Notes:

Filters:

A simple example of an FIR filter is the moving average filter given by $H_0(z) = (1+z^{-1})/2$.

A simple example of an IIR filter is the fading memory filter given by $H(z) = k/(1-az^{-1})$, or
$H(z) = k(1+z^{-1})/(1-az^{-1})$.

Note: There are no *genuine* high-pass filters in DSP!

Filter Characteristics:

- Lowpass:

$$H(f) = \begin{cases} 1 & \text{if } |f| \leq f_c \\ 0 & \text{if } |f| > f_c \end{cases}$$

- Bandpass

$$H(f) = \begin{cases} 1 & \text{if } f_l \leq |f| \leq f_u \\ 0 & \text{otherwise} \end{cases}$$

Notes:
Note: the DSP differentiator is band-limited, and so is feasible.

Notes:

- Differentiator:

$$H(f) = jCf$$

- Hilbert transformer

$$H(f) = \begin{cases} -j & \text{if } f < 0 \\ j & \text{if } f > 0 \end{cases}$$

- Notch
- Matched, Wiener, etc.

Notes:

Filter Design:

- FIR: sections 4.3 – 4.6
- IIR: section 5.3

Slide 42

FIR Filter Properties:

- linear-phase capability;
- easily implemented
- easily scaled
- easily controlled roundoff and truncation errors
- simple to design
- high-order for sharp transitions
- high delay (for linear phase)

Slide 43

Notes:

Notes:

IIR Filter Properties:

- nonlinear-phase
- possible instability
- difficult to scale
- difficult to control roundoff and truncation
- complex design
- low-order, sharp transitions

Slide 44

Notes:

FIR filter design methods:

- Windowing: secs. 4.6.1 – 4.6.4
- Equiripple (Computer-Aided): sec. 4.6.5

Notes:

Windowing:

- Translate given requirements to discrete-time (normalized, angular) frequency domain, using $\theta = 2\pi f / f_S$
- Apply inverse discrete-time Fourier transform to the resulting frequency reponse, $G(\theta)$, to get a noncausal impulse reponse (g_n).

See sections 4.5 (p. 131), and 4.6 (p. 135).

Slide 46

Notes:

- This response will normally be infinite; to make it finite, multiply by a *window function* $w_N(n)$, which is zero for $|n| > N$. We then have a finite, noncausal impulse response, $(\tilde{g}_n)$, given by

$$\tilde{g}_n = \begin{cases} g_n w_N(n) & \text{if } |n| \le N \\ 0 & \text{if } |n| > N \end{cases}$$

- Then delay this response by N units to get a causal, implementable impulse response (of order $2N$):

$$h_n = \tilde{g}_{n-N}$$

Notes:

Effect of window function:

Multiplying by g_n by $w_N(n)$ is equivalent to convolving $G(\theta)$ with $W_N(\theta)$ in the θ-domain, where $W_N(\theta)$ is the DTFT of $w_N(n)$. This in turn is equivalent to replacing each edge in the ideal reponse by the indefinite integral of $W_N(\theta)$.

This has two effects:

- Sharp edges are replaced by finite-width transition regions
- Ripple (approximation error) appears in the passbands and stopbands (the same level of ripple in both)

For this reason, the effect of a window is usually described in terms of $W_N(\theta)$; the important quantities are the *width of the main lobe*, and the *peak approximation error*, or *ripple*. (The peak level of the sidelobes is often given also.)

The *shape* of the window is determined by the desired level of ripple; the *order* of the window is then determined by the desired width of the transition regions.

Notes:
For next six slides, see secs. 4.6.1 – 4.6.4 (p. 135)

Notes:

Some windows:

Rectangular (Truncation):

$$w_N(n) = \begin{cases} 1 & \text{if } |n| \le N \\ 0 & \text{if } |n| > N \end{cases}$$

Peak Approximation Error: -21 dB

Approximate Main Lobe Width: $2\pi/N$

Notes:

Triangular (Bartlett):

$$w_N(n) = \begin{cases} 1 - \frac{|n|}{N} & \text{if } |n| \leq N \\ 0 & \text{if } |n| > N \end{cases}$$

Peak Approximation Error: -25 dB

Approximate Main Lobe Width: $4\pi/N$

Slide 51

Notes:

Hanning:

$$w_N(n) = \begin{cases} \frac{1}{2}\left(1 + \cos\frac{n\pi}{N}\right) & \text{if } |n| \le N \\ 0 & \text{if } |n| > N \end{cases}$$

Peak Approximation Error: -44 dB

Approximate Main Lobe Width: $4\pi/N$

Notes:

Hamming:

$$w_N(n) = \begin{cases} 0.5435 + .4565 \cos \frac{n\pi}{N} & \text{if } |n| \le N \\ 0 & \text{if } |n| > N \end{cases}$$

Peak Approximation Error: -53 dB

Approximate Main Lobe Width: $4\pi/N$

Slide 53

Blackman:

$$w_N(n) = \begin{cases} .42 + .5\cos\frac{n\pi}{N} + .08\cos\frac{2n\pi}{N} & \text{if } |n| \le N \\ 0 & \text{if } |n| > N \end{cases}$$

Peak Approximation Error: -74 dB

Approximate Main Lobe Width: $6\pi/N$

Slide 54

Notes:
For next slide, see section 4.6.4 (p.137)
Note that the book uses Q for N, and a for β below.

Notes:

Kaiser:

$$w_N(n) = \begin{cases} \frac{I_0(\beta\sqrt{1-(n/N)^2})}{I_0(\beta)} & \text{if } |n| \le N \\ 0 & \text{if } |n| > N \end{cases}$$

where β is a parameter which determines the peak approximation error, and $I_0(x)$ is the modified Bessel function of the first kind and order 0.

Empirical equations for selecting β:

$$\beta = \begin{cases} .1102(A-8.7) & A > 50 \\ .584(A-21)^{0.4} + .0789(A-21) & 21 \le A \le 50 \\ 0 & A < 21 \end{cases}$$

and

$$N = \frac{A-8}{4.57\Delta}$$

where $A = -20\log_{10}\delta$ is the peak approximation error in dB, and Δ is the transition width in terms of the discrete-time angular frequency θ.

Impulse responses of some ideal filters:

The IDTFT of the ideal low-pass filter with normalized angular cutoff frequency α is

$$h_{ideal}(n) = \begin{cases} \frac{\alpha}{\pi} & \text{if } n = 0 \\ \frac{1}{n\pi}\sin(n\alpha) & \text{if } n \neq 0 \end{cases}$$

Others:

- High-pass: ideal response is $h_{ideal}(n) = \delta(n) - h_{ideal,lowpass}(n)$
- Band-pass: get ideal response by difference of two low-pass or high-pass
- Band-limited differentiator: $h_0 = 0$, and ideal response for $n \neq 0$ is:
$$h_n = \frac{\alpha}{n\pi}\cos(n\alpha) - \frac{1}{n^2\pi}\sin(n\alpha)$$
- Band-limited Hilbert transformer (from α up to β): $h_0 = 0$, and ideal response for $n \neq 0$ is:
$$h_n = \frac{1}{n\pi}\left[\cos(n\alpha) - \cos(n\beta)\right]$$

Slide 56

Notes: Note: next example should be done using each filter design technique that we cover.

For now, use Kaiser window.

Notes:

Example:

Output filter for CD player with 2 times oversampling.

Slide 57

Notes:

Equiripple (Computer-Aided): section 4.6.5

As before, the first and last steps are:

- Translate given requirements to discrete-time frequency domain, using $\theta = 2\pi f T$
- Use procedure below to get a finite, symmetric, noncausal response g_n
- Then delay this response by N units to get a causal, implementable impulse response (of order $2N$):

$$h_n = g_{n-N}$$

We assume that the g_n are symmetric.

Notes:

To find the noncausal impulse reponse, solve the minimax optimization problem:

Find the g_n coefficients which minimize the functional

$$I(g_0, g_1, \ldots, g_N) = Max_{\theta}[|E(\theta)|]$$

where the error function $E(\theta)$ is given by

$$E(\theta) = W(\theta)\left(D(\theta) - \sum_{n=-N}^{N} g_n e^{-jn\theta}\right)$$

and $W(\theta)$ is a positive weight funtion, and $D(\theta)$ is the desired response.

The optimum has the *equiripple* property:

$E_{opt}(\theta)$ must achieve the peak error at at least $N+2$ points, and the errors must alternate in sign.

Notes:

Do the 2× oversampling CD example using the MATLAB equiripple FIR design.

This optimization problem can be solved iteratively:

- Remez exchange algorithm
- Parks-McClellan algorithm

Slide 61

Notes:
Next start on IIR design.