Digital Signal Processing

ESE 547

Slide 1

Notes:

Text: Mitra, S.J., “Digital Signal Processing: A Computer-Based Approach”, 3rd ed., 2006, McGraw-Hill

ISBN: 0-07-286546-6 (may be part of 0-07-304837-2, bundled with DSP lab experiments).

Part 1

System Overview

Sampled-Signal Processing Basics

Frequency-Domain and Transforms

Signal Sampling and Reconstruction

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Notes:
Chapter 1 in book; especially sec. 1.3, 1.4, 1.5.

1.1 System Overview

- Applications
- Drawbacks
- Advantages
- System Structure

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Notes:
See section 1.4;

Notes:

1.1.1 Applications

- Consumer:
  - CD
  - DAT
  - HDTV (Future)
  - Speech Recognition
- Military:
  - Radar
  - Sonar
  - Guidance
  - Fundamental Technology

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Notes:

Applications (Continued)

- Industrial:
  - Control
  - Medical Imaging
  - Medical Signals (EEG, ECG, EMG)
  - Geophysical Exploration
  - Nondestructive Testing
  - Mechanical Monitoring and Diagnosis
- COMMUNICATIONS

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1.1.2 Drawbacks

- Processing Bottlenecks
  - A/D
  - Processor
- High Initial Cost; justified by
  - High performance
  - High complexity
  - Leverage
  - Flexibility

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Notes:

See section 1.5;

Leverage refers to situations where expensive equipment is spread over many customers, for example, CD mastering, or HDTV generation.

1.1.3 Advantages

- Precision
- Flexibility
- Low incremental Cost
- High capability

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Notes:

An example of flexibility is the ability to modify processing by ROM changes; an example of low incremental cost is the logarithmic growth of requirements for resolution (1 bit doubles resolution), as against the exponential growth in analog.

Notes:
See section 4.1 (p.171)

1.1.4 Digital Signal Processing System Structure

- Anti-Aliasing Filter
- Sample-and-Hold (or Track-and-Hold)
- A-to-D Converter
- Digital Processor
- D/A converter
- Smoothing Filter

Filters and conversion: extremely important; will cover later.

Conversion process sets a limit on the signal bandwidth possible – usually $< 1/2$ sampling frequency.

Present focus on processing algorithms.

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1.2 Sampled-Signal Processing Basics

- Input-Output Description and Impulse Response
- FIR & IIR Systems
- Processing Elements and Difference Equations

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Notes:

See chapter 2, especially sections 2.1 and 2.2

Homework: Modify the MATLAB referred to on programs on pp. 63 – 64 to take a sampling period and time-length instead of “length of sequence”, and to take real frequency parameters (Hz); then use this to plot various sinusoids, exponentials, and exponential sinusoids, and to verify that aliasing occurs.

The next slide is of no practical value, but may clarify some fundamental system-theory concepts.

Notes:

1.2.1 Input-Output Description

Input: a sequence $u = (u_n)$; output $y = (y_n)$

$$y = B(u)$$

Linear:

$$\begin{aligned} B(u+v) &= B(u) + B(v) \\ B(\alpha u) &= \alpha B(u) \end{aligned}$$

for all constants α.

Time-Invariant:

If

$$(y_n) = B((u_n))$$

then

$$(y_{n+k}) = B((u_{n+k}))$$

for all k.

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Notes:

Input-Output Description (continued):

Fundamental fact: If the processing is *linear* and *time-invariant* then it must be a *convolution* – so convolution is the main task of DSP chips.

Convolution is defined by

$$(y_n) = (u_n) * (h_n)$$

where

$$y_n = \sum_{k=-\infty}^{\infty} h_k u_{n-k}$$

and h_n is the *impulse response* given by:

$$(h_n) = B((\delta_n))$$

where (δ_n) is the discrete-time unit (im)pulse:

$$\delta_n = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

Causal: $h_n = 0$ for $n < 0$.

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Notes:

1.2.2 FIR & IIR Systems

Special convolution cases:

1. FIR filtering – Finite Impulse Response, usually causal, simplest; a direct convolution whose impulse response is given by the filter coefficients:

$$\begin{aligned} y_n &= h_0 u_n + h_1 u_{n-1} + h_2 u_{n-2} \cdots + h_M u_{n-M} \\ &= \sum_{i=0}^{M} h_i u_{n-i} \end{aligned}$$

2. IIR filtering – Infinite Impulse Response, causal, more complex than FIR; uses past outputs:

$$\begin{aligned} y_n &= b_0 u_n + b_1 u_{n-1} + \cdots + b_M u_{n-M} \\ &\quad -a_1 y_{n-1} - a_2 y_{n-2} - \cdots - a_N y_{n-N} \\ &= \sum_{i=0}^{M} b_i u_{n-i} - \sum_{k=1}^{N} a_k y_{n-k} \end{aligned}$$

3. FFT – Fast Fourier Transform: usually noncausal, can be used for causal, usually FIR

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FIR & IIR Systems (continued):

FIR Terminology:

$$y_n = b_0 u_n + b_1 u_{n-1} + \cdots + b_M u_{n-M} = \sum_{i=0}^{M} b_i u_{n-i}$$

— Finite Discrete Convolution

— Transversal Filter

— Tapped Delay Line

Other Algorithms:

1. FFT is also widely used for spectral analysis and spectral estimation
2. Adaptive processing is nonlinear/time-varying
3. Many coding/modulation and decoding/demodulation and detection techniques are nonlinear.

Note: a DSP chip's architecture must have fast multiply-adds, efficient shifting (circular buffers), and zero-overhead (non-branching) looping; also usually has bit-reversed addressing.

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Notes:

1.2.3 Processing Elements and Difference Equations

For Linear Time-Invariant processing, the basic processing elements are:

- Unit Delays
- Constant gains
- Adders

Mathematically, interconnections of these elements give difference equations.

Schematically, we get block diagrams or signal flow graphs.

Note: z^{-1} is used to denote a unit delay, even in a time-domain block diagram or signal flow graph!

Examples: FIR: moving average filter:

$$y_n = 0.5u_n + 0.5u_{n-1}$$

IIR: (digital) integrator:

$$y_n = u_n + y_{n-1}$$

Block Diagrams: note that the FIR has no (directed) loops; if there are loops, they must have at least one delay.

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Notes:
See figures 8.2 – 8.4 (p.430).

Elementary sampled signals:

(Recall) unit (im)pulse:

$$\delta_n = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

Note: the unit impulse is an ordinary function in DSP. When simulating continuous-time systems, be careful about its normalization!

Unit step:

$$U_n = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Check unit step is impulse response of (digital) integrator.

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Notes:

Sampled Signal Processing Basics end here, and Frequency Domain and Z-Transforms begin.

1.3 Frequency-Domain and Transforms

- Sampled Sinusoids
- Frequency, Amplitude, and Phase Responses
- DTFT, Z-transforms and Properties
- Poles and Zeros

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Notes:

See Chapter 3; note that, for most purposes, the frequency-domain is more appropriate for describing *signals*, while the Z-transform is more appropriate for describing *systems* (or *operators which act on signals* to produce signals.

Notes:

1.3.1 Sampled Sinusoids

Continuous-time:

$$x(t) = A\cos(2\pi ft + \phi)$$

where A is the amplitude, and ϕ is the phase.

If signal is sampled every T seconds, the sampling instants are $t = nT$, and so the sampled (discrete-time) sinusoid is:

$$x_n = A\cos(2\pi fnT + \phi)$$

where A is the amplitude, and ϕ is the phase.

Amplitude: A (note: not necessarily peak value!)

Phase: ϕ

Sampling period: T

Sampling frequency: $f_S = 1/T$

Angular sampling frequency: $\omega_S = 2\pi/T$

Note: the sequence (x_n) is unchanged if the frequency f is changed to $f + k/T$, for any integer k (aliasing).

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Sampled Sinusoids (continued):

Sinusoid in complex form:

$$\begin{aligned} g_n &= Ae^{j(2\pi f_0 nT+\phi)} \\ &= Ae^{j\phi}\left(e^{j2\pi f_0 T}\right)^n \end{aligned}$$

where A is the amplitude, and ϕ is the phase.

For a decaying sinusoid:

Continuous-time:

$$g(t) = Ae^{\sigma_0 t}e^{j(2\pi f_0 t+\phi)}$$

Sampled:

$$\begin{aligned} g_n &= Ae^{\sigma_0 nT}e^{j(2\pi f_0 nT+\phi)} \\ &= Ae^{j\phi}z_0^n \end{aligned}$$

where

$$\begin{aligned} z_0 &= e^{(\sigma_0+j\omega_0)T} \\ &= e^{s_0 T} \end{aligned}$$

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Notes:
See sections 3.8 and 3.9, for the next 6 slides.

Notes:

Sampled Sinusoids (continued):

Consequence:

The relationship between the (continuous-time) s-domain and the (discrete-time) z-domain is given by:

$$z = e^{sT}$$

Restrict to frequency domain:

$$z = e^{j2\pi fT} = e^{j\theta}$$

So the discrete-time *angular frequency* is given by

$$\theta = 2\pi fT = \omega T$$

and the frequency information is on the unit circle.

Also, stability is equivalent to the absence of poles *on or outside the unit circle*.

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Notes:

1.3.2 Frequency, Amplitude, and Phase Responses

Suppose we have an FIR system with input $u_n = e^{j2\pi f n T}$. Then, with $\omega = 2\pi f$:

$$\begin{aligned} y_n &= b_0 e^{j\omega nT} + b_1 e^{j\omega(n-1)T} + \cdots + b_M e^{j\omega(n-M)T} \\ &= e^{j\omega nT}\left(b_0 + b_1 e^{-j\omega T} + \cdots + b_M e^{-Mj\omega T}\right) \\ &= e^{j\omega nT} K(\omega T) \end{aligned}$$

So if u_n is a (complex) sampled sinusoid of angular frequency ω,

$$\begin{aligned} y_n &= K(\omega T) e^{j\omega nT} \\ &= K(\omega T) u_n \end{aligned}$$

where the complex number $K(\omega T)$ is given by

$$K(\omega T) = b_0 + b_1 e^{-j\omega T} + \cdots + b_M e^{-Mj\omega T}$$

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Notes:

Frequency Response (continued)

For an IIR system, if we also assume that the output y_n is given by $y_n = Ke^{j2\pi fnT}$, where K is a complex constant:

$$Ke^{j\omega nT} + a_1 Ke^{j\omega(n-1)T} + \cdots + a_N Ke^{j\omega(n-N)T}$$
$$= b_0 e^{j\omega nT} + b_1 e^{j\omega(n-1)T} + \cdots + b_M e^{j\omega(n-M)T}$$

or

$$Ke^{j\omega nT}\left(1 + a_1 e^{-j\omega T} + \cdots + a_N e^{-Nj\omega T}\right)$$
$$= e^{j\omega nT}\left(b_0 + b_1 e^{-j\omega T} + \cdots + b_M e^{-Mj\omega T}\right)$$

so that

$$\begin{aligned} K(\omega T) &= H(e^{j\omega T}) \\ &= \frac{b_0 + b_1 e^{-j\omega T} + \cdots + b_M e^{-Mj\omega T}}{1 + a_1 e^{-j\omega T} + \cdots + a_N e^{-Nj\omega T}} \end{aligned}$$

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Notes:

Frequency Response (continued)

For a general convolution, (with $\omega = 2\pi f$)

$$\begin{aligned} y_n &= \sum_{k=0}^{\infty} h_k e^{j\omega(n-k)T} \\ &= e^{j\omega nT} \sum_{k=0}^{\infty} h_k e^{-j\omega kT} \\ &= e^{j\omega nT} H(e^{j\omega T}) \end{aligned}$$

The function $H(e^{j\omega T})$ is called the *frequency* response of the system;

The function $A(\omega T) = |H(e^{j\omega T})|$ is called the *amplitude* response;

The function $\phi(\omega T) = \angle H(e^{j\omega T})$ is called the *phase* response.

Note:

- These are periodic with period $2\pi/T = 2\pi f_S = \omega_S$
- They depend only on the *ratio* $f/f_S = \omega/\omega_S$

For this reason, digital signal processing often works with a normalized frequency f/f_S.

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Notes:

1.3.3 DTFT and Z-transforms

Discrete-Time Fourier Transform (DTFT):

Based on the formula for the frequency response, the Discrete-Time Fourier Transform of a (bounded) sequence (g_n), is defined by:

$$G(\theta) = \sum_{k=-\infty}^{\infty} g_k e^{-jk\theta}$$

The inverse transform is given by:

$$g_n = 1/2\pi \int_{-\pi}^{\pi} G(\theta) e^{jn\theta} d\theta$$

Note 1: *Boundednes* means that there is some constant M such that $|g_n| \leq M$ for all n.

Note 2: If (g_n) is the impulse response of a system, the frequency response of the system is given in terms of the DTFT of (g_n) by

$$G(e^{j2\pi fT}) = G(e^{j\theta})|_{\theta=2\pi fT}$$

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Notes:

DTFT and Z-transforms (continued)

Two-sided Z-transform:

The *two-sided* Z-transform of a bounded sequence (g_n) is defined by

$$\begin{aligned} G(z) &= \mathcal{Z}\{g_n\} \\ &= \sum_{n=-\infty}^{\infty} g_n z^{-n} \\ &= \cdots + g_{-1} z + g_0 + g_1 z^{-1} + g_2 z^{-2} + \cdots \end{aligned}$$

for all complex numbers z for which the series converges – assumed to include the unit circle.

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Two-Sided Z-transform properties:

- Linearity
- Shifting: $\mathcal{Z}\{g_{n-k}\} = z^{-k}G(z)$ for all k
- Convolution: If f_n and g_n are two bounded sequences, then $\mathcal{Z}\{(f_n) * (g_n)\} = F(z)G(z)$

— The Z-transform transforms convolution into multiplication.

Examples:

1. $a^n U_n$ with $|a| < 1$
2. $-a^n U_{-n-1}$ with $|a| > 1$
3. sinusoids

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Notes:

Notes:

DTFT and Z-transforms (continued)

Relation to frequency response and DTFT:

Frequency response is given by

$$K(2\pi fT) = \sum_{k=-\infty}^{\infty} h_k e^{-j2\pi kfT}$$

so that

$$\begin{aligned} K(2\pi fT) &= G(e^{j2\pi fT}) \\ &= G(e^{j\theta})|_{\theta=2\pi f/f_S} \\ &= G(z)|_{z=e^{j2\pi f/f_S}} \end{aligned}$$

The main use of the two-sided Z-transform is in calculating the inverse DTFT of functions which are rational in z.

The inverse DTFT of some other types of functions of θ is found by the direct inverse formula (will see later).

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Notes:

DTFT and Z-transforms (continued)

The *one-sided* Z-transform of a sequence (g_n) is defined by

$$\begin{aligned} G(z) &= \mathcal{Z}\{g_n\} \\ &= \sum_{n=0}^{\infty} g_n z^{-n} \\ &= g_0 + g_1 z^{-1} + g_2 z^{-2} + \cdots \end{aligned}$$

for all complex numbers z for which the series converges (region of convergence, or ROC).

Differs from the two-sided Z-transform and DTFT:

- assumes that (g_n) is one-sided
- does not assume that (g_n) is bounded
- therefore does not necessarily converge on unit circle
- converges in a region of the form $\{z \mid |z| > R\}$ for some R.
- handles initial conditions and transients.

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DTFT and Z-transforms (continued)

Relation to frequency response, for a causal, *stable* system :

As before, frequency response is given by

$$K(2\pi fT) = \sum_{k=0}^{\infty} h_k e^{-j2\pi fTk}$$

so that

$$\begin{aligned} K(2\pi fT) &= H(e^{j2\pi fT}) \\ &= H(z)|_{z=e^{j2\pi fT}} \end{aligned}$$

Here, stable means BIBO stable, which is equivalent to h_n being absolutely summable, i.e., $\sum_{k=0}^{\infty} |h_k|$ is finite (stronger than bounded).

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Notes:
See section 6.7 (p.335)

Notes:

DTFT and Z-transforms (continued)

If g_n is the impulse response, h_n, of a causal system, its one-sided Z-transform

$$H(z) = \mathcal{Z}\{h_n\} = \sum_{n=0}^{\infty} h_n z^{-n}$$

is called the *transfer function* of the system

Examples:

1. $a^n U_n$
2. sinusoids
3. exponentially decaying and growing sinusoids

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Notes:

DTFT and Z-transforms (continued)

(One-sided) Z-transform properties:

- Linearity
- Shifting: $\mathcal{Z}\{g_{n-k}\} = z^{-k}G(z) \qquad$ for $k \geq 0$
- Non-Causal Shift: $\mathcal{Z}\{g_{n+1}\} = zG(z) - zg_0$
- Convolution: If $f_n = 0$ and $g_n = 0$ for $n < 0$, then $\mathcal{Z}\{(f_n) * (g_n)\} = F(z)G(z)$

— *The Z-transform transforms convolution into multiplication.*

Note 1: The noncausal shifting property enables the Z-transform to handle initial conditions and transients.

Note 2: Because the signals are one-sided, the convolution here is given by:

$$p_n = \sum_{k=0}^{n} f_k g_{n-k}$$

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Notes:

DTFT and Z-transforms (continued)

Inverse one-sided Z-transform:

Theoretical formula (rarely used, actually a contour integral):

$$g_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} G(Re^{j\theta})(Re^{j\theta})^n \, d\theta$$

where R is such that all poles of $G(z)$ are inside the circle of radius R.

The normal method is to use partial fractions, but the calculations are simpler if $G(z)/z$, instead of $G(z)$, is written in terms of z (not z^{-1}) and is then expanded in partial fractions.

Examples:

1. $1/(1 - az^{-1})$ with $|a| < 1$
2. $1/(1 - az^{-1})$ with $|a| > 1$
3. real poles – stable
4. real poles – unstable
5. complex poles – stable and unstable
6. comples poles on unit circle – second-order oscillator; (see p.394)

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Notes:

1.3.4 Poles and Zeros

Terminology (for rational functions with no common factors between numerator and denominator):

1. Pole: a value of z where $H(z)$ is infinite, i.e., where the denominator is zero.
2. Simple Pole: a is a simple pole if there is a factor $(z - a)$ in the denominator, and no factor of $(z - a)^k$ with $k > 1$.
3. Multiple Pole: a is a multiple pole if there is a factor $(z - a)^k$ in the denominator, with $k > 1$.
4. Zero: a value of z where $H(z)$ is zero
5. Residue: the coefficient A in the term $A/(z - a)$ in the partial fraction expansion.

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Notes:

Poles and Zeros (continued)

Notes:

1. All rational functions can be expanded as product of first-order factors with complex coefficients.
2. All rational functions can be expanded as product of first- and second-order factors with real coefficients.
3. To have a partial fraction expansion, we must have $H(z) \to 0$ as $z \to \infty$
4. Partial fraction expansions get more complicated with multiple poles; the basic formulas can be found by differentiating the equation

$$\frac{1}{1 - az^{-1}} = \sum_{n=0}^{\infty} a^n z^{-n}$$

5. Knowing the poles gives us a good idea of what the impulse response will look like.
6. Knowing the poles and using the initial- and final-value theorems gives a good idea of the step response.
7. The poles and zeros (product expansion) can be used to give a good idea of the frequency response.

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Notes:

1.4 Signal Sampling and Reconstruction

- Theory
- Antialiasing and Smoothing Filters
- Signal Conversion

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Notes:

1.4.1 Sampling and Reconstruction Theory

Theoretically, sampling is a multiplication operation:

Give a sampl*ing* signal $s(t)$ which is periodic with period T, and a continuous-time signal $g(t)$, define the sampl*ed* signal $g_S(t)$ by

$$g_S(t) = s(t)g(t)$$

T is called the *sampling period* and $f_S = 1/T$ is the *sampling frequency.*

Normally, $s(t)$ will approximate a periodic sequence of δ-functions, and a further integration step is used to obtain a specific value.

Note that the sampled function is still a continuous-time function.

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Notes:

Sampling and Reconstruction Theory (continued)

For *ideal* sampling, we assume that $s(t)$ is a periodic sequence of δ-functions:

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

and the sampling operation then has two steps:

- Multiply by the sequence of δ-functions:

$$\begin{aligned} g_S(t) &= \sum_{n=-\infty}^{\infty} \delta(t - nT) g(t) \\ &= \sum_{n=-\infty}^{\infty} g(nT) \delta(t - nT) \end{aligned}$$

- Pick off numbers representing samples

$$g_n = g(nT)$$

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Notes:

Sampling and Reconstruction Theory (continued)

Effect of sampling in frequency domain:

$$G_S(2\pi jf) = \sum_{k=-\infty}^{\infty} c_k G(2\pi j(f - kf_S))$$

where c_k are the complex Fourier coefficients of the $s(t)$:

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} s(t)e^{-j2\pi kt/T}\, dt$$

Here $2\pi f_S = 2\pi/T = \omega_S$ is called the *sampling angular frequency.*

Also, $f_S/2$ is sometimes called the *folding frequency*.

For ideal sampling, we get

$$G_S(2\pi jf) = 1/T \sum_{k=-\infty}^{\infty} G(2\pi j(f - kf_S))$$

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Notes:

Sampling and Reconstruction Theory (continued)

Classical Sampling Criterion:

Must Sample at a rate higher than the *Nyquist rate* $= 2f_{MAX}$

WARNING!1! Nonlinear operations internal to the signal processing can increase the bandwidth, and require much higher sampling rates.

WARNING!2! Aliased components are not just a mathematical abstraction, but very real; you can see them on an oscilloscope, and hear them.

WARNING!3! Higher sampling rates can also be needed to reduce phase-shift, for example, in a feedback loop.

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Sampling and Reconstruction Theory (continued)

To reconstruct: use lowpass filter

Reconstruction is also possible in other cases:

- Bandpass
- Periodic

In time-domain:

$$g_R(t) = \sum_{n=-\infty}^{\infty} g(nT)l(t - nT)$$

where $l(t)$ is the impulse response of the reconstruction (smoothing) filter.

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Notes:

For a band-pass signal in the band $f_L \leq f \leq f_H$, any sampling frequency, f_S which allows ideal reconstruction satisfies, for some integer $k \geq 0$

$$\begin{aligned} kf_S &\leq 2f_L \\ (k+1)f_S &\geq 2f_H \end{aligned}$$

Then the possible sampling frequencies are given by

$$\frac{2f_H}{k+1} \leq f_S \leq \frac{2f_L}{k}$$

for each integer $k \geq 0$ for which the interval is non-empty, where the second inequality is assumed to vanish for $k = 0$ (the broad-band case).

These intervals are disjoint, since otherwise $2f_L/k \geq 2f_H/((k-1)+1)$ would imply $f_L \geq f_H$.

The integers k for which the intervals are non-empty must satisfy

$$\frac{2f_H}{k+1} \leq \frac{2f_L}{k}$$

or, equivalently,

$$k(f_H - f_L) \leq f_L$$

and so

$$0 \leq k \leq k_0 = \left\lfloor \frac{f_L}{f_H - f_L} \right\rfloor$$

The smallest such sampling frequency is given taking the largest k, and the smallest frequency in the corresponding interval:

$$f_{S(min)} = \frac{2f_H}{k_0 + 1}$$

See section 4.10 (p.221)

Notes:

Sampling and Reconstruction Theory (continued):

"Ideal" reconstruction: ideal low-pass filter with theoretical gain T; unrealizable since

$$\begin{aligned} l(t) &= \frac{\sin(\pi t/T)}{\pi t/T} \\ &= \operatorname{sinc}(t/T) \end{aligned}$$

(Note that this vanishes at all sample points except $t = 0$, and so gives the correct value at the sample points.)

Actual D/A conversion: zero-order hold followed by smoothing filter. Frequency response of zero-order hold is given by

$$\begin{aligned} H(f) &= e^{-j\pi f/f_S} \frac{\sin(\pi f/f_S)}{\pi f/f_S} \\ &= e^{-j\pi x} \frac{\sin(\pi x)}{\pi x} \end{aligned}$$

where x is the normalized frequency.

It follows that the zero-order hold has a group delay of $T/2$, and has a rough lowpass characteristic with infinite attenuation at non-zero multiples of the sampling frequency, and an attenuation of $2/\pi$ or about 3.9 dB at $f_S/2$.

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Notes:

1.4.2 Antialiasing (Input) and Smoothing (Output) Filters:

- Needed from sampling theorem
- To reject spurious signals and noise at input
- To reject aliased components at output

Problem: Need high order *analog* filter, especially at output

Can alleviate requirements at output:

- Upconvert, filter digitally, high-rate D/A converter,
- Upconvert, high-rate D/A converter, switched-capacitor filter, final simple smoothing filter.

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Notes:

Antialiasing and Smoothing Filters (continued)

Can also alleviate requirements at input:

- Gentle analog filter, very high sampling rate – low accuracy
- Use high-frequency, low word-width Digital lowpass filter
- Use DSP to Down-convert to low sampling rate, high word-width – noise-shaping
- Σ-Δ converters.

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Notes:

1.4.3 Signal Conversion

Track and Hold Amplifier:

- Tracks signal until strobed, and holds constant value until released.
- Stores charge on capacitor
- Properties
  - Aperture
  - Acquisition time
  - Step
  - Droop
  - Jitter

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Notes:

Signal Conversion (continued)

A/D Converters:

- Successive Approximation
- Flash
- Sigma-Delta
- Dual-Slope, Counting, usually too slow

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Signal Conversion (continued)

D/A Converters:

Switched current source; numerous other types.

Virtually always use zero-order hold.

Zero-order hold amplitude response: “$\sin(x)/x$”

Zero-order hold delay: 1/2 sample period.

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Notes:
Example:
What are the requirements for the smoothing filter in a CD player?