NOTE: IF $\dot{\underline{x}}(t) = A(t)\underline{x}(t)$ WITH $\underline{x}(t_0) = \underline{x}_0$

THEN

$$\underline{x}(t) = \Phi(t, t_0)\underline{x}_0$$

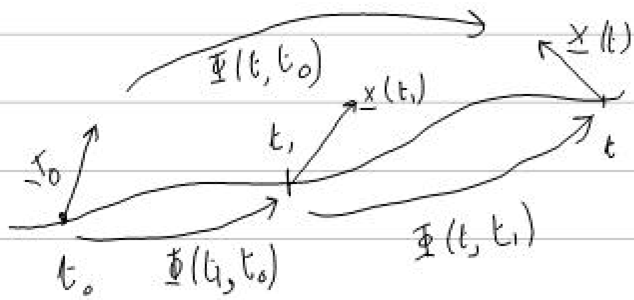
PROOF: IF $\tilde{\underline{x}}(t) = \Phi(t, t_0)\underline{x}_0$

THEN

$$\begin{aligned}\dot{\tilde{\underline{x}}}(t) &= \frac{\partial}{\partial t}\Phi(t, t_0)\underline{x}_0 \\ &= A(t)\Phi(t, t_0)\underline{x}_0 \\ &= A(t)\tilde{\underline{x}}(t)\end{aligned}$$

AND $\tilde{\underline{x}}(t_0) = \Phi(t_0, t_0)\underline{x}_0 = \underline{x}_0$

SO, BY UNIQUENESS, $\tilde{\underline{x}}(t) = \underline{x}(t)$



$$\underline{x}(t) = \Phi(t, t_0)\underline{x}(t_0) + \int_{t_0}^{t}\Phi(t, \sigma)B(\sigma)\underline{u}(\sigma)\,d\sigma$$

$$\begin{aligned}\Rightarrow \quad \dot{\underline{x}}(t) &= A(t)\Phi(t, t_0)\underline{x}(t_0) + \Phi(t, t)B(t)\underline{u}(t) \\ &\quad + \int_{t_0}^{t} A(t)\Phi(t, \sigma)B(\sigma)\underline{u}(\sigma)\,d\sigma \\ &= A(t)\left[\Phi(t, t_0)\underline{x}(t_0) + \int_{t_0}^{t}\Phi(t, \sigma)B(\sigma)\underline{u}(\sigma)\,d\sigma\right] \\ &\quad + B(t)\underline{u}(t) \\ &= A(t)\underline{x}(t) + B(t)\underline{u}(t)\end{aligned}$$

$$Q = X^{-1}(0)\, X(T) \quad = e^{\bar{A}t}$$

$$X(t+T) = X(t)\, Q \longleftarrow \text{PROVE}$$

$$P(t) = e^{\bar{A}t} X^{-1}(t)$$

$$\dot{X}(t+T) = A(t+T)\, X(t+T) \qquad \& \quad X(0+T) = X(T)$$

$$= A(t)\, X(t+T) \qquad\qquad (t=0)$$

$$\& \quad \frac{d}{dt}\left(X(t)Q\right) = A(t)\, X(t)\, Q \qquad \& \quad X(0)Q = X(0) X^{-1}(0) X(T)$$

$$= X(T)$$

SO $X(t+T)$ & $X(t)Q$ SATISFY THE SAME D.E. WITH THE SAME INITAL CONDITION $\Rightarrow X(t+T) = X(t)Q$

NEXT, $P(t)$ IS PERIODIC:

$$P(t+T) = e^{\bar{A}(t+T)}\, X^{-1}(t+T)$$

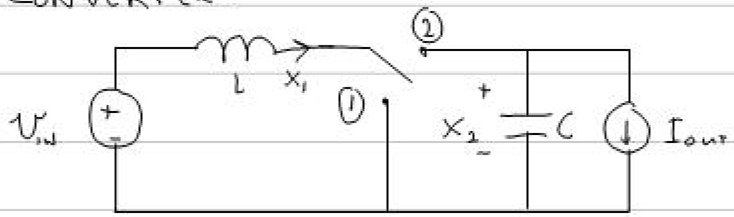
$$= e^{\bar{A}t} e^{\bar{A}T}\, Q^{-1} X^{-1}(t)$$

$$= e^{\bar{A}t}\, X^{-1}(t)$$

$$= P(t)$$

EXAMPLE (PERIODICALLY TIME-VARYING):

BOOST CONVERTER:



STATE EQUATIONS:

FOR $nT < t < nT + DT$

$$\dot{x}_1 = \frac{1}{L} v_{IN}$$

$$\dot{x}_2 = \frac{-1}{C} I_{LOAD}$$

FOR $nT + DT < t < (n+1)T$

$$\dot{x}_1 = \frac{1}{L}\left(v_{IN} - x_2\right)$$

$$\dot{x}_2 = \frac{1}{C}\left(x_1 - I_{OUT}\right)$$

So

$$\dot{\underline{x}} = A(t)\,\underline{x}(t) + \begin{pmatrix} 1/L & 0 \\ 0 & -\frac{1}{C} \end{pmatrix}\begin{pmatrix} v_{IN} \\ I_{OUT} \end{pmatrix}$$

WHERE

$$A(t) = \begin{cases} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & nT < t < nT + DT \\ \begin{pmatrix} 0 & -1/L \\ 1/C & 0 \end{pmatrix} & nT + DT < t < (n+1)T \end{cases}$$

FUND. MATRIX: $\dot{X}(t) = A(t)X(t)$

Assume $X(0) = I$

So $X(t) = I \qquad 0 < t < DT$

& $X(t) = e^{\begin{pmatrix} 0 & -1/L \\ 1/C & 0 \end{pmatrix}(t-DT)} \qquad DT < t < T$

$$\Rightarrow X(T) = \exp\left((1-D)T\begin{pmatrix} 0 & -1/L \\ 1/C & 0 \end{pmatrix}\right) = Q = e^{\bar{A}T}$$

$$\Rightarrow \bar{A} = (1-D)\begin{pmatrix} 0 & -1/L \\ 1/C & 0 \end{pmatrix}$$

$$y(t) = \int_{-\infty}^{\infty} u(t-\tau)\, h(\tau)\, d\tau$$

$$\Rightarrow y(0) = \int_{-\infty}^{\infty} u(-\tau)\, h(\tau)\, d\tau$$

Let $u(\tau) = \begin{cases} -1 & \text{IF } h(-\tau) < 0 \\ 1 & \text{IF } h(-\tau) \geq 0 \end{cases}$

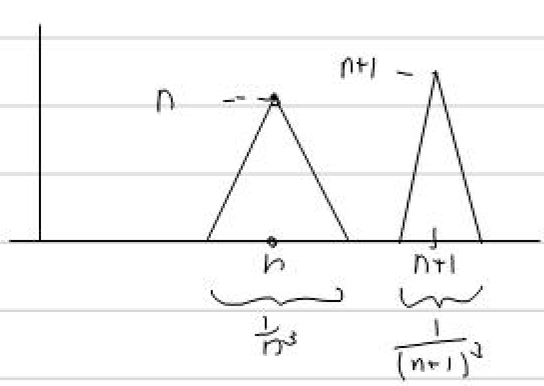
Then $u(-\tau)h(\tau) = \begin{cases} -h(\tau) & \text{IF } h(\tau) < 0 \\ h(\tau) & \text{IF } h(\tau) \geq 0 \end{cases}$

$$= |h(\tau)|$$

So

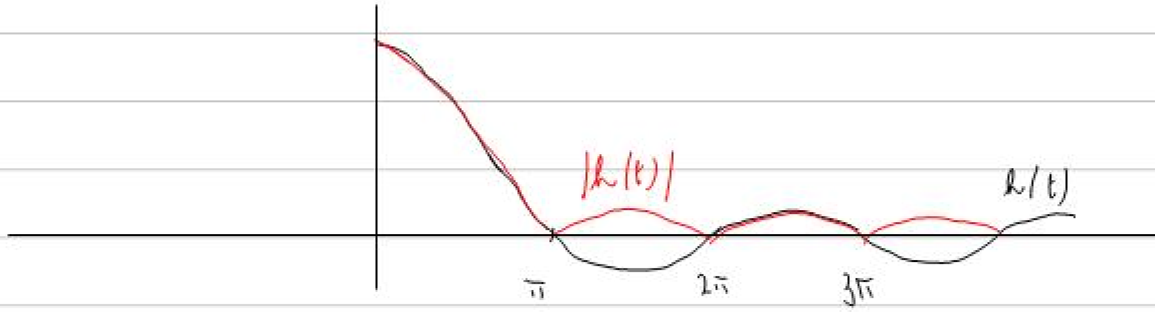
$$y(0) = \int_{-\infty}^{\infty} |h(\tau)|\, d\tau = \infty$$

If $f(t)$ is given by



<u>NOTE:</u> THE IDEAL LOW-PASS FILTER IS BIBO UNSTABLE.

<u>PROOF:</u> $h(t) = \frac{\sin(t)}{t}$



FOR THE REGION $n\pi \leq t \leq (n+1)\pi$,

$$\left|\frac{\sin(t)}{t}\right| \leq \left|\frac{\sin t}{n\pi}\right|$$

So $$\int_{n\pi}^{(n+1)\pi} \left|\frac{\sin(t)}{t}\right| dt \leq \frac{1}{n\pi}\int_{n\pi}^{(n+1)\pi} |\sin t|\, dt = \frac{2}{\pi}\cdot\frac{1}{n}$$

& $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$, so $\int_{\pi}^{\infty} |h(t)|\, dt = \infty$

$$\Rightarrow \int_{-\infty}^{\infty} |h(t)|\, dt = \infty$$