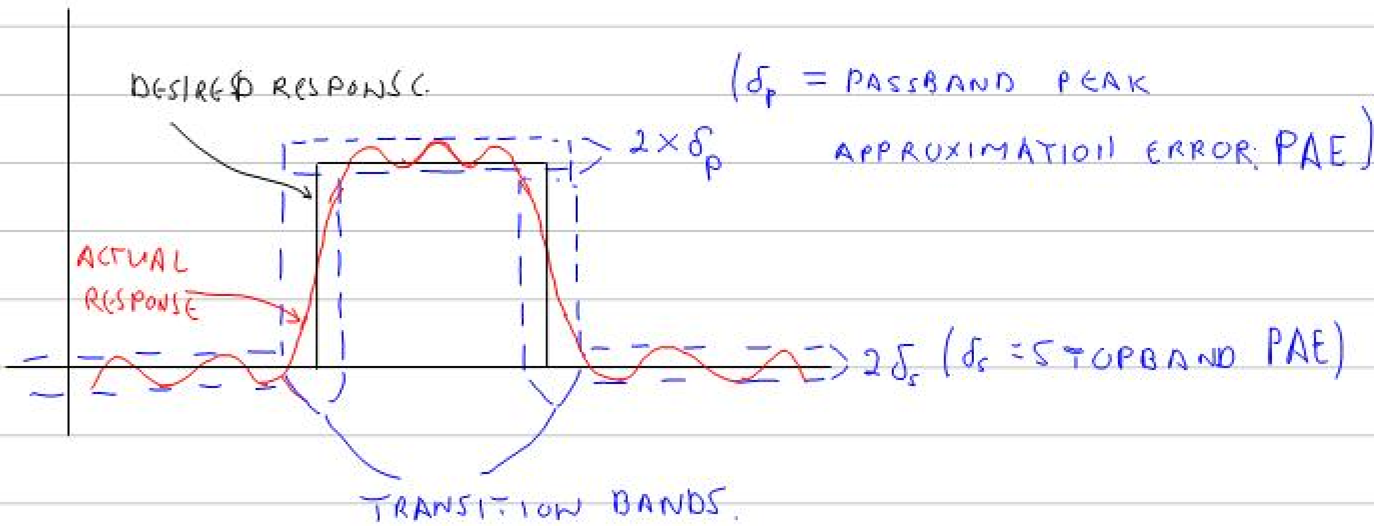
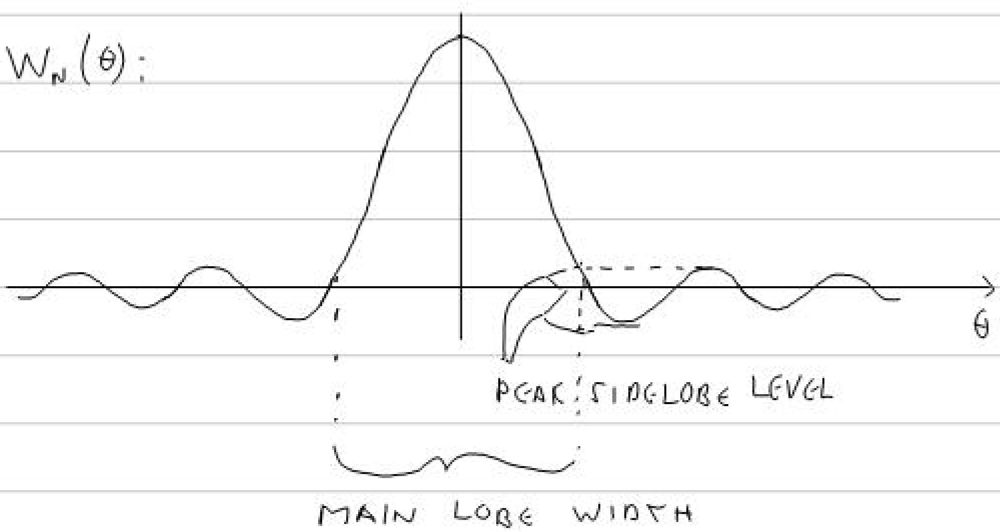
FILTER SPECIFICATION:



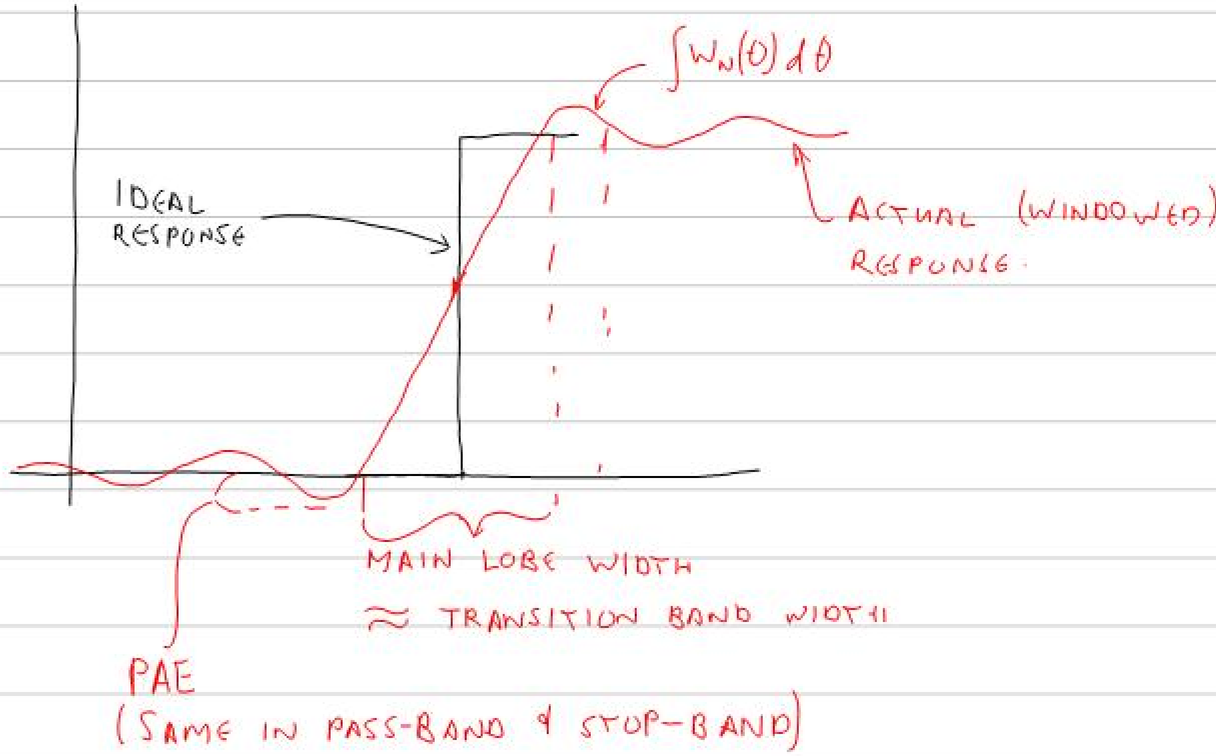
SPECIFICATIONS: δ_p, δ_s, STOPBAND EDGE(S), PASSBAND EDGE(S)

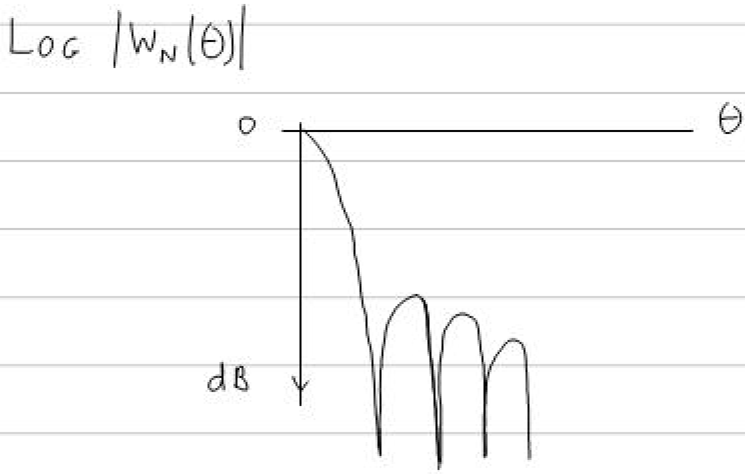
WINDOW FUNCTIONS (GENERAL):

$W_N(\theta)$:



EFFECT OF WINDOW FUNCTION:





FOR A WINDOWED FIR DESIGN, WE TAKE THE CUTOFF FREQUENCY OF THE ORIGINAL IDEAL FILTER TO BE HALF-WAY BETWEEN THE PASS-BAND EDGE & THE STOP-BAND EDGE.

SO FOR THE PREVIOUS EXAMPLE:

$$\theta_c = \frac{\theta_p + \theta_r}{2} = \frac{220.5}{441}\pi = \frac{\pi}{2}$$

SO THE IDEAL RESPONSE WOULD BE:

$$H_{IDEAL}(\theta) = \begin{cases} 1 & |\theta| < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < |\theta| \leq \pi \end{cases}$$

THEN THE IDEAL IMPULSE RESPONSE IS (WITH $\alpha = \frac{\pi}{2}$)

$$h_{IDEAL}(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{IDEAL}(\theta)\, e^{jn\theta}\, d\theta$$

$$= \frac{1}{2\pi}\int_{-\alpha}^{\alpha} e^{jn\theta}\, d\theta$$

FOR $n = 0$: $$h_{IDEAL}(0) = \frac{\alpha}{\pi}$$

FOR $n \neq 0$: $$h_{IDEAL}(n) = \frac{1}{2\pi jn}\left(e^{jn\alpha} - e^{-jn\alpha}\right)$$

$$= \frac{1}{n\pi}\sin(n\alpha)$$

SO $$h_{IDEAL}(n) = \begin{cases} \frac{1}{2} & n = 0 \\ \frac{1}{n\pi}\sin\left(n\frac{\pi}{2}\right) & n \neq 0 \end{cases}$$

LOW-ORDER EXAMPLE:

IN A SYSTEM WITH $f_s = 24\,kHz$, FIND COEFFICIENTS FOR A 7-COEFFICIENT SYMMETRIC LOW-PASS FIR FILTER WITH NOMINAL CUTOFF FREQUENCY $f_c = 4\,kHz$. USE A HAMMING WINDOW.

FIRST: $\theta_c = \frac{2\pi f_c}{f_s} = \frac{\pi}{3}$

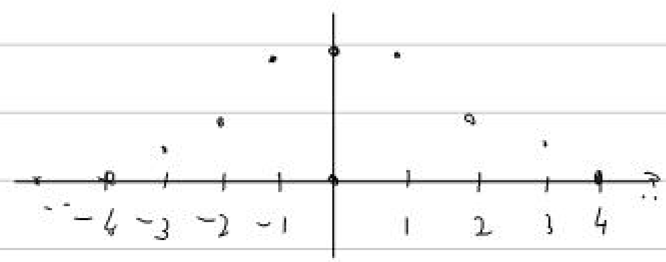
SO $$H_{IDEAL}(\theta) = \begin{cases} 1 & |\theta| < \frac{\pi}{3} \\ 0 & \frac{\pi}{3} < |\theta| \le \frac{\pi}{2} \end{cases}$$

THEN

$$h_{IDEAL}(n) = \begin{cases} \frac{1}{3} & n = 0 \\ \frac{1}{n\pi} \sin n\frac{\pi}{3} & n \neq 0 \end{cases}$$

WINDOW: 7 COEFFICIENTS:

$\Rightarrow N = 3$



THEN

$$w_3(n) = \begin{cases} 0.54 + 0.46 \cos \frac{n\pi}{3} & |n| \le 3 \\ 0 & |n| > 3 \end{cases}$$

WINDOWED IMPULSE RESPONSE:

$$h_{NC}(n) = \begin{cases} \frac{1}{3} & n = 0 \\ 0 & |n| > 3 \\ \left(0.54 - 0.46 \cos \frac{n\pi}{3}\right) \frac{1}{n\pi} \sin n\frac{\pi}{3} & 0 < |n| \le 3 \end{cases}$$

So: $h_{NC}(n)$

$n<-3$	0	
$n=-3$	0	$=h_0=0$
$n=-2$	$\left(0.54+0.46\cos\frac{2\pi}{3}\right)\frac{1}{2\pi}\sin\left(\frac{2\pi}{3}\right) = .0427$	$=h_1$
$n=-1$	$\left(0.54+0.46\cos\frac{\pi}{3}\right)\frac{1}{\pi}\sin\left(\frac{\pi}{3}\right) = .2123$	$=h_2$
$n=0$	$\frac{1}{3}$	$=h_3$
$n=1$	$h_{NC}(-1) = .0427$	$=h_4$
$n=2$	$h_{NC}(-2) = .2123$	$=h_5$
$n=3$	0	$=h_6=0$
$n>3$	0	

- ONLY 5 COEFFICIENTS.

IMPLEMENT: (DELAY BY 2):

$$y_n = h_1 u_n + h_2 u_{n-1} + \frac{1}{3}u_{n-2} + h_2 u_{n-3} + h_1 u_{n-4}$$

TRANSFER FUNCTION:

$$H(z) = h_1 + h_2 z^{-1} + \frac{1}{3}z^{-2} + h_2 z^{-3} + h_1 z^{-4}$$

$$= z^{-2}\left(\frac{1}{3} + h_2(z+z^{-1}) + h_1(z^2+z^{-2})\right)$$

FREQUENCY RESPONSE:

$$H(e^{j\theta}) = e^{-2j\theta}\left(\frac{1}{3}+2h_2\cos\theta + 2h_1\cos(2\theta)\right)$$

AMPLITUDE RESPONSE:

$$|H(e^{j\theta})| = \left|\frac{1}{3}+2h_2\cos\theta + 2h_1\cos(2\theta)\right|$$

PHASE RESPONSE:

$$\angle H(e^{j\theta}) = -2\theta \ (\pm\pi) \quad \text{- LINEAR FUNCTION OF } \theta$$

PHASE DELAY: $-\frac{\phi(\theta)}{\theta} = 2$; GROUP DELAY $= -\frac{d}{d\theta}(\phi(\theta)) = 2$

"LINEAR PHASE"