Absorption of light (in bulk material)

Direct transitions

\[ h\nu = E_g + E_h(\tilde{k}) + E_e(\tilde{k}') \]
\[ \tilde{k}' = \tilde{k} + \tilde{\kappa} \approx \tilde{k} \]
\[ \hbar\tilde{\kappa} \text{ - photon momentum } \approx 0 \]

Indirect transitions

\[ h\nu = E_g + E_h(\tilde{k}) + E_e(\tilde{k}') + E_q \]
\[ \tilde{k}' = \tilde{k} + \tilde{q} \]
\[ E_q \text{ and } \tilde{q} \text{ - phonon energy and momentum} \]
Absorption of light (in bulk material)

\[ \alpha (\text{cm}^{-1}) \propto \rho(E) \propto \sqrt{\hbar \nu - E_g} \]

* direct transitions

\[ \rho \] – density of states

\[ E = E_e + E_h = (\hbar \nu - E_g) \] from energy conservation law
Absorption and amplification of light

Nonequilibrium case: electrons and holes are created by injection

The photons with energy \( h\nu > E_g + (E_e)_F + (E_h)_F \) can be absorbed
Absorption and amplification of light

Nonequilibrium case: electrons and holes are created by injection

Absorption:
\[ \alpha \propto \rho(k) \cdot \left\{ \left[ 1 - f_h \left( E_h(k) \right) \right] \cdot \left[ 1 - f_e \left( E_e(k) \right) \right] - \left( f_e \cdot f_h \right) \left( E_h(k) \right) \right\} \]

Stimulated emission:
\[ \alpha \propto \rho(k) \cdot \left\{ \left[ 1 - f_h \left( E_h(k) \right) \right] - \left( f_e \cdot f_h \right) \left( E_h(k) \right) \right\} \]

\[ h\nu = E_g + E_e(k) + E_h(k), \quad E_e(k) = \frac{\hbar^2 k^2}{2m_e}, \quad E_h(k) = \frac{\hbar^2 k^2}{2m_h} \]

\[ \rho(k) \propto k \propto E \propto \sqrt{hv - E_g} \]

Light intensity as a function of the coordinate in the media with \( \alpha \) (\( J_0 \) – intensity at the input)
\[ J(x) = J_0 \cdot e^{-\alpha \cdot x} \]

Absorption: \( \alpha > 0 \quad \left( 1 - f_h - f_e \right) > 0 \quad \text{or} \quad \left( f_e + f_h \right) < 1 \quad \text{or} \quad (E_e)_F + (E_h)_F + E_g < h\nu \)

Amplification: \( \alpha < 0 \quad \left( 1 - f_h - f_e \right) < 0 \quad \text{or} \quad \left( f_e + f_h \right) > 1 \quad \text{or} \quad (E_e)_F + (E_h)_F + E_g > h\nu \)

\( (-\alpha) \equiv G \) – material gain

Condition \( \left( f_e + f_h \right) > 1 \) is called population inversion condition and can be rewritten as
\[ f_e^{CB} - (1 - f_h)^{VB} > 0 \quad \text{or} \quad f_e^{CB} > f_e^{VB} \]
Absorption and amplification of light

\[ \alpha \propto \sqrt{h\nu - E_g} \]

\( T > 0 \)

the region of light absorption \((h\nu_2)\)

\( (f_h + f_e) < 1 \)

\[ h\nu = (E_e)_F + (E_h)_F + E_g \]

the region of light amplification \((h\nu_1)\)

\( (f_e + f_h) > 1 \)
Semiconductor Quantum Well (QW)

Quantum well (QW): thin layer of the semiconductor between two barriers with the width $L_W$ of the order of the electron wavelength ($L_W \approx \lambda = 2\pi k = \hbar / m_e^* V_e$, i.e. in the range 50-200Å). QW is usually originated if the $E_g$ of inner semiconductor is smaller than $E_g$ of semiconductor comprising barrier layers.

The QW potential reminds the atomic potential. The energy of carriers and their movement along axis OZ are quantized. The movement of carriers in the layer plane is free. The first subband electron energy can be written for $L_W >>> a$:

$$E_{e1} \approx \frac{\hbar^2}{2m_e^*} \left( \frac{\pi}{L_W} \right)^2; \quad E_e(k_{||}) = E_{e1} + \frac{\hbar^2 k_{||}^2}{2m_e^*}$$
Energy of quantization

For infinite barrier energy

\[ \frac{2 \cdot L_w}{\lambda_{\text{electron}}} = m \quad m = 1, 2, 3 \ldots \]
\[ \lambda_{\text{electron}} = 2 \cdot L_w \cdot m \]

\[ k_z = \frac{2\pi}{\lambda_{\text{electron}}} = \frac{\hbar \cdot \pi}{L_w} \cdot m \]

\[ E_{e_m} = \frac{\hbar^2 k_z^2}{2m_e} = \frac{\hbar^2}{2m_e} \cdot \left( \frac{\pi}{L_w} \right)^2 m^2 \quad m = 1, 2, 3 \ldots \]
Density of states of 2D carriers in QW

In $k$-space for layer with radius $k_\parallel$ and thickness $dk_\parallel$ the number of electron states can be written as:

$$dN_s = \frac{2(\text{spin}) \cdot 2\pi \cdot k_\parallel \cdot dk_\parallel}{(2\pi^2)^2 \frac{L_x L_y}{S}}$$

- the area per one state in $k$-space
- $(L_x, L_y$ are crystal dimensions, $S=L_x L_y$ is area of QW)

In terms of energy: $E_{em} = \frac{\hbar^2 k_\parallel^2}{2m^*_e} + E_{e1} \cdot m^2 \quad m = 1,2,3...$

The density of states for first subband: $\rho_{e1} = \frac{1}{S} \cdot \frac{dN_s}{dE_{e1}} = \frac{m^*_e}{\pi \cdot \hbar^2} , \left[ \frac{1}{cm^2 eV} \right]$

For $m$ subbands: $\rho = \sum_{m=1}^{m} \rho_{em} = \frac{m^*_e}{\pi \cdot \hbar^2} \sum_{m} \Theta(E - E_{em})$

The density of states per unit volume is $\rho_v = \rho/L_W$
Density of states in QW

\[ \rho = \sum_{m=1}^{\infty} \rho_{e_m} = \frac{m_e^*}{\pi \cdot \hbar^2} \sum_{m} \Theta(E - E_{e_m}) \]

\[ \rho_{e_1} = \frac{1}{S} \cdot \frac{d\mathcal{N}_s}{dE_{e_1}} = \frac{m_e^*}{\pi \cdot \hbar^2} \left[ \frac{1}{\text{cm}^2 \text{eV}} \right] \]
Absorption of light in QW

Selection rule: $\Delta m=0$

$$E_{em} = \frac{\hbar^2}{2m_e} \left( \frac{\pi}{L_W} \right)^2 m^2, \quad m = 1, 2, \ldots$$

$$E_{hm} = \frac{\hbar^2}{2m_h} \left( \frac{\pi}{L_W} \right)^2 m^2, \quad m = 1, 2, \ldots$$

Energy conservation: $E_g + E_{em} + E_{hm} = h\nu$

Momentum conservation: $k' = k + \kappa \approx k, \kappa \ll k$
Absorption of light in QW

Nonequilibrium case: there are injected electrons and holes

Absorption:

\[
\alpha = \rho(E) \cdot \left\{ 1 - f_e \left( E_e (k \|) \right) - f_h \left( E_h (k \|) \right) \right\}
\]

Absorption: \( \alpha > 0 \) \( (1 - (f_h + f_e)) > 0 \) or \( (f_e + f_h) < 1 \) for \( h\nu > E_g + E_{e1} + E_{h1} + (E_e)_F + (E_h)_F \)

Amplification: \( \alpha < 0 \) \( (1 - (f_h + f_e)) < 0 \) or \( (f_e + f_h) > 1 \) for \( h\nu < E_g + E_{e1} + E_{h1} + (E_e)_F + (E_h)_F \)