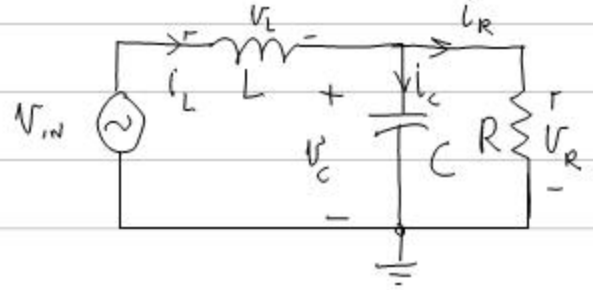
L. C. Circuit:



State:

$x_1 = i_L$

$x_2 = v_C$

State Equations:

$$\dot{x}_1 = \frac{d}{dt} i_L = \frac{1}{L} v_L = \frac{1}{L}\left(v_{in} - v_C\right) = \frac{1}{L}\left(v_{in} - x_2\right)$$

$$\dot{x}_2 = \frac{d}{dt} v_C = \frac{1}{C} i_C = \frac{1}{C}\left(i_L - i_R\right) = \frac{1}{C}\left(x_1 - \frac{1}{R} x_2\right)$$

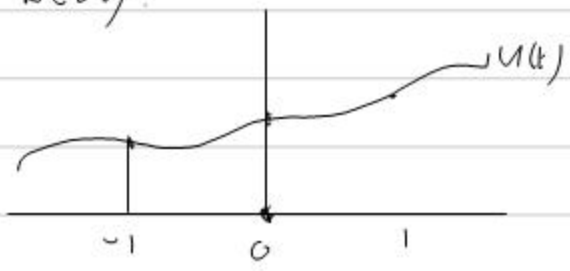
Matrix Form:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & -1/L \\ \frac{1}{C} & -\frac{1}{RC} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1/L \\ 0 \end{pmatrix} v_{in}$$

Output $\quad v_R = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

Continuous-time Unit Delay:

$y(t) = u(t-1)$



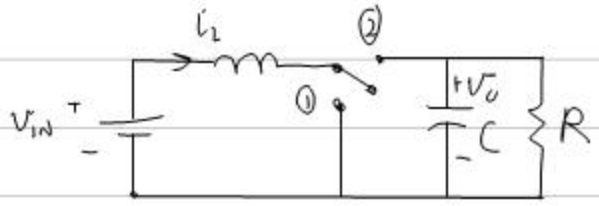
$y(t)$ for $t \geq 0$?

$y(t)$ for $0 \leq t \leq 1$: need to know $u(t)$ for $-1 \leq t \leq 0$

Infinite set of variables

EXAMPLE II.

BOOST CONVERTER IN CONTINUOUS CONDUCTION MODE:

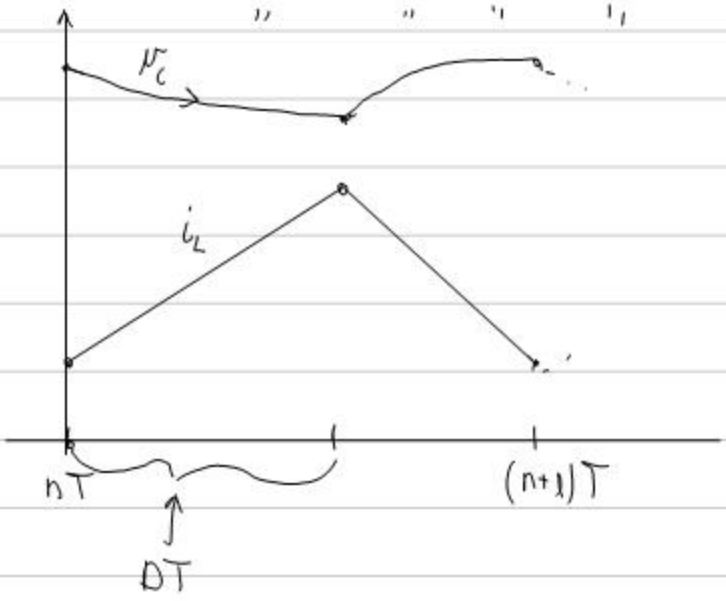


OPERATION: SWITCH OPERATES PERIODICALLY WITH PERIOD T.

DUTY CYCLE: D

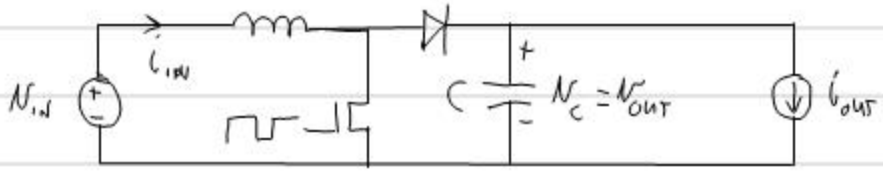
SWITCH IS IN POSITION 1 FOR A TIME DT

" " " " 2 " " " (1-D)T



PERIODICALLY TIME-VARYING LINEAR SYSTEM.

MORE REALISTIC:



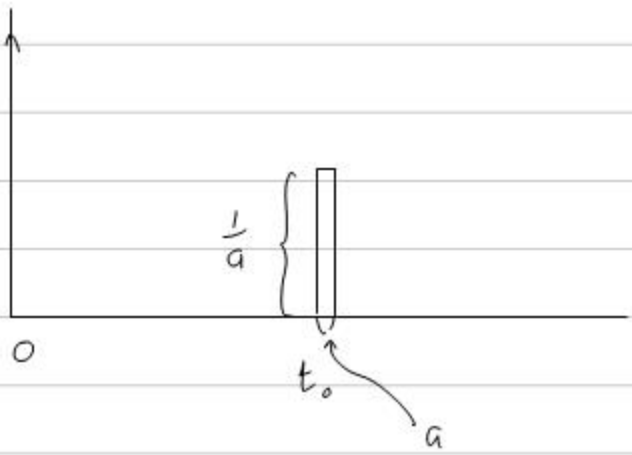
INPUTS V_{IN} & i_{OUT} } LINEAR SYSTEM.

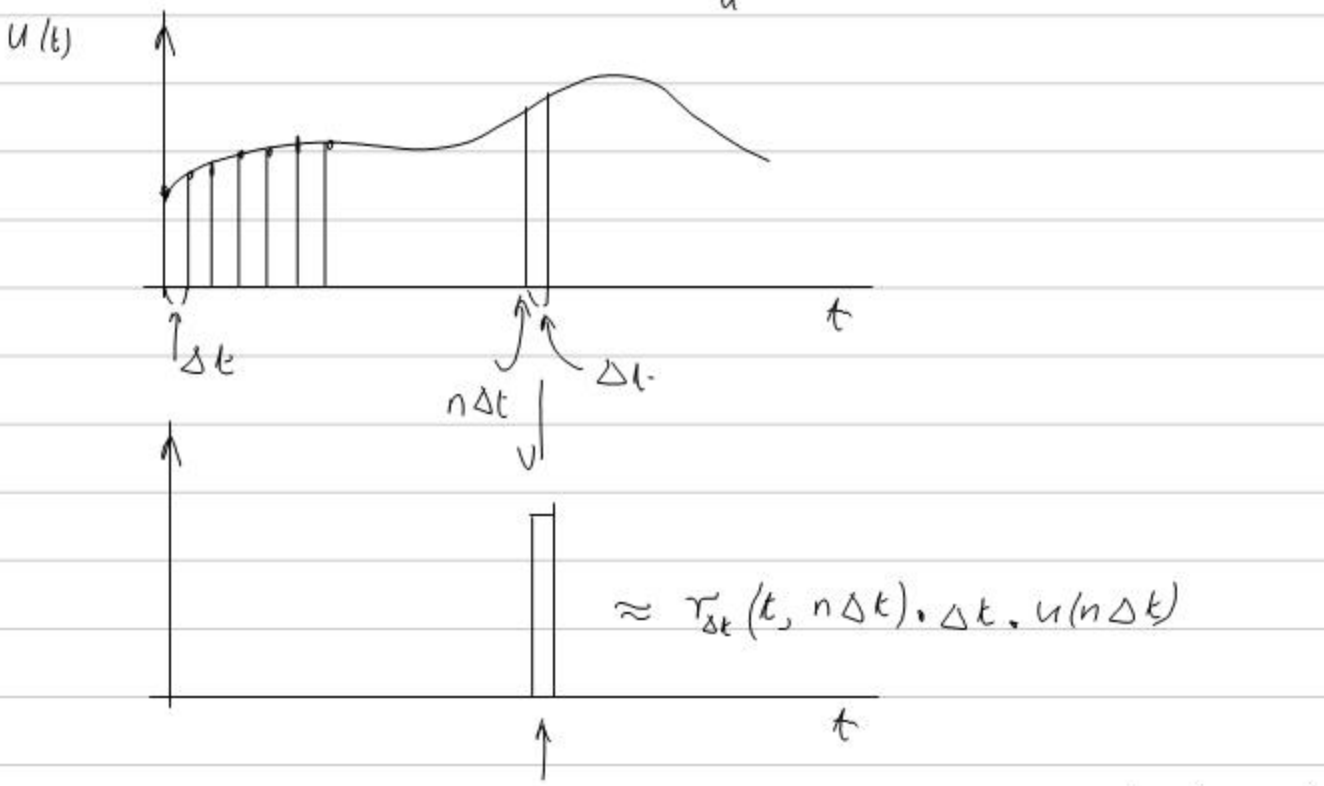
OUTPUTS V_{OUT} & i_{IN} }

REAL INPUT: D

- NONLINEAR.

$r_a(t-t_0)$:





$\approx r_{\Delta t}(t, n\Delta t) \cdot \Delta t \cdot u(n\Delta t)$

INPUT: $\Rightarrow$ OUTPUT $= \Delta t\, u(n\Delta t)\, h_{\Delta t}(t, n\Delta t)$

TAKE SUM, & LET $\Delta t \to 0$.

IF SYSTEM IS TIME-INVARIANT, THEN RESPONSE TO $r_a(t-t_0)$ IS $h_a(t-t_0)$, WHERE $h_a(t)$ IS THE RESPONSE TO $r_a(t)$

PROOF OF CONVOLUTION FORMULA:

$$\int_0^\infty \int_0^t k(t,\tau)\, d\tau\, dt$$

$$= \int_0^\infty \int_\tau^\infty k(t,\tau)\, dt\, d\tau$$