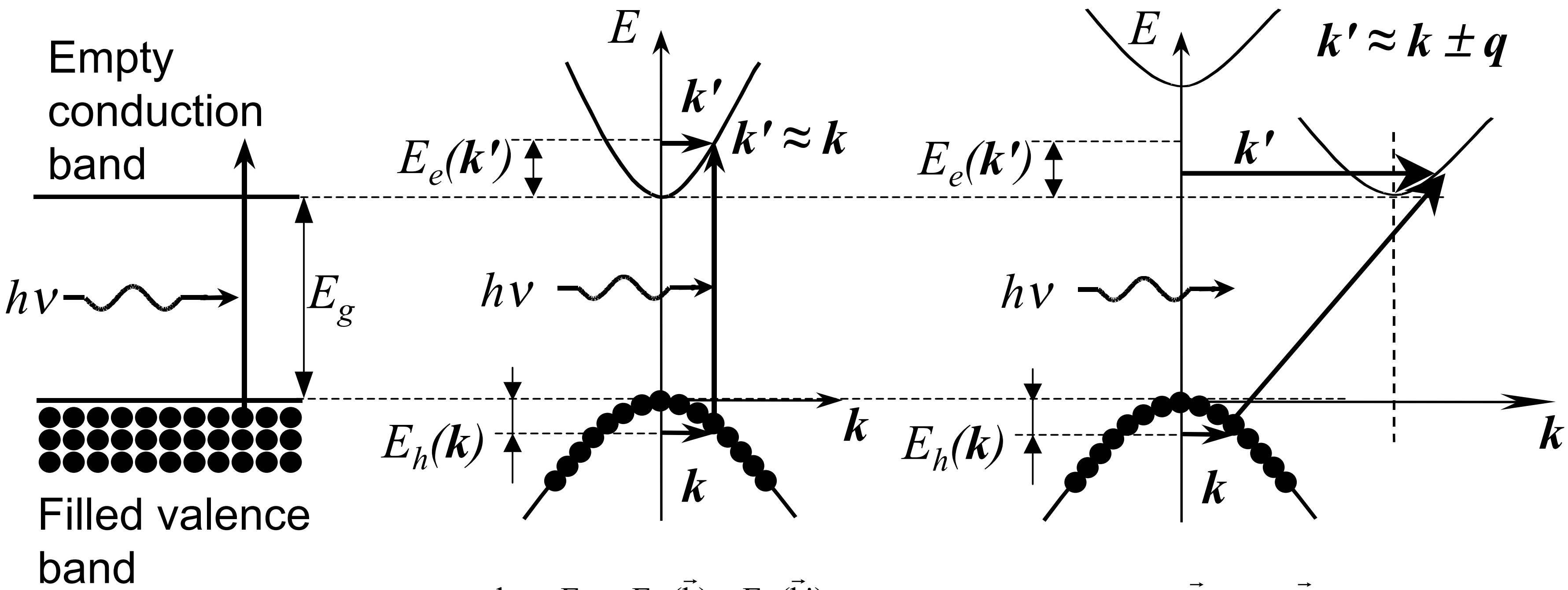
Absorption of light (in bulk material)

Direct transitions | Indirect transitions



$$h\nu = E_g + E_h(\vec{k}) + E_e(\vec{k}')$$

$$\vec{k}' = \vec{k} + \vec{\kappa} \approx \vec{k}$$

$\hbar\vec{\kappa}$ - photon momentum ≈ 0

$$h\nu = E_g + E_h(\vec{k}) + E_e(\vec{k}') \pm E_q$$

$$\vec{k}' = \vec{k} + \vec{q}$$

E_q and $\vec{q}$ - phonon energy and momentum

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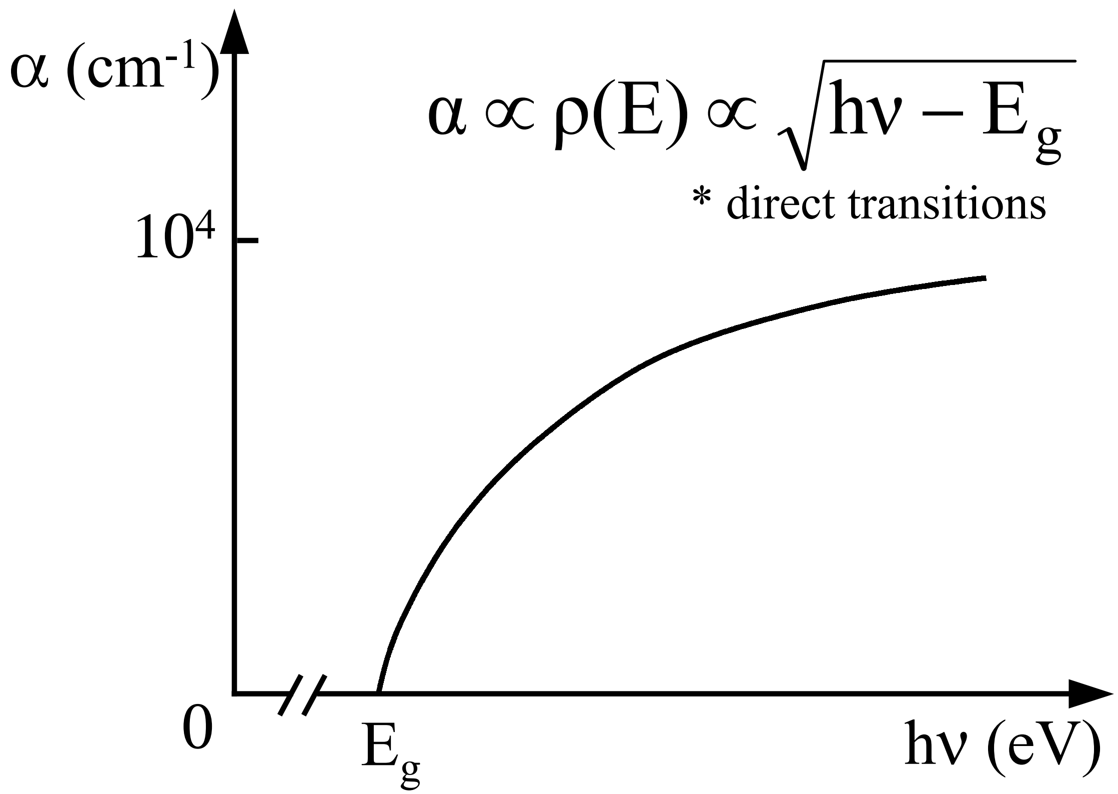
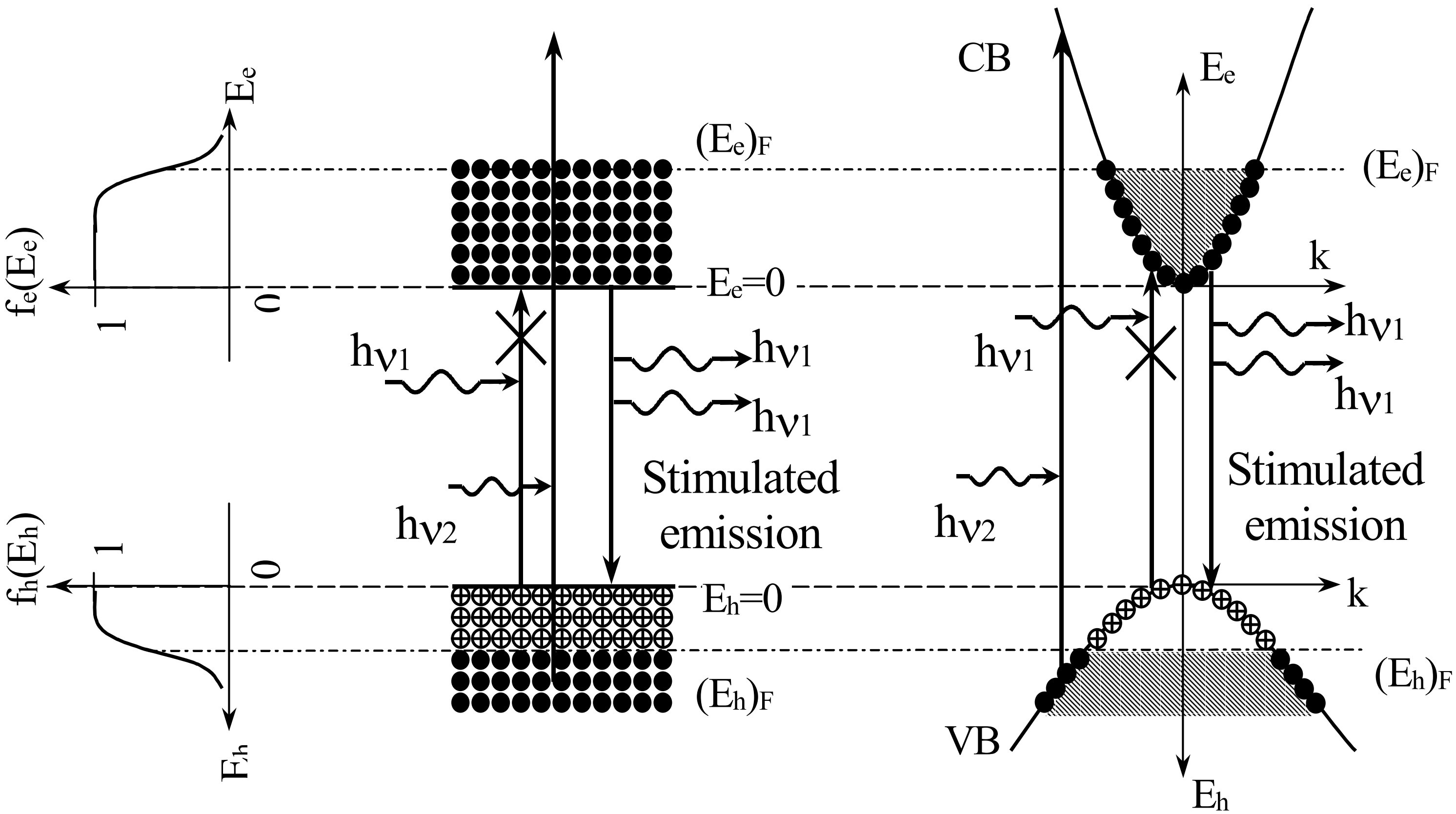


ρ – density of states

$\mathbf{E=E_e+E_h=(h\nu-E_g)}$ from energy conservation law

Absorption and amplification of light

Nonequilibrium case: electrons and holes are created by injection



The photons with energy $\mathbf{h\nu > E_g + (E_e)_F + (E_h)_F}$ can be absorbed

Absorption and amplification of light

Nonequilibrium case: electrons and holes are created by injection

absorption *stimulated emission*

$$\alpha \propto \rho(k)\cdot\{[1-f_h(E_h(k))]\cdot[1-f_e(E_e(k))]-f_e(E_e(k))\cdot f_h(E_h(k))\}$$

$$\alpha \propto \rho(k)\cdot\{1-f_h(E_h(k))-f_e(E_e(k))\}$$

$$h\nu = Eg + E_e(k) + E_h(k),\quad E_e(k)=\frac{\hbar^2 k^2}{2m_e},\quad E_h(k)=\frac{\hbar^2 k^2}{2m_h}$$

$$\rho(k)\propto k \propto \sqrt{E} \propto \sqrt{h\nu - E_g}$$

Light intensity as a function of the coordinate in the media with α (J_0 – intensity at the input)

$$J(x) = J_0 \cdot e^{-\alpha\cdot x}$$

Absorption: $\alpha > 0 \quad (1-f_h-f_e)>0 \quad \text{or} \quad (f_e+f_h)<1 \quad \text{or} \quad (E_e)_F+(E_h)_F+E_g<h\nu$

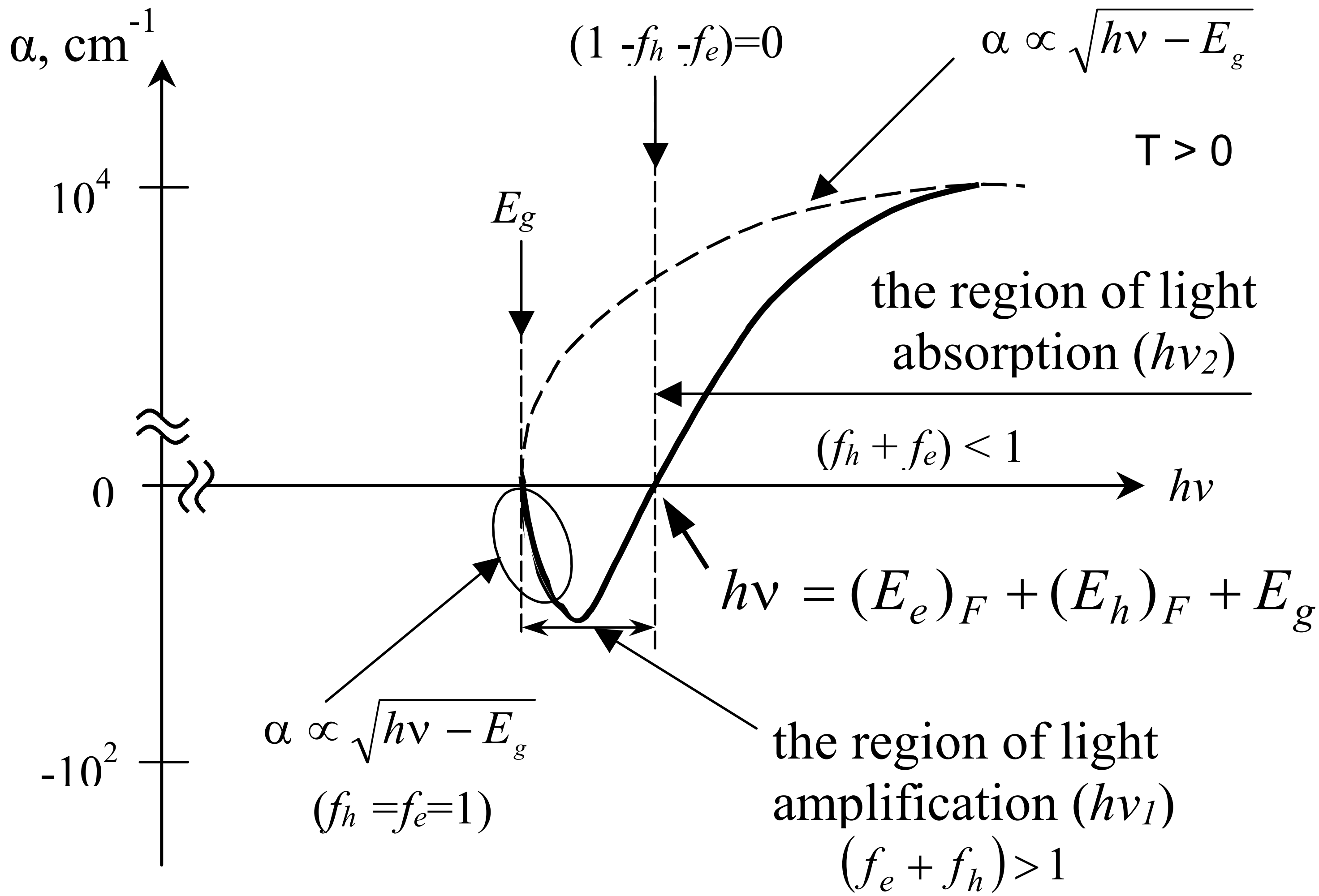
Amplification: $\alpha < 0 \quad (1-f_h-f_e)<0 \quad \text{or} \quad (f_e+f_h)>1 \quad \text{or} \quad (E_e)_F+(E_h)_F+E_g>h\nu$

$(-\alpha) \equiv G$ – material gain

Condition $(f_e+f_h)>1$ is called population inversion condition and can be rewritten as

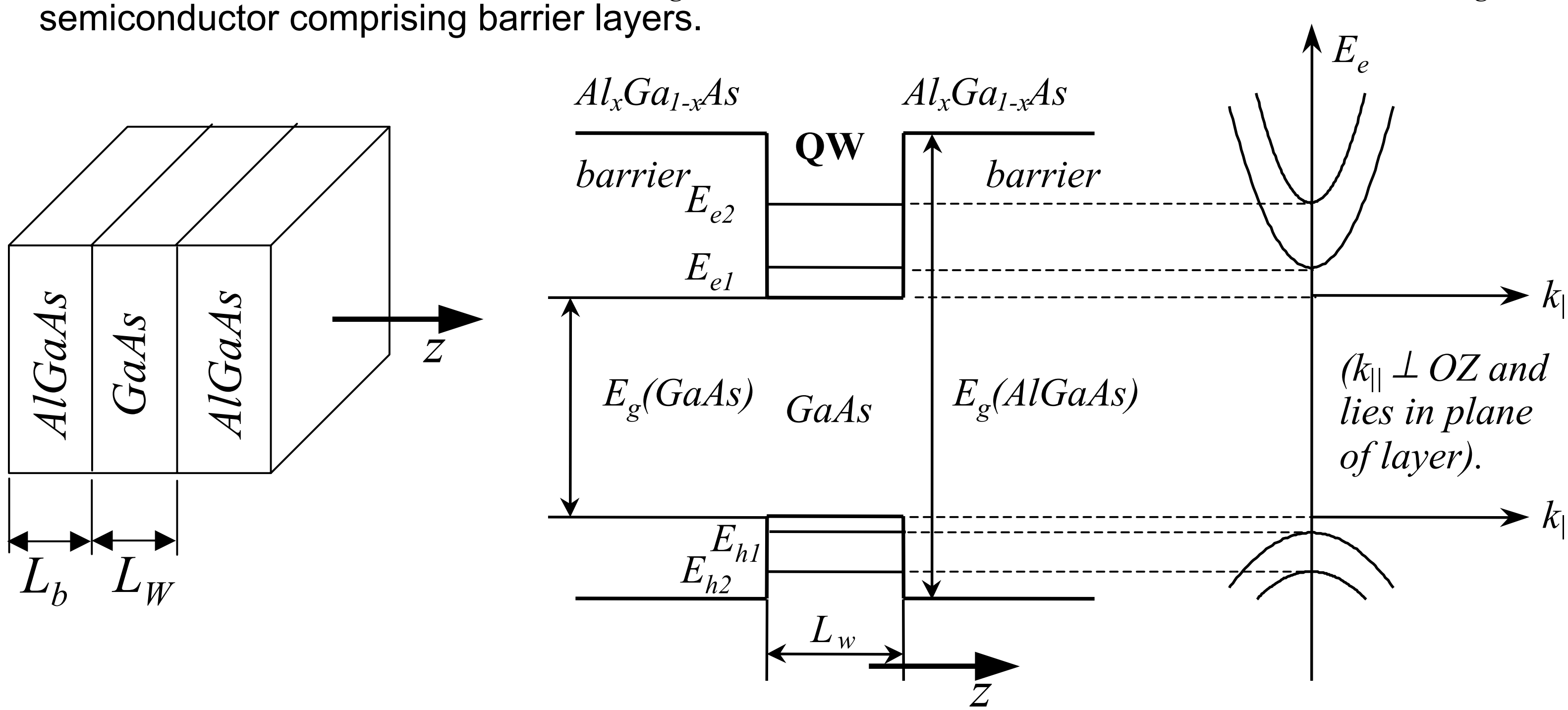
$$f_e^{CB} - (1-f_h)^{VB} > 0 \quad \text{or} \quad f_e^{CB} > f_e^{VB}$$

Absorption and amplification of light



Semiconductor Quantum Well (QW)

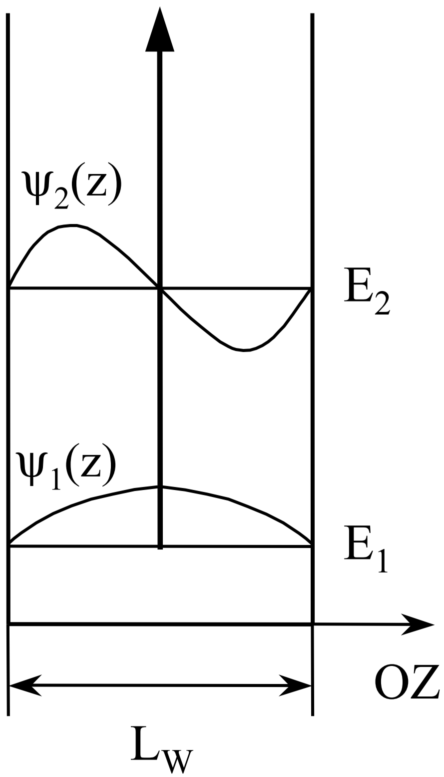
Quantum well (QW): thin layer of the semiconductor between two barriers with the width L_W of the order of the electron wavelength ($L_W \approx \lambda=2\pi/k=h/m_e{*}V_e$, i.e. in the range 50-200A). QW is usually originated if the E_g of inner semiconductor is smaller than E_g of semiconductor comprising barrier layers.



The QW potential reminds the atomic potential. The energy of carriers and their movement along axis OZ are quantized. The movement of carriers in the layer plane is free. The first subband electron energy can be written for $L_W >> a$:

$$E_{e1} \approx \frac{\hbar^2}{2m_e{*}} \cdot \left(\frac{\pi}{L_W}\right)^2; \qquad E_e\left(k_{||}\right) = E_{e1} + \frac{\hbar^2 k_{||}^{\,2}}{2m_e{*}}$$

Energy of quantization



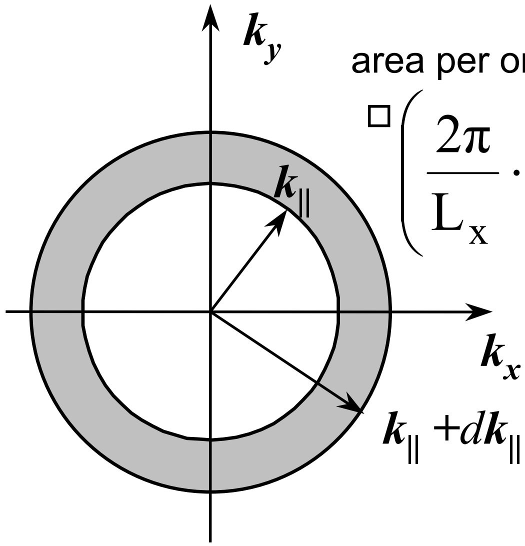
For infinite barrier energy

$$\frac{2\cdot L_W}{\lambda_{electron}} = m \quad m = 1,2,3... \qquad \lambda_{electron} = 2\cdot L_W \cdot m$$

$$k_z = \frac{2\pi}{\lambda_{electron}} = \hbar \cdot \frac{\pi}{L_w} \cdot m$$

$$E_{e\,m} = \frac{\hbar^2 k_z^2}{2m_e} = \frac{\hbar^2}{2m_e} \cdot \left(\frac{\pi}{L_W}\right)^2 m^2 \quad m = 1,2,3...$$

Density of states of 2D carriers in QW



area per one state □ $\left(\frac{2\pi}{L_x} \cdot \frac{2\pi}{L_y}\right)$

In $\boldsymbol{k}$-space for layer with radius $\boldsymbol{k}_{||}$ and thickness $d\boldsymbol{k}_{||}$ the number of electron states can be written as:

$$dN_s = \frac{2(\text{spin}) \cdot 2\pi \cdot k_{||} \cdot dk_{||}}{\frac{(2\pi)^2}{L_x L_y}}$$

$$\frac{4\pi^2}{L_x L_y} = \frac{4\pi^2}{S}$$ - the area per one state in $\boldsymbol{k}$-space (L_x, L_y are crystal dimensions, $S=L_xL_y$ is area of QW)

In terms of energy: $E_{em} = \frac{\hbar^2 k_{||}^2}{2m_e^*} + E_{e1} \cdot m^2 \quad m = 1,2,3...$

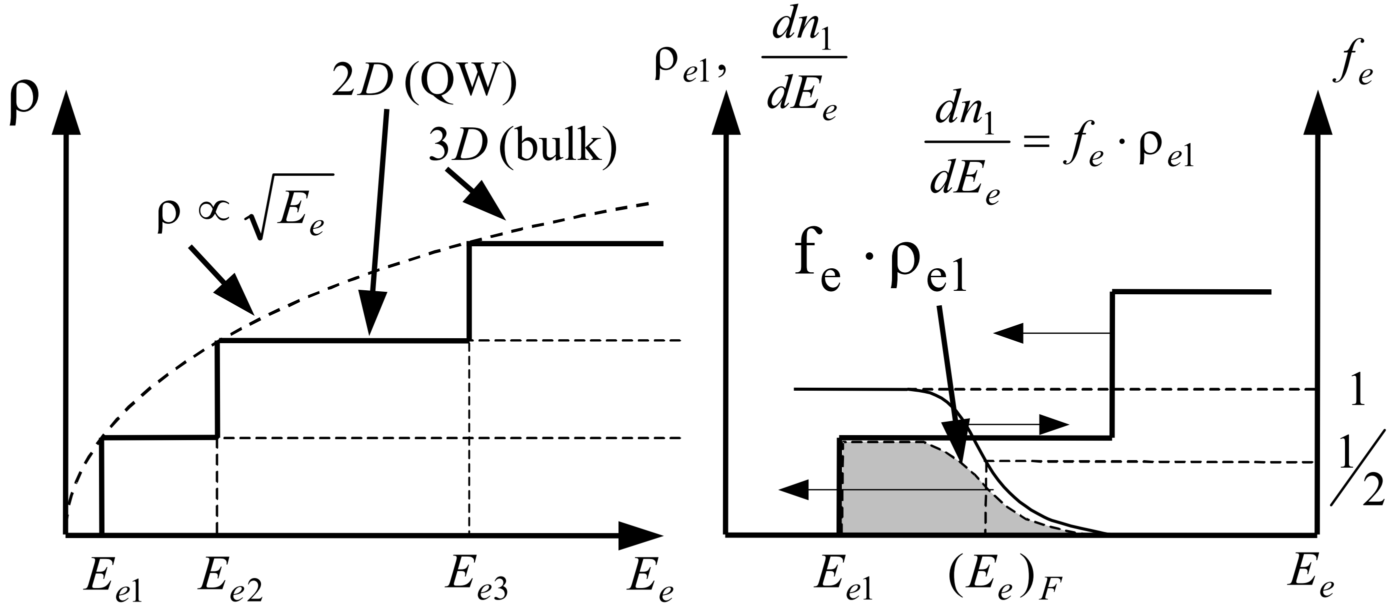
The density of states for first subband: $\rho_{e1} = \frac{1}{S} \cdot \frac{dN_s}{dE_{e1}} = \frac{m_e^*}{\pi \cdot \hbar^2}, \left[\frac{1}{cm^2 eV}\right]$

For m subbands: $\rho = \sum_{m=1} \rho_{em} = \frac{m_e^*}{\pi \cdot \hbar^2} \sum_m \Theta(E - E_{em})$

$$*\Theta = \begin{cases} 1 & (E \geq E_{em}) \\ 0 & (E \leq E_{em}) \end{cases}$$

The density of states per unit volume is $\rho_V = \rho / L_W$

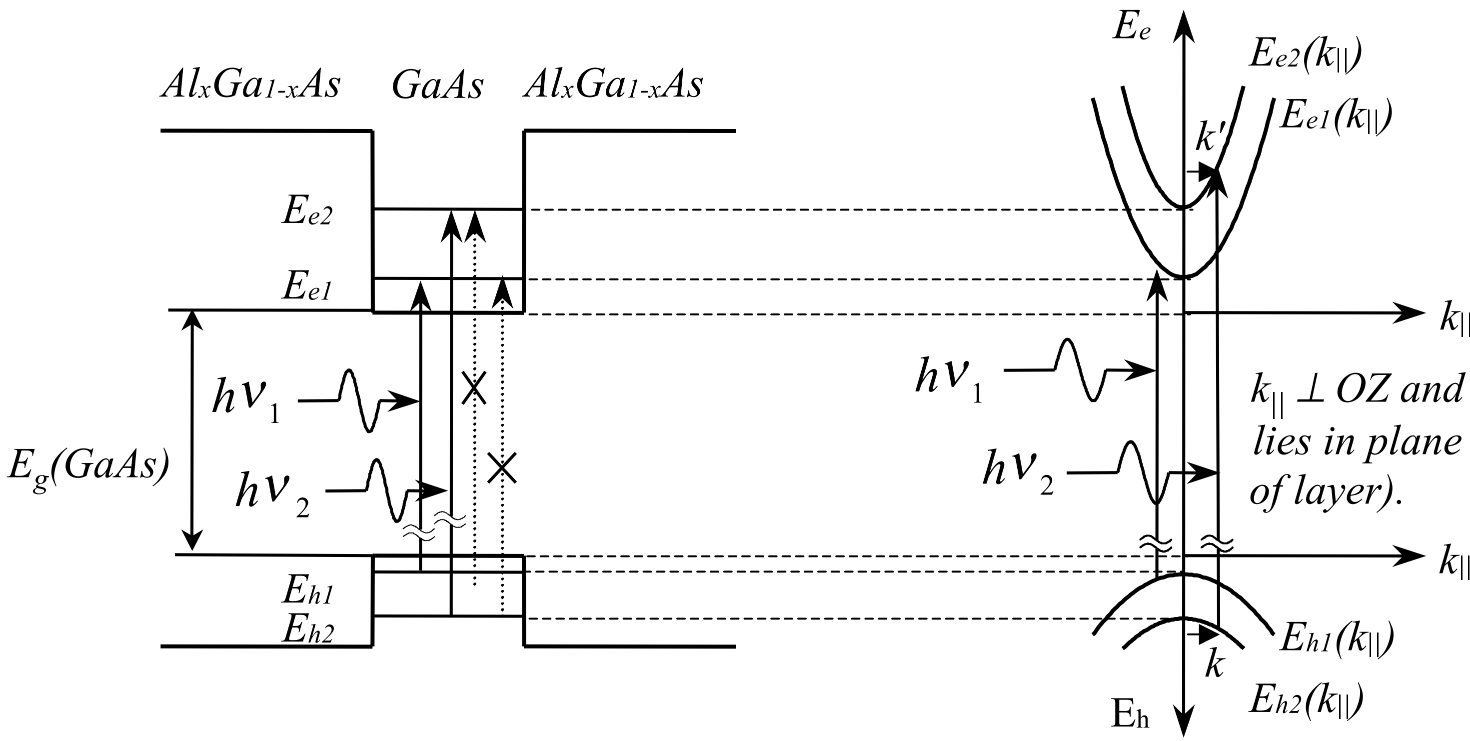
Density of states in QW



$$\rho_{e1} = \frac{1}{S} \cdot \frac{dN_s}{dE_{e1}} = \frac{m_e^*}{\pi \cdot \hbar^2}, \left[\frac{1}{cm^2 eV}\right]$$

$$\rho = \sum_{m=1} \rho_{em} = \frac{m_e^*}{\pi \cdot \hbar^2} \sum_m \Theta(E - E_{em})$$

Absorption of light in QW



Selection rule: $\Delta m=0$

$$E_{e\,m} = \frac{\hbar^2}{2m_e}\cdot\left(\frac{\pi}{L_W}\right)^2\cdot m^2, \qquad m = 1,2\ldots$$

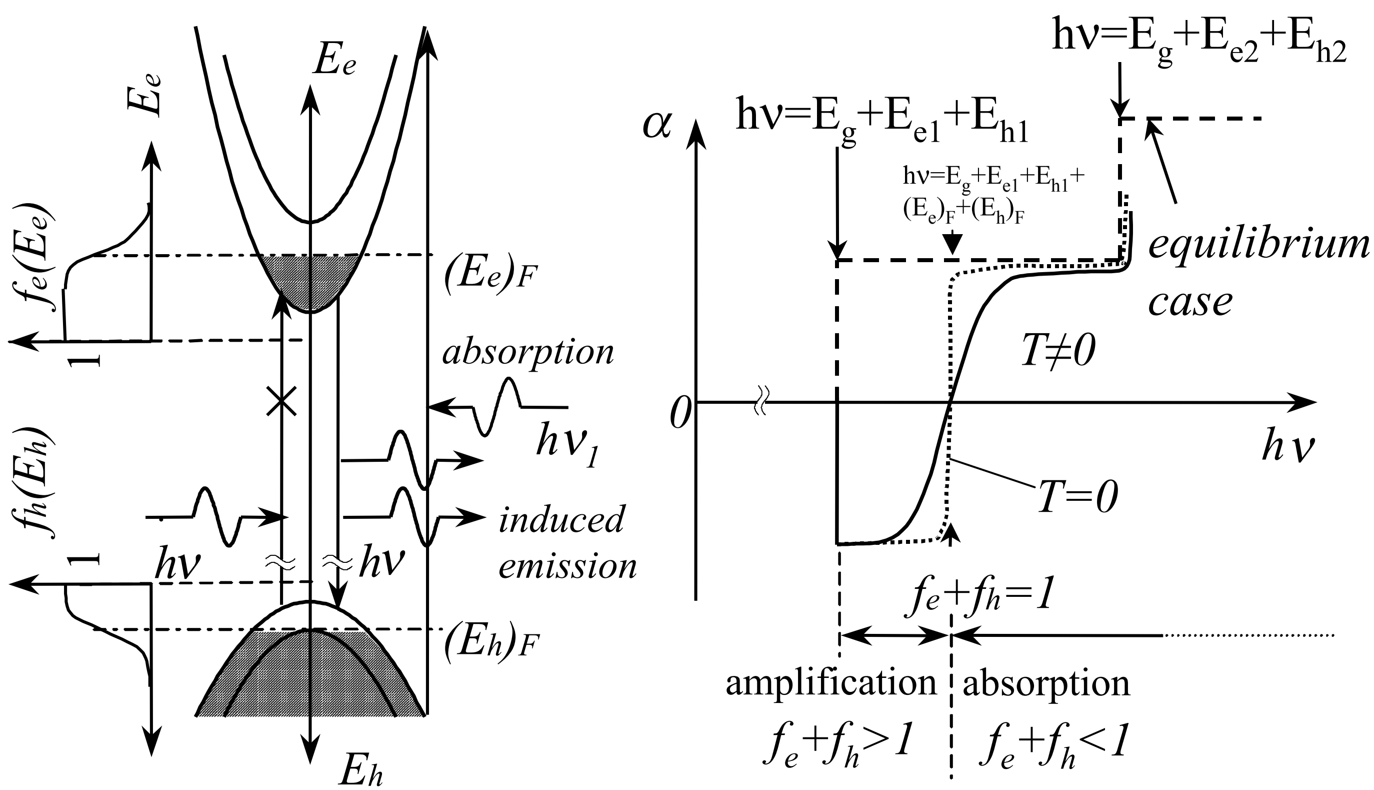
$$E_{h\,m} = \frac{\hbar^2}{2m_h}\cdot\left(\frac{\pi}{L_W}\right)^2\cdot m^2, \qquad m = 1,2\ldots$$

Energy conservation: $E_g + E_{em} + E_{hm} = h\nu$

Momentum conservation: $k' = k + \kappa \approx k$, $\kappa << k$

Absorption of light in QW

Nonequilibrium case: there are injected electrons and holes



$$\alpha = \rho(E)\cdot\left\{1 - f_e\left(E_e(k_{\|})\right) - f_h\left(E_h(k_{\|})\right)\right\}$$

Absorption: $\alpha > 0 \quad (1-(f_h+f_e)) > 0 \quad \text{or} \quad (f_e+f_h) < 1 \text{ for } h\nu > E_g + E_{e1} + E_{h1} + (E_e)_F + (E_h)_F$

Amplification: $\alpha < 0 \quad (1-(f_h+f_e)) < 0 \quad \text{or} \quad (f_e+f_h) > 1 \text{ for } h\nu < E_g + E_{e1} + E_{h1} + (E_e)_F + (E_h)_F$