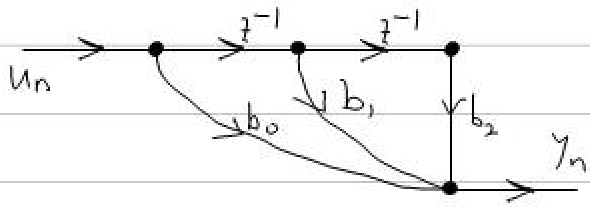
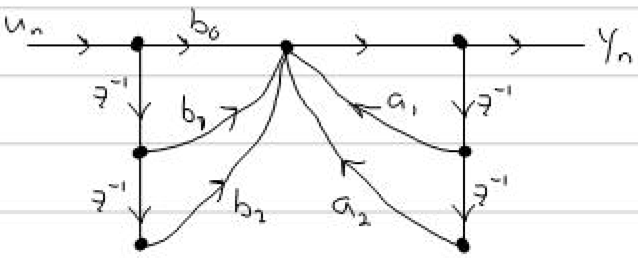
SFG:

2ND-ORDER FIR: $y_n = b_0 u_n + b_1 u_{n-1} + b_2 u_{n-2}$



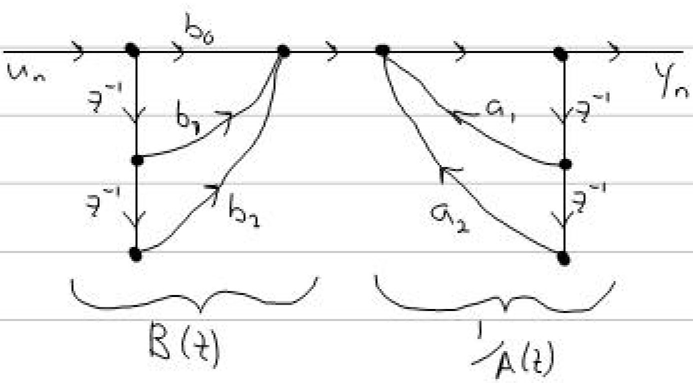
2ND-ORDER IIR: $y_n = b_0 u_n + b_1 u_{n-1} + b_2 u_{n-2} + a_1 y_{n-1} + a_2 y_{n-2}$

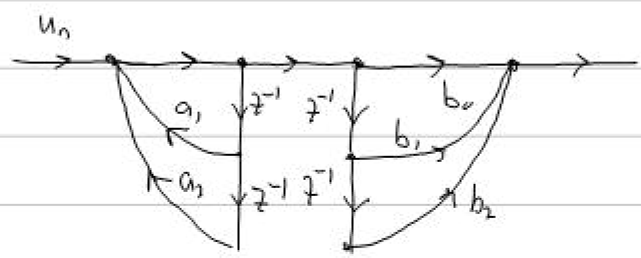


DIRECT FORM I

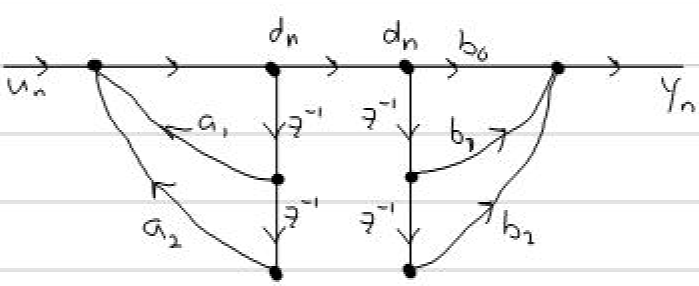
$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}} = \frac{B(z)}{A(z)}$$

REDRAW DIRECT FORM I:

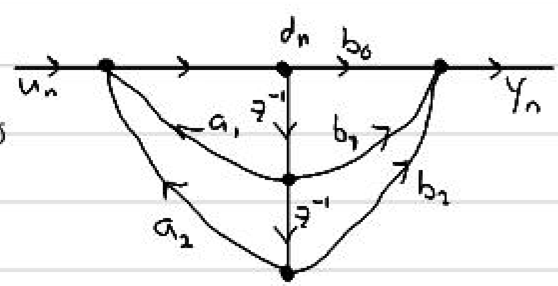




SINCE LTI: CHANGE ORDER:

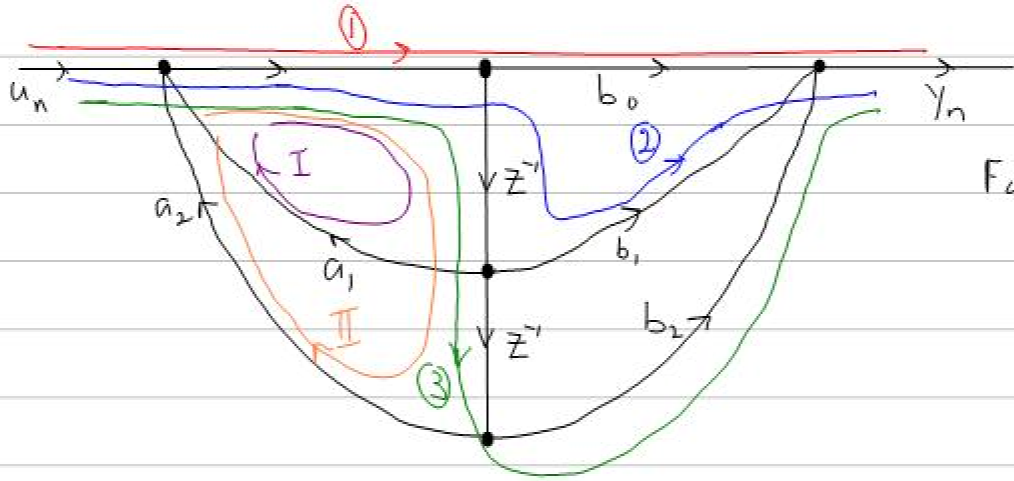


$\Longleftrightarrow$



DIRECT FORM II

Transfer Function of DF II (Mason).



Forward Paths:

$P_1(z) = b_0$

$P_2(z) = b_1 z^{-1}$

$P_3(z) = b_2 z^{-2}$

Loops:

I: $L_I(z) = a_1 z^{-1}$

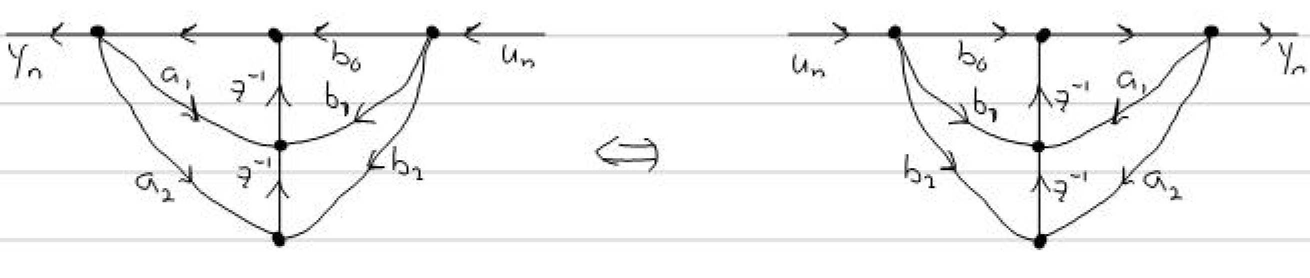
II: $L_{II}(z) = a_2 z^{-2}$

Transfer Function: $\Delta = 1 - (a_1 z^{-1} + a_2 z^{-2})$

$\Delta_1 = \Delta_2 = \Delta_3 = 1$

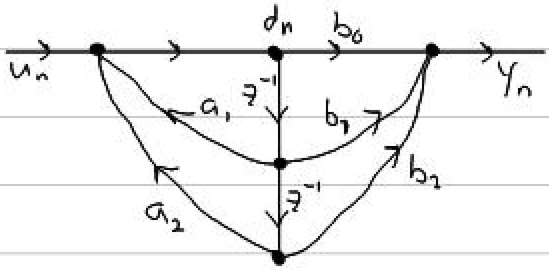
$$\Rightarrow \quad H(z) = \frac{\sum P_i \Delta_i}{\Delta} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}}$$

Transposed DF II:



Transposed DF I: Homework.

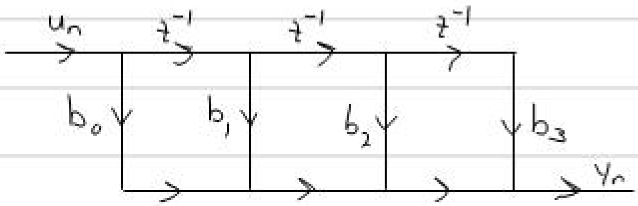
DF II DIFFERENCE EQUATIONS:



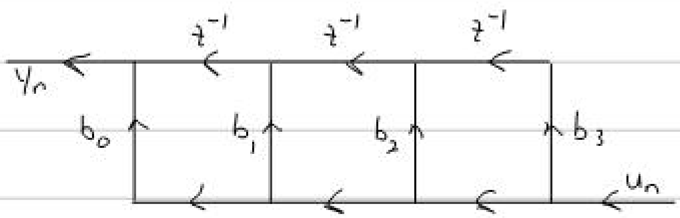
$$d_n = u_n + a_1 d_{n-1} + a_2 d_{n-2}$$
$$y_n = b_0 d_n + b_1 d_{n-1} + b_2 d_{n-2}$$

TRANSPOSED FIR FILTER:

ORIGINAL



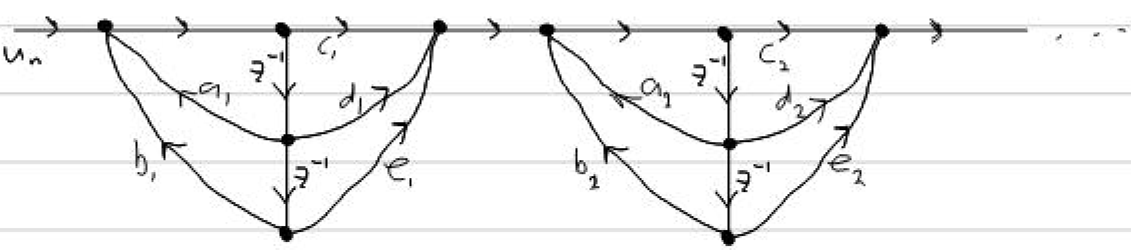
TRANSPOSE:



← CAN BE IMPLEMENTED IN PARALLEL

CASCADE FORMS: FACTOR THE TRANSFER FUNCTION NUMERATOR & DENOMINATOR INTO FIRST- & SECOND-ORDER FACTORS.

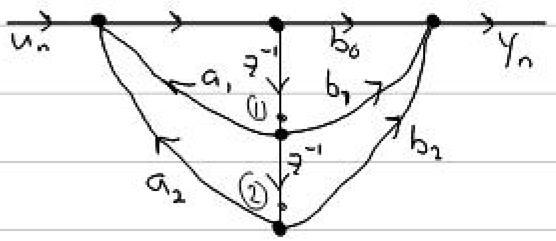
E.G.:



– NOT UNIQUE!! – NORMAL WAY OF IMPLEMENTING A HIGH-ORDER FILTER.

PARALLEL FORMS: SPLIT TRANSFER FUNCTION INTO PARTIAL FRACTIONS.

State Equations for DF II:



$$x_1(n+1) = u_n + a_1 x_1(n) + a_2 x_2(n)$$

$$x_2(n+1) = x_1(n)$$

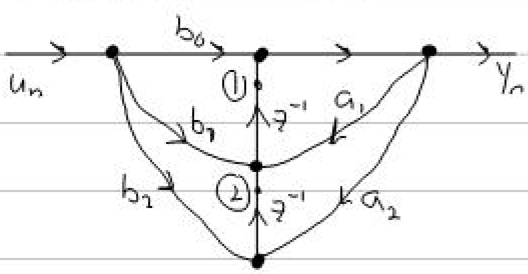
$$y_n = b_2 x_2(n) + b_1 x_1(n) + b_0\left(u_n + a_1 x_1(n) + a_2 x_2(n)\right)$$

Matrix Form:

$$\begin{pmatrix} x_1(n+1) \\ x_2(n+1) \end{pmatrix} = \begin{pmatrix} a_1 & a_2 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1(n) \\ x_2(n) \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u_n$$

$$y_n = \left(b_0 a_1 + b_1,\ b_0 a_2 + b_2\right)\begin{pmatrix} x_1(n) \\ x_2(n) \end{pmatrix} + b_0 u_n$$

Transposed DF II:



$$x_1(n+1) = b_1 u_n + x_2(n) + a_1\left(b_0 u_n + x_1(n)\right)$$

$$x_2(n+1) = b_2 u_n + a_2\left(b_0 u_n + x_1(n)\right)$$

$$y_n = b_0 u_n + x_1(n)$$

Matrix Form:

$$\underline{x}(n+1) = \begin{pmatrix} a_1 & 1 \\ a_2 & 0 \end{pmatrix}\underline{x}(n) + \begin{pmatrix} b_0 a_1 + b_1 \\ b_0 a_2 + b_2 \end{pmatrix} u_n$$

$$y_n = (1 \quad 0)\,\underline{x}(n) + b_0 u_n$$

ALGORITHM FOR TDF II:

```
LOOP FOREVER
    { INPUT UN;
      T1 = a1 * X1 + X2 + k1*UN      // k_1 = b_0 a_1 + b_1
      T2 = a2 * X1      + k2* UN     // k_2 = b_0 a_2 + b_2
      YN =      X1 + b0* UN
      OUTPUT YN
      X1 = T1
      X2 = T2
    }
```

STATE EQUATIONS:
$$\underline{x}(n+1) = A\underline{x}(n) + Bu_n$$
$$y_n = C\underline{x}(n) + d u_n$$

Z-TRANSFORM:
$$z\underline{X}(z) - z\underline{x}(0) = A\underline{X}(z) + B\,U(z)$$
$$Y(z) = C\underline{X}(z) + d\,U(z)$$

$$\Rightarrow \quad (zI - A)\underline{X}(z) = B\,U(z) + z\underline{x}(0)$$

$$\Rightarrow \quad \underline{X}(z) = (zI-A)^{-1}B\,U(z) + z(zI-A)^{-1}\underline{x}(0)$$

$$\Rightarrow \quad Y(z) = \left[C(zI-A)^{-1}B + d\right]U(z) + z\,C(zI-A)^{-1}\underline{x}(0)$$

FOR TRANSFER FUNCTION, $\underline{x}(0) = 0$.

THEN
$$H(z) = C(zI-A)^{-1}B + d$$

EXAMPLE: TDF II:

$$\underline{x}(n+1) = \begin{pmatrix} a_1 & 1 \\ a_2 & 0 \end{pmatrix} \underline{x}(n) + \begin{pmatrix} b_0 a_1 + b_1 \\ b_0 a_2 + b_2 \end{pmatrix} u_n$$

$$y_n = (1 \quad 0)\, \underline{x}(n) + b_0 u_n$$

$$\Longrightarrow \quad H(z) = (1 \quad 0) \begin{pmatrix} z - a_1 & -1 \\ -a_2 & z \end{pmatrix}^{-1} \begin{pmatrix} b_0 a_1 + b_1 \\ b_0 a_2 + b_2 \end{pmatrix} + b_0$$

$$= \frac{1}{z^2 - a_1 z - a_2} (1 \quad 0) \begin{pmatrix} z & 1 \\ a_2 & z - a_1 \end{pmatrix} \begin{pmatrix} b_0 a_1 + b_1 \\ b_0 a_2 + b_2 \end{pmatrix} + b_0$$

$$= \frac{1}{z^2 - a_1 z - a_2} (z \quad 0) \begin{pmatrix} b_0 a_1 + b_1 \\ b_0 a_2 + b_2 \end{pmatrix} + b_0$$

$$= \frac{\cancel{b_0 a_1 z} + b_1 z + \cancel{b_0 a_2} + b_2 + b_0 z^2 - \cancel{b_0 a_1 z} - \cancel{b_0 a_2}}{z^2 - a_1 z - a_2}$$

$$= \frac{b_0 z^2 + b_1 z + b_2}{z^2 - a_1 z - a_2}$$

$$= \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}}$$

ORIGINAL SFG: $\begin{pmatrix} A & \underline{b} \\ \underline{c}^T & d \end{pmatrix}$; TRANSPOSED: $\begin{pmatrix} A^T & \underline{c} \\ \underline{b}^T & d \end{pmatrix}$

So

$$H_{ORIGINAL}(z) = \underline{c}^T (zI - A)^{-1} \underline{b} + d$$

$$\text{q} \quad H^T_{ORIGINAL}(z) = \underline{b}^T (zI - A^T)^{-1} \underline{c} + d = H_{TRANSPOSED}(z)$$

$$\|$$

$H_{ORIGINAL}(z)$ FOR SISO.