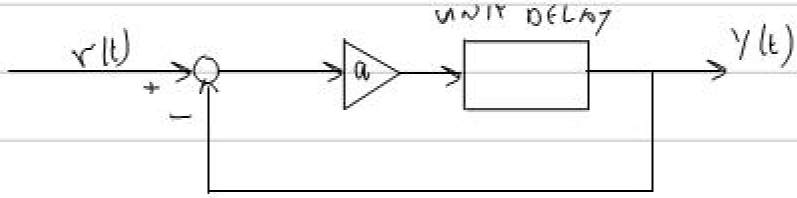
Ex. 2.3:



IMPULSE RESPONSE:

$$h_c(t) = a\,\delta(t-1) - a^2\,\delta(t-2) + a^3\,\delta(t-3) \ldots$$

STABILITY: STABLE $\Leftrightarrow$ $|a|<1$

Ex. 2.4: TRANSFER FUNCTION OF UNIT DELAY:

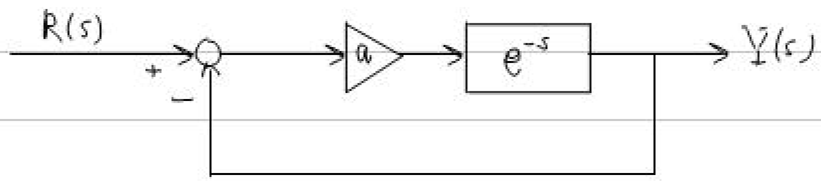
$$H(s) = \mathcal{L}\{\delta(t-1)\} = \int_0^\infty \delta(t-1)\,e^{-st}\,dt = e^{-s}$$

Ex. 2.5: TRANSFER FUNCTION OF CLOSED-LOOP SYSTEM: (Ex. 2.3)

$$H_c(s) = \mathcal{L}\{h_c(t)\} = a e^{-s} - a^2 e^{-2s} + a^3 e^{-3s} + \ldots$$

$$= \frac{a e^{-s}}{1+a e^{-s}}$$

OR:



$$Y(s) = e^{-s}.a.(R(s) - Y(s))$$

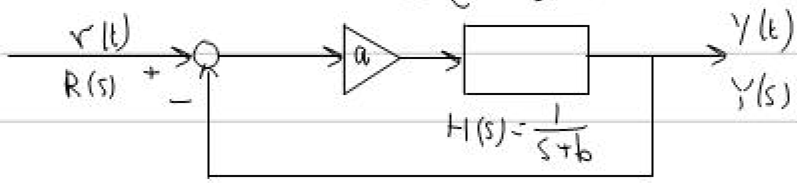
$$\Rightarrow (1+ae^{-s})\,Y(s) = ae^{-s}R(s)$$

$$\Rightarrow Y(s) = \frac{a e^{-s}}{1+a e^{-s}} \cdot R(s)$$

$$\Rightarrow \frac{Y(s)}{R(s)} = H_c(s) = \frac{a e^{-s}}{1+a e^{-s}}$$

EXAMPLE:

IMPULSE RESPONSE $= e^{-bt}\,U(t)$



TRY IN TIME-DOMAIN?

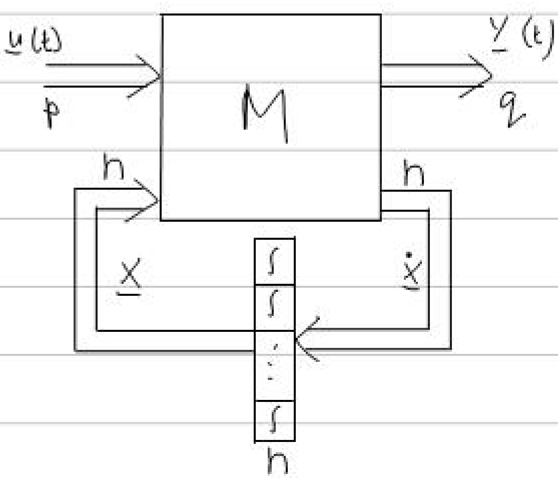
IN TERMS OF s:

$$Y(s) = a \cdot \frac{1}{s+b}\left(R(s) - Y(s)\right)$$

$$\Rightarrow (s+a+b)\,Y(s) = aR(s)$$

$$\Rightarrow H_{cl}(s) = \frac{a}{s+a+b}$$

STATE VARIABLE SYSTEM:



NONLINEAR:

$$\begin{pmatrix} \dot{\underline{x}}(t) \\ \underline{y}(t) \end{pmatrix} = \begin{pmatrix} f(\underline{x}(t), \underline{u}(t), t) \\ g(\underline{x}(t), \underline{u}(t), t) \end{pmatrix}$$

NONLINEAR, TIME-INV.

$$\begin{pmatrix} \dot{\underline{x}}(t) \\ \underline{y}(t) \end{pmatrix} = \begin{pmatrix} f(\underline{x}(t), \underline{u}(t)) \\ g(\underline{x}(t), \underline{u}(t)) \end{pmatrix}$$

LINEAR:

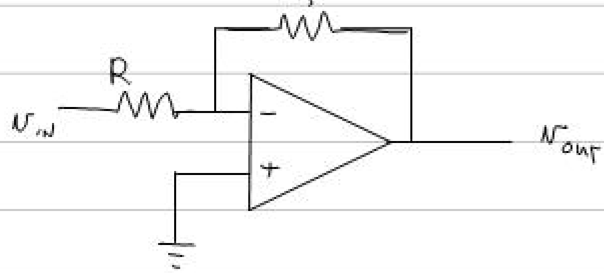
$$\begin{pmatrix} \dot{\underline{x}}(t) \\ \hline \underline{y}(t) \end{pmatrix} = M(t)\begin{pmatrix} \underline{x}(t) \\ \hline \underline{u}(t) \end{pmatrix} = \left(\begin{array}{c:c} A(t) & B(t) \\ \hdashline C(t) & D(t) \end{array}\right)\begin{pmatrix} \underline{x}(t) \\ \hline \underline{u}(t) \end{pmatrix}$$

[$M(t)$, $A(t)$, $B(t)$, $C(t)$, $D(t)$ ARE ALL MATRICES]

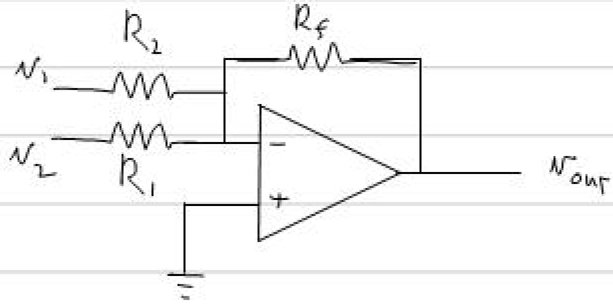
LINEAR, TIME-INVARIANT: $M(t)$, $A(t)$, $B(t)$, $C(t)$, $D(t)$ ARE <u>CONSTANT</u>

OP-AMP IMPLEMENTATION:

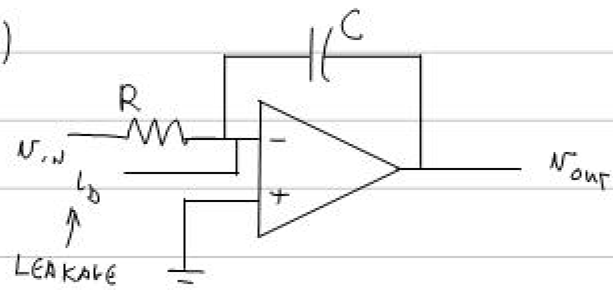
CONSTANT GAIN:



ADDER (WITH GAINS):

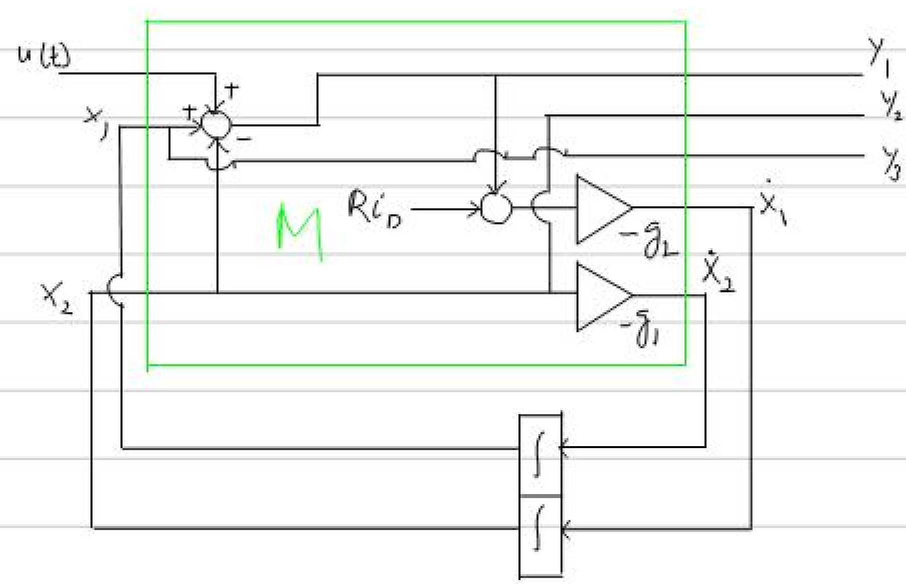


INTEGRATOR: (WITH GAIN)



$$V_{out}(s) = \frac{-1}{sRC} V_{in} - \frac{1}{sC} i_D$$

EXAMPLE: STATE-VARIABLE "BIQUAD" FILTER.



STATE-VARIABLE EQUATIONS FOR BIQUAD:

$$\dot{x}_1 = -g_1 x_2$$

$$\dot{x}_2 = -g_2 (u + x_2 - x_1 + R\, i_b)$$

$$y_1 = u + x_2 - x_1$$

$$y_2 = x_2$$

$$y_3 = x_1$$

So

$$\underline{\dot{x}} = \begin{pmatrix} 0 & -g_1 \\ g_2 & -g_2 \end{pmatrix} \underline{x} + \begin{pmatrix} 0 & 0 \\ -g_2 & -Rg_2 \end{pmatrix} \begin{pmatrix} u \\ i_b \end{pmatrix}$$

$$\underline{y} = \begin{pmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \underline{x} + \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u \\ i_b \end{pmatrix}$$

So

$$H(s) = \begin{pmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} s & g_1 \\ -g_2 & s+g_2 \end{pmatrix}^{-1} \begin{pmatrix} 0 & 0 \\ -g_2 & -Rg_2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

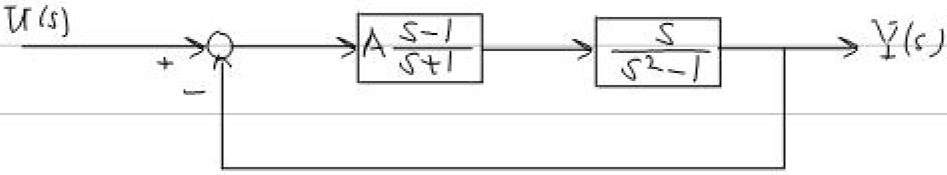
$$= \frac{1}{s^2 + sg_2 + g_1 g_2} \begin{pmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} s+g_2 & -g_1 \\ g_2 & s \end{pmatrix} \begin{pmatrix} 0 & 0 \\ -g_2 & -Rg_2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \frac{1}{s^2 + sg_2 + g_1 g_2} \begin{pmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} g_1 g_2 & Rg_1 g_2 \\ -sg_2 & -Rsg_2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \frac{1}{s^2 + sg_2 + g_1 g_2} \begin{pmatrix} -sg_2 - g_1 g_2, & -R(sg_2 + g_1 g_2) \\ -sg_2 & , -sg_2 R \\ g_1 g_2 & , Rg_1 g_2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

EXAMPLES:

A:



TRANSFER FUNCTION:

$$Y(s) = \frac{s}{s^2-1} \cdot A\frac{s-1}{s+1}\left(U(s) - Y(s)\right)$$

$$= \frac{As}{(s+1)^2}\left(U(s) - Y(s)\right)$$

$$\Rightarrow \left(s^2+(2+A)s+1\right)Y(s) = As\,U(s)$$

$$\Rightarrow H(s) = \frac{As}{s^2+(2+A)s+1}$$