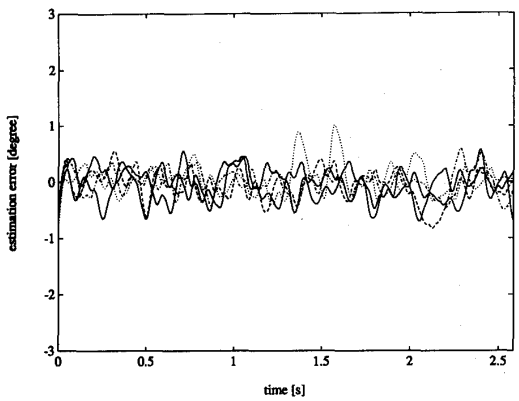

Fig. 7. Estimation error of $\Delta\phi[n]$. The filter used is IIR (Bessel). The filter order is 4, its cutoff frequency is 6 Hz, and attenuation at 120 Hz is 88 dB.

In addition to the Bessel filter, other IIR filters were tried, in particular, some elliptic filters. They are characterized by a magnitude response that is equiripple in the passband and the stopband. In addition, for given order and ripple specifications, they have the narrowest transition of all filters. They have good performance when the phase variations are not significant. But when they are as large as in our example, the nonconstant group delay for low frequencies starts to make impact, and the error quickly becomes unacceptable.

VI. Conclusions

Tracking of transient phase angles in power networks by demodulation is a very promising technique. It maintains accuracy within the required tolerances of 1 degree while maintaining a relatively moderate computational burden on the measurement hardware. A very important issue for successful estimation of the instantaneous phase angle is the type of the applied low-pass filter and the choice of its parameters. We found that carefully designed FIR filters and the Bessel filter provide excellent filtering performance. The Bessel filter is particularly attractive since it easily achieves significant attenuation of the 120 Hz component, has a narrow transition bandwidth, and has a constant group delay for low frequencies. The implementation of this technique would likely be around a DSP processor capable of providing real-time operation. Given the relatively low frequency of power networks and slowness of the typical transient, real-time operation is not expected to present a major problem.

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