Ex. 4.1

$$A = \begin{pmatrix} 0 & -1 \\ 1 & -2 \end{pmatrix}$$

FIND $(sI-A)^{-1}$

Ex 4.4 EIGENVALUES: $0 = \begin{vmatrix} \lambda & 1 \\ -1 & \lambda+2 \end{vmatrix} = (\lambda+1)^2$

$\lambda_0 = -1$, MULT. 2

THEN $(sI-A)^{-1} = \beta_0(s) I + \beta_1(s) A$

WHERE $(s-\lambda)^{-1} = \beta_0(s) + \beta_1(s)\lambda$ ON SPECTRUM

i.e.,

$$(s+1)^{-1} = \beta_0(s) - \beta_1(s)$$

$$\& \quad \frac{d}{d\lambda}\left((s-\lambda)^{-1}\right)\Big|_{\lambda=-1} = \beta_1(s)$$

$$\Rightarrow \quad \beta_1(s) = \frac{1}{(s+1)^2}$$

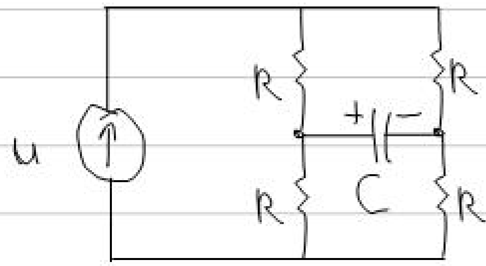
So

$$(sI-A)^{-1} = \left(\frac{1}{s+1} + \frac{1}{(s+1)^2}\right) I + \frac{1}{(s+1)^2}\begin{pmatrix} 0 & -1 \\ 1 & -2 \end{pmatrix}$$

Ex. 4.2:

$$A = \begin{pmatrix} 0 & -1 \\ 1 & -2 \end{pmatrix}$$

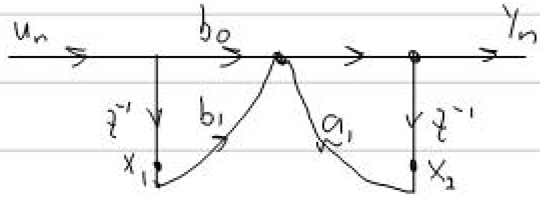
FIND e^{At}, & FIND SOLUTION



EXAMPLE: ZERO-STATE EQUIVALENCE $\not\Rightarrow$ ALGEBRAIC EQUIVALENCE.

1^{st}-ORDER DIGITAL FILTER:

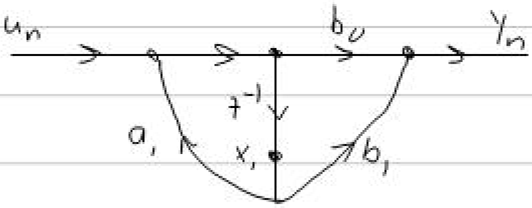
DF I:



$$y_n = b_0 u_n + b_1 u_{n-1} + a_1 y_{n-1}$$

$$\Rightarrow \frac{Y(z)}{U(z)} = H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$$

DF II.



$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$$

DF II STATE EQUATIONS.

$$x_1(n+1) = u_n + a_1 x_1(n)$$
$$y_n = b_0 \left(u_n + a_1 x_1(n)\right) + b_1 x_1(n)$$

$$\left.\begin{aligned} x(n+1) &= a_1 x(n) + 1 \cdot u_n \\ y_n &= (b_0 a_1 + b_1) x(n) + b_0 u_n \end{aligned}\right\} D = b_0 \text{ \& } CA^m B = (b_0 a_1 + b_1) a_1^m$$

DF I STATE EQUATIONS:

$$x_1(n+1) = u_n$$
$$x_2(n+1) = b_0 u_n + b_1 x_1(n) + a_1 x_2(n)$$
$$y_n = b_0 u_n + b_1 x_1(n) + a_1 x_2(n)$$

$$\Rightarrow \quad \left.\begin{aligned} \underline{x}(n+1) &= \begin{pmatrix} 0 & 0 \\ b_1 & a_1 \end{pmatrix} \underline{x}(n) + \begin{pmatrix} 1 \\ b_0 \end{pmatrix} u_n \\ y_n &= \begin{pmatrix} b_1 & a_1 \end{pmatrix} \underline{x}(n) + b_0 u_n \end{aligned}\right\} \begin{aligned} &D = b_0 \ \& \\ &CA^m B = \, ? \end{aligned}$$

$$CA^m B = \begin{pmatrix} b_1 & a_1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ b_1 & a_1 \end{pmatrix}^m \begin{pmatrix} 1 \\ b_0 \end{pmatrix}$$

TRANSFER FUNCTION: $H(z) = \begin{pmatrix} b_1 & a_1 \end{pmatrix} \begin{pmatrix} z & 0 \\ -b_1 & z-a_1 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ b_0 \end{pmatrix} + b_0$

$$= \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$$

POLE: $z = a_1$

EIGENVALUES OF A: $\lambda_1 = a_1$ & $\lambda_2 = 0$

$$sZ = AZ + B$$

$$\Rightarrow \begin{pmatrix} sZ_1 \\ sZ_2 \\ sZ_3 \\ \vdots \\ sZ_r \end{pmatrix} = \begin{pmatrix} -\alpha_1 I_p & -\alpha_2 I_p & \cdots & -\alpha_r I_p \\ I_p & O_p & \cdots & O_p \\ O_p & I_p & \cdots & \\ \vdots & & & \vdots \\ O_p & & I_p & O_p \end{pmatrix} \begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ \vdots \\ Z_r \end{pmatrix} + \begin{pmatrix} I_p \\ O_p \\ \vdots \\ O_p \end{pmatrix}$$

FOR $2 \leq k \leq r$:

$$sZ_2 = Z_1 \Rightarrow Z_2 = \frac{1}{s} Z_1$$

$$sZ_3 = Z_2 \Rightarrow Z_3 = \frac{1}{s^2} Z_1$$

$$sZ_4 = Z_3 \Rightarrow Z_4 = \frac{1}{s^3} Z_1$$

$$\vdots$$

$$Z_r = \frac{1}{s^{r-1}} Z_1$$

FIRST ROW (BLOCK):

$$sZ_1 = -\alpha_1 Z_1 - \alpha_2 Z_2 \cdots - \alpha_r Z_r + I_p$$

$$= -\alpha_1 Z_1 - \frac{\alpha_2}{s} Z_1 - \frac{\alpha_3}{s^2} \cdots - \frac{\alpha_r}{s^{r-1}} Z_1 + I_p$$

$$\Rightarrow \left(s^r + \alpha_1 s^{r-1} + \alpha_2 s^{r-2} + \cdots \alpha_r \right) Z_1 = s^{r-1} I_p$$

$$\Rightarrow Z_1 = \frac{s^{r-1}}{d(s)} I_p$$

$$\text{\&} \quad Z_2 = \frac{s^{r-2}}{d(s)} I_p$$

$$\vdots$$

$$Z_r = \frac{1}{d(s)} I_p$$

Example 4.6:

$$G(s) = \begin{pmatrix} \dfrac{4s-10}{2s+1} & \dfrac{3}{s+2} \\ \dfrac{1}{(2s+1)(s+2)} & \dfrac{s+1}{(s+2)^2} \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} \dfrac{-6}{s+\frac{1}{2}} & \dfrac{3}{s+2} \\ \dfrac{1/2}{(s+\frac{1}{2})(s+2)} & \dfrac{s+1}{(s+2)^2} \end{pmatrix}$$

So

$$G_{sp}(s) = \begin{pmatrix} \dfrac{-6}{s+\frac{1}{2}} & \dfrac{3}{s+2} \\ \dfrac{\frac{1}{2}}{(s+\frac{1}{2})(s+2)} & \dfrac{s+1}{(s+2)^2} \end{pmatrix}$$

$$= \frac{1}{(s+\frac{1}{2})(s+2)^2} \begin{pmatrix} -6(s+2)^2 & 3(s+\frac{1}{2})(s+2) \\ \frac{1}{2}(s+2) & (s+1)(s+\frac{1}{2}) \end{pmatrix}$$

$$= \frac{1}{s^3+\frac{9}{2}s^2+6s+2}\left[\begin{pmatrix} -6 & 3 \\ 0 & 1 \end{pmatrix}s^2 + \begin{pmatrix} -24 & \frac{15}{2} \\ \frac{1}{2} & \frac{3}{2} \end{pmatrix}s + \begin{pmatrix} -24 & 3 \\ 1 & \frac{1}{2} \end{pmatrix}\right]$$

$p=2$ & $q=2$: State equations: $(n = rp = 3\times 2 = 6)$

$$\dot{\underline{x}} = \begin{pmatrix} -9/2 & 0 & -6 & 0 & -2 & 0 \\ 0 & -9/2 & 0 & -6 & 0 & -2 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}\underline{x} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -6 & 3 & -24 & 15/2 & -24 & 3 \\ 0 & 1 & \frac{1}{2} & \frac{3}{2} & 1 & \frac{1}{2} \end{pmatrix}\underline{x} + \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

Ex. 4.7: SINGLE-INPUT SYSTEMS:

$$G_1(s) = \begin{pmatrix} \dfrac{4s-10}{2s+1} \\ \dfrac{1}{(2s+1)(s+2)} \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} \dfrac{-6}{s+\frac{1}{2}} \\ \dfrac{1/2}{(s+\frac{1}{2})(s+2)} \end{pmatrix} \qquad (p=1)$$

$$G_{SP,1}(s) = \begin{pmatrix} \dfrac{-6}{s+\frac{1}{2}} \\ \dfrac{1/2}{(s+\frac{1}{2})(s+2)} \end{pmatrix} = \frac{1}{(s+\frac{1}{2})(s+2)} \begin{pmatrix} -6(s+2) \\ \frac{1}{2} \end{pmatrix}$$

$$= \frac{1}{s^2+\frac{5}{2}s+1} \left[\begin{pmatrix} -6 \\ 0 \end{pmatrix} s + \begin{pmatrix} -12 \\ \frac{1}{2} \end{pmatrix} \right]$$

REALIZATION:

$$\dot{\underline{x}}_1 = \begin{pmatrix} -\frac{5}{2} & -1 \\ 1 & 0 \end{pmatrix} \underline{x}_1 + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u_1$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -6 & -12 \\ 0 & \frac{1}{2} \end{pmatrix} \underline{x}_1 + \begin{pmatrix} 2 \\ 0 \end{pmatrix} u_1$$

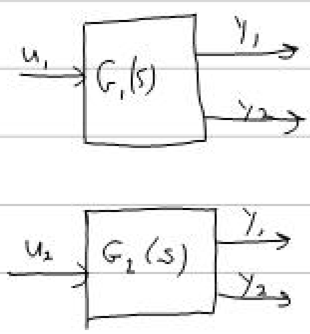
$$G_2(s) = G_{SP,2}(s) = \begin{pmatrix} \dfrac{3}{s+2} \\ \dfrac{s+1}{(s+2)^2} \end{pmatrix} = \frac{1}{s^2+4s+4} \begin{pmatrix} 3(s+2) \\ s+1 \end{pmatrix}$$

$$= \frac{1}{s^2+4s+4} \left[\begin{pmatrix} 3 \\ 1 \end{pmatrix} s + \begin{pmatrix} 6 \\ 1 \end{pmatrix} \right]$$

REALIZATION:

$$\dot{\underline{x}}_2 = \begin{pmatrix} -4 & -4 \\ 1 & 0 \end{pmatrix} \underline{x}_2 + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u_2$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 1 & 1 \end{pmatrix} \underline{x}_2$$



$$Y(s) = G(s)\begin{pmatrix} U_1(s) \\ U_2(s) \end{pmatrix}$$

$$= G(s)\begin{pmatrix} U_1(s) \\ 0 \end{pmatrix} + G(s)\begin{pmatrix} 0 \\ U_2(s) \end{pmatrix}$$

$$= G_1(s)\,U_1(s) + G_2(s)\,U_2(s)$$

SO THE REALIZATION OF $G(s)$ OBTAINED FROM $G_1(s)$ & $G_2(s)$ IS:

$$\begin{pmatrix} \dot{\underline{x}}_1(t) \\ \underline{x}_2(t) \end{pmatrix} = \begin{pmatrix} -\frac{5}{2} & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -4 & -4 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \left(\begin{array}{cc:cc} -6 & -12 & 3 & 6 \\ 0 & \frac{1}{2} & 1 & 1 \end{array}\right) \begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$