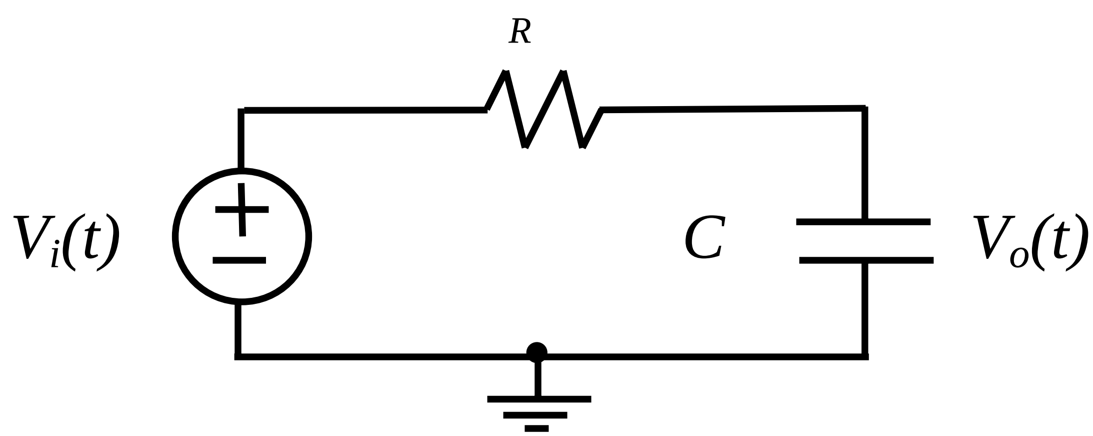
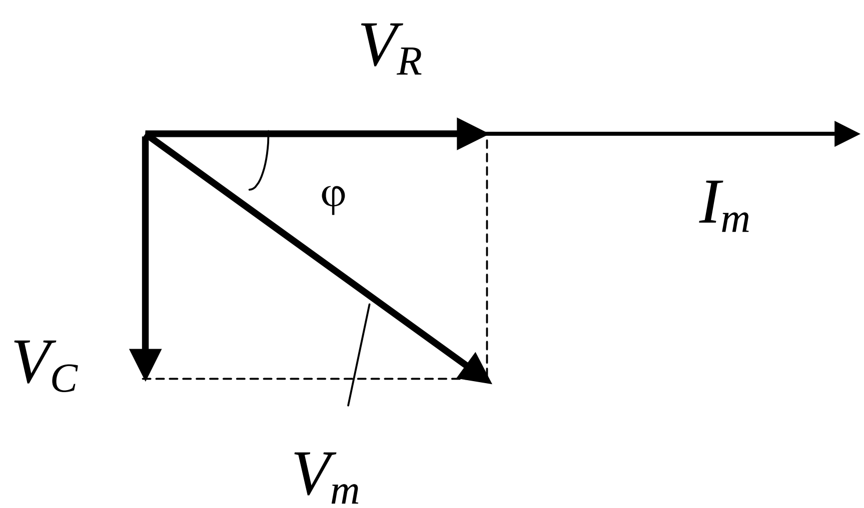
Phasor Diagram of an *RC* Circuit

$V(t) = V_m \sin(\omega t)$



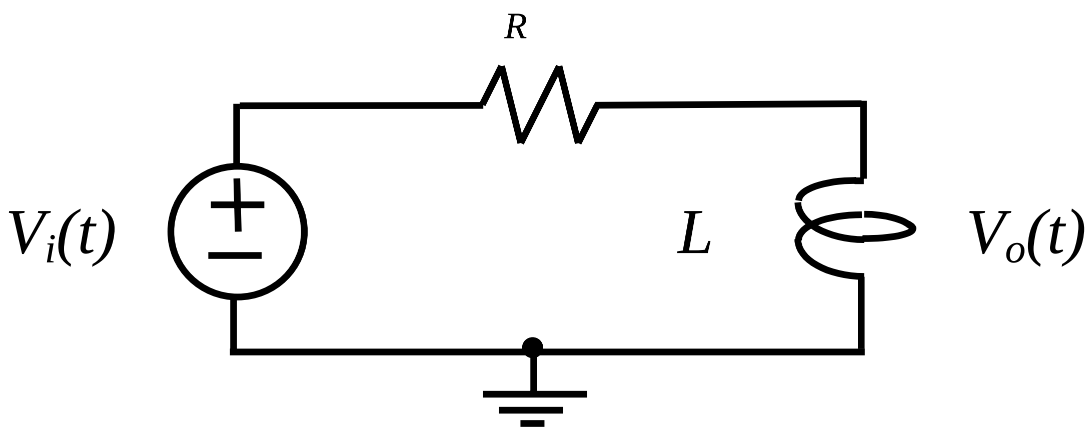
- Current is a reference in *series* circuit

$$\text{KVL: } \overline{V_m} = \overline{V_R} + \overline{V_C}$$

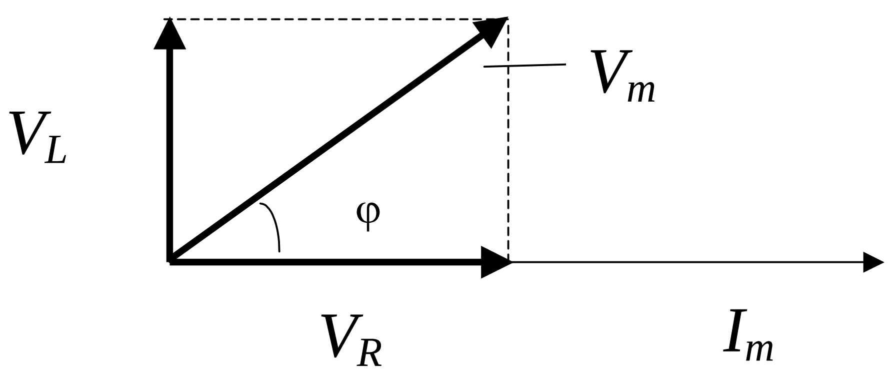


Phasor Diagram of an *RL* Circuit

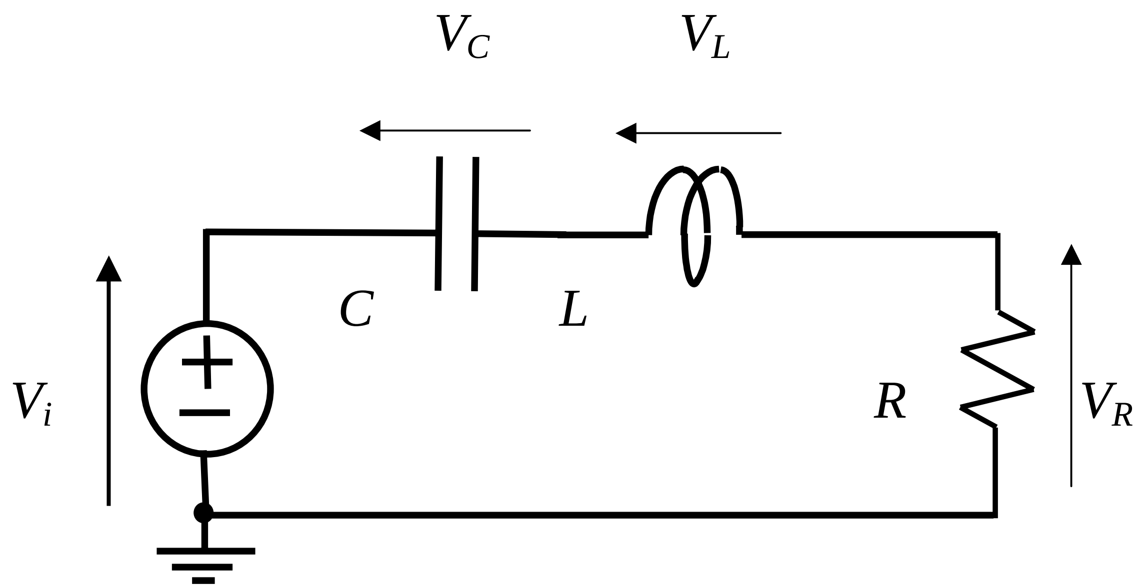
$V(t)=V_m\sin(\omega t)$ $\qquad$ $V_R\,(t)$



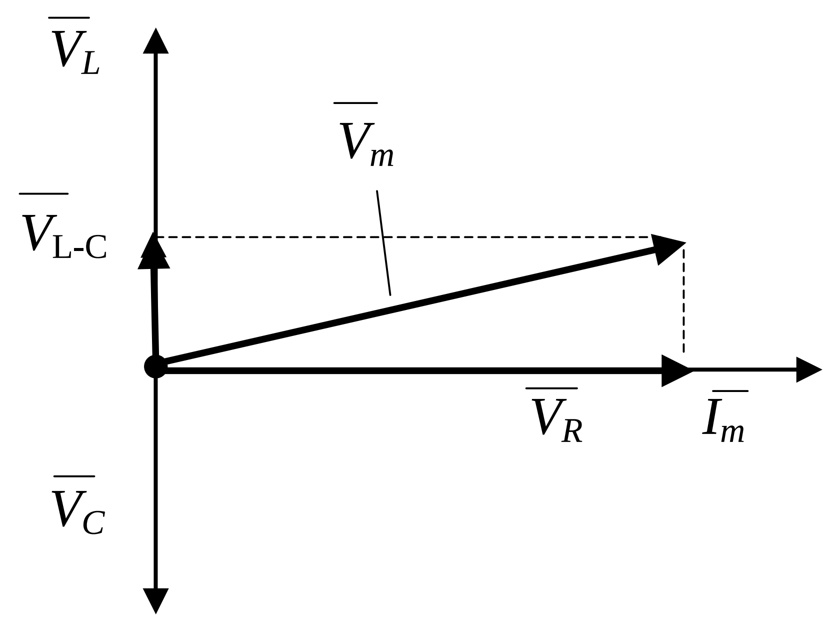
KVL: $V_m = V_R + V_L$



Phasor Diagram of a Series *RLC* Circuit

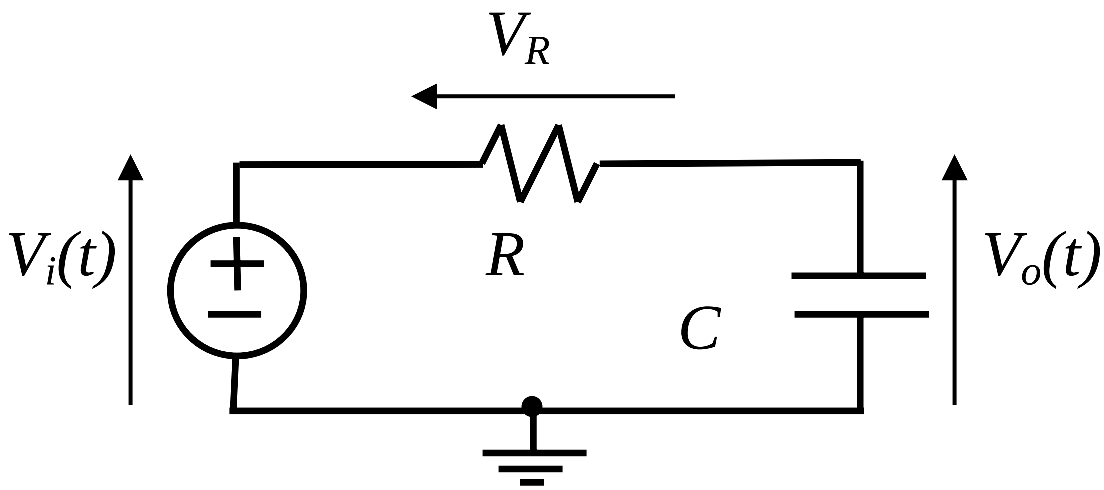


KVL: $\overline{V_m} = \overline{V_C} + \overline{V_L} + \overline{V_R}$



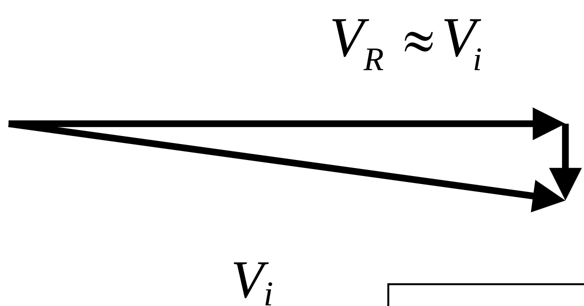
- Voltages across capacitor and inductor *compensate each other*

Integrating Circuit



$$\overline{V}_i = \overline{V}_R + \overline{V}_C \quad \text{or} \quad |V_i| = \sqrt{|V_R|^2 + |V_C|^2}$$

- Assume high frequency $1/\omega C << R$ then $V_R >> V_C$

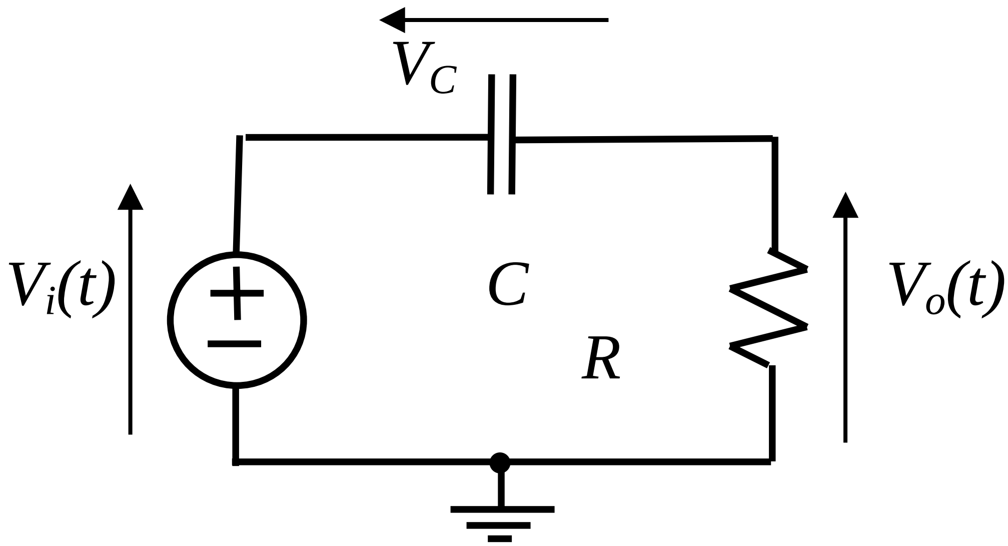


$$V_O = \frac{1}{C}\int_0^t I_i dt \approx \frac{1}{RC}\int_0^t V_i dt$$

$$\boxed{V_O \propto \int V_i dt}$$

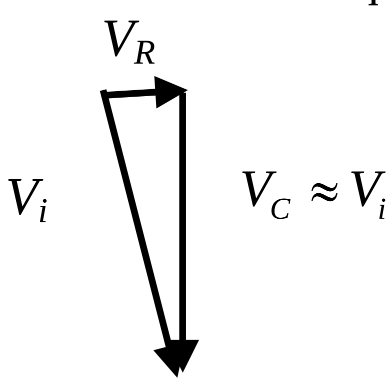
- *Accurate* integrating can be obtained at *high frequency* that leads to a *low* output signal

Differentiating Circuit



$$\overline{V}_i = \overline{V}_R + \overline{V}_C \quad \text{or} \quad |V_i| = \sqrt{|V_R|^2 + |V_C|^2}$$

- Assume *low* frequency $1/\omega C >> R$ then $V_R << V_C$



$$V_i \approx \frac{1}{C}\int_0^t I_i dt \Rightarrow I_i = C\frac{d}{dt}V_i$$

$$V_O = V_R = I_i R \approx RC\frac{d}{dt}V_i$$

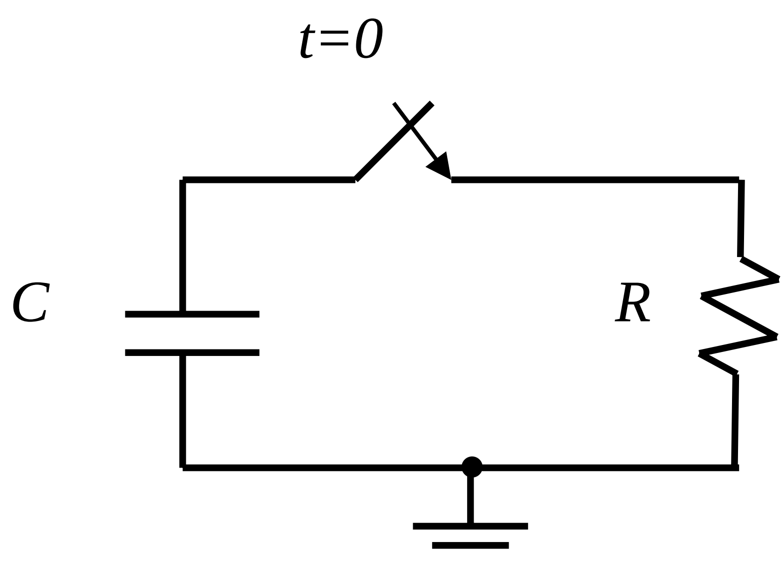
$$\boxed{V_O \propto \frac{d}{dt}V_i}$$

- An accurate result can be achived at low frequency.

Transient Processes in Passive Circuits

RC Circuit without a Source

- The circuit response is due only to the energy *stored* in the capacitor



- The capacitor C is precharged to the voltage V_0

Applying KCL:

$$C\, dV/dt + V/R = 0$$

- This is the 1st order differential equation

Solution of the Differential Equation

$$dV/dt + V/RC = 0$$

$$dV/V = -dt/RC$$

$$\ln V = -t/RC + a$$

$$V = A\, e^{-t/RC}, \quad A = e^{+a}$$

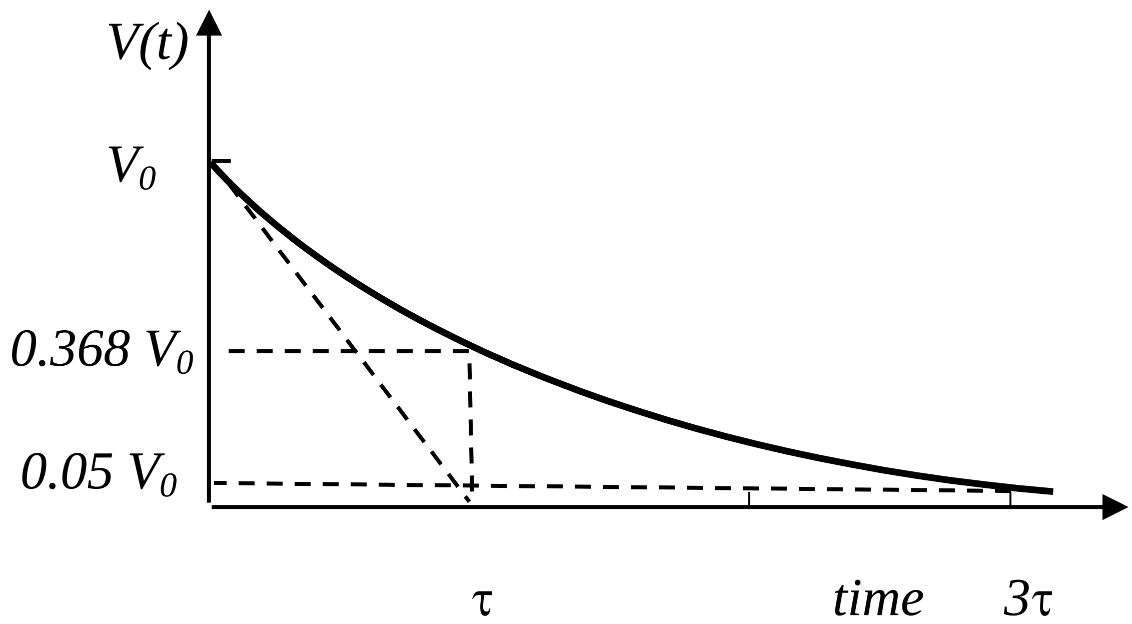
- e is the base of the natural logarithms, $e = 2.718\ldots$
- *Continuity* requires the initial condition:

$$\text{At } t = 0 \qquad V(0) = V_0$$

$$\boxed{V(t) = V_0\, e^{-t/RC}}$$

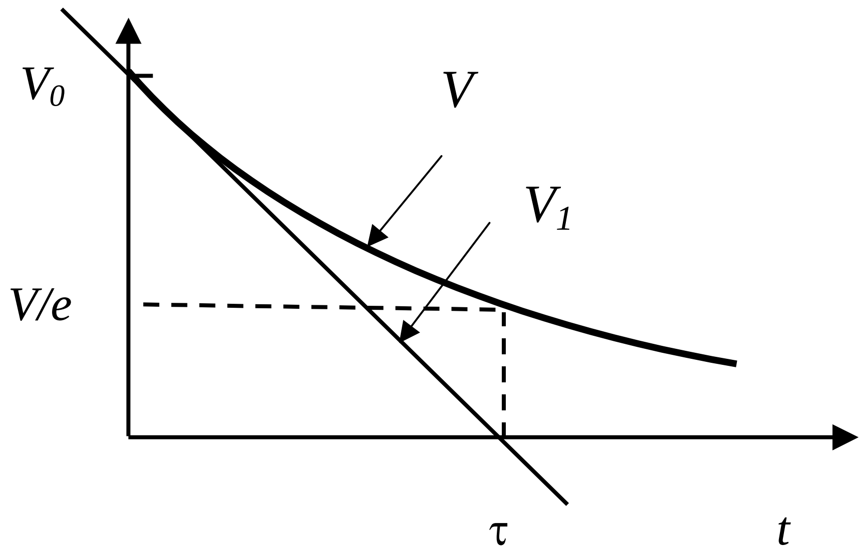
- The response governed by the elements themselves without external force is called a *natural response*

The Time Constant



- The *time constant* $\tau = RC$ is the rate at which the natural response decays to zero
- Time τ is required to decay by a factor of $1/e=0.368$
- After the time period of 3τ the transient process is considered to be completed

Determination of the Time Constant



1) *From the solution of the equation*

By definition: as the time when the signal decreases by 1/e

$$V(t+\tau)/V(t) = 1/e$$

The time t can be chosen arbitrarily

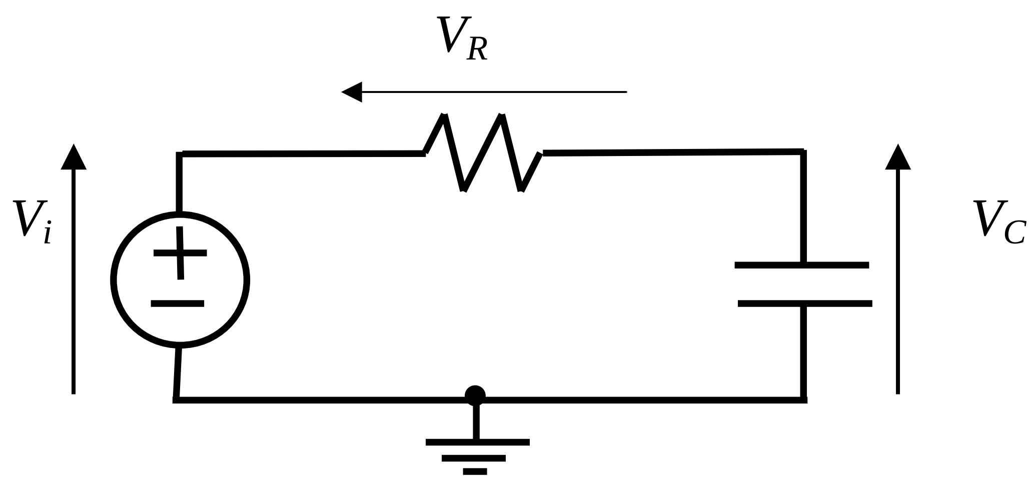
2) *From the slope of the line at t=0*

Differentiating the solution

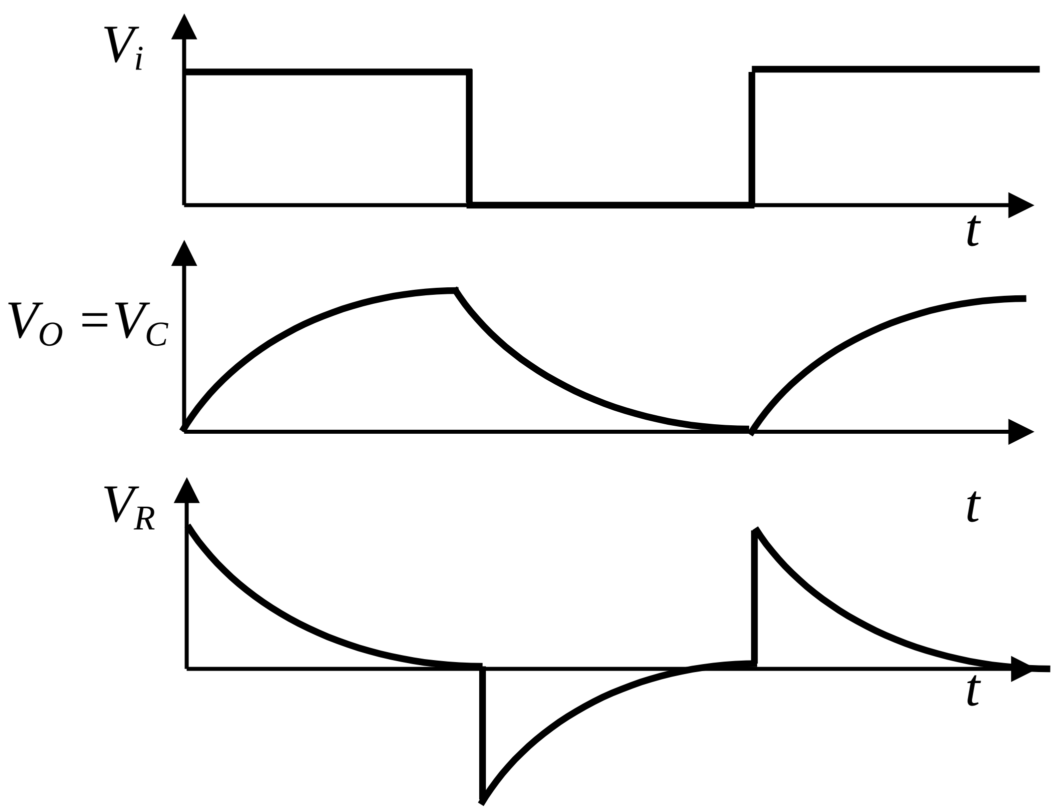
$$dV/dt = -\{V_0/\tau\}\, e^{-t/\tau}$$

$$V_1(t) = -\{V_0/\tau\}t + V_0 \qquad \boxed{V_1(\tau) = 0}$$

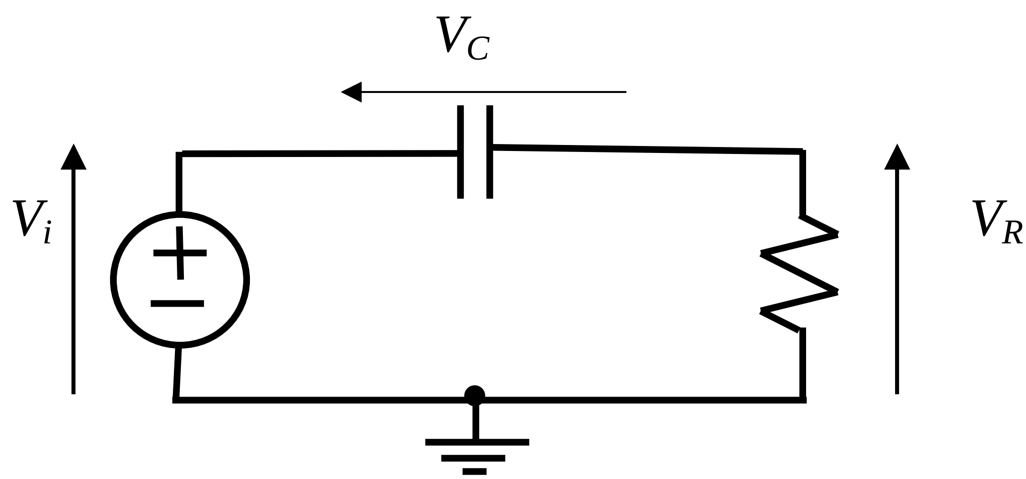
Transient Response of an *Integrating RC* Circuits



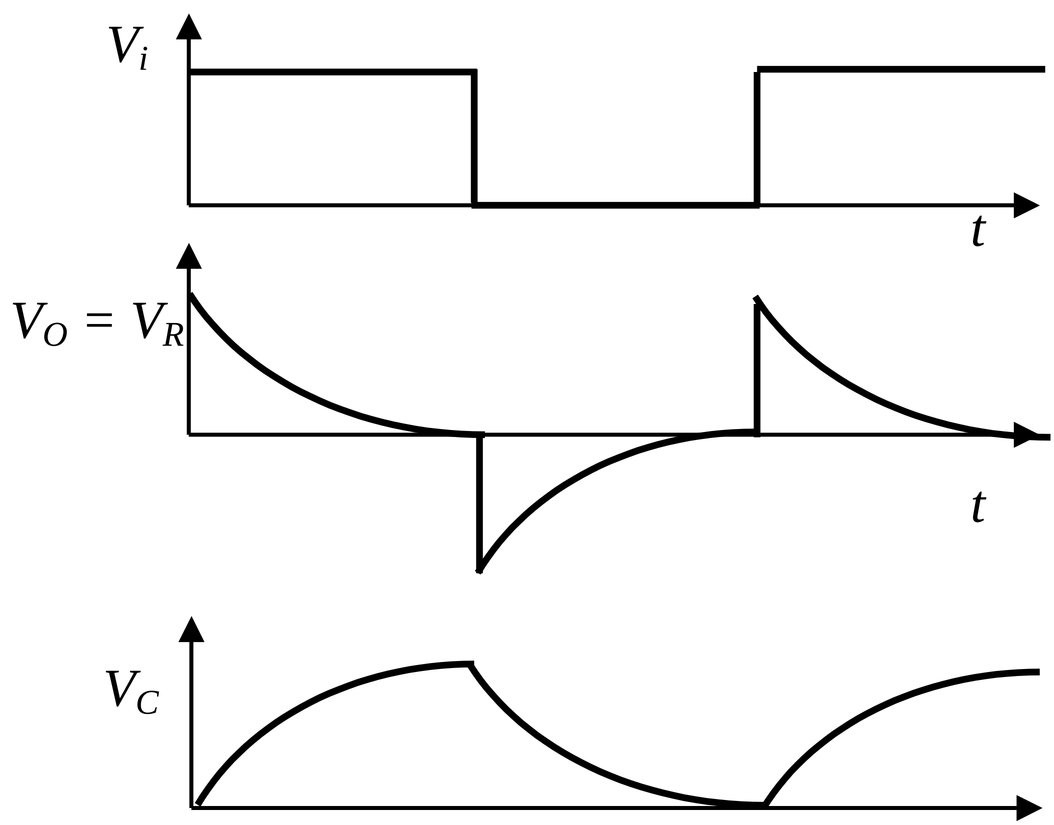
KVL: $\overline{V_i} = \overline{V_R} + \overline{V_C}$



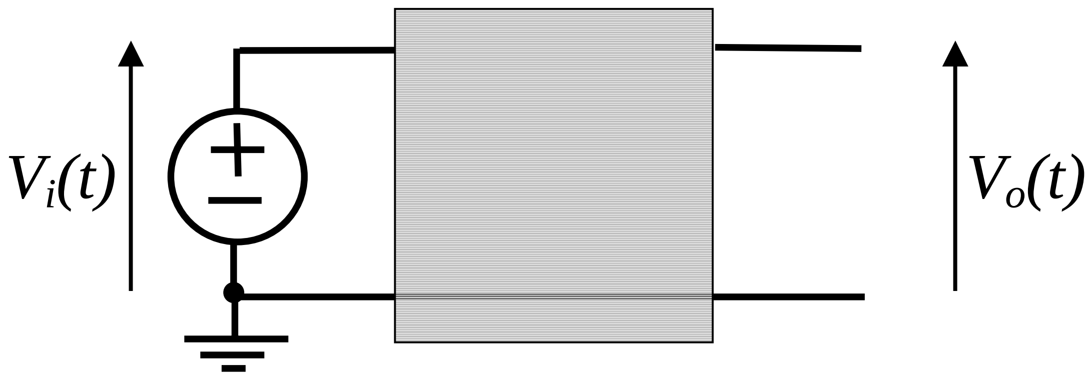
Transient Response of a *Differentiating RC* Circuits



KVL: $\overline{V_i} = \overline{V_C} + \overline{V_R}$



Transfer Function



$$V_i(t) = V_{im}\cos(\omega t) \qquad V_o(t) = V_{om}\cos(\omega t+\varphi)$$

$$V_{om}(\omega) \qquad \varphi(\omega)$$

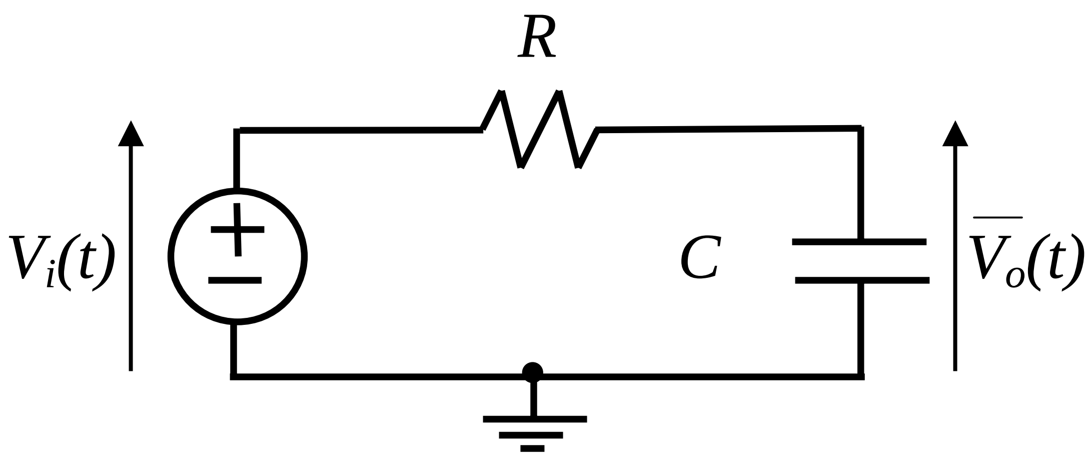
$$V_o(t) = Re\{\,\overline{V_o}\, e^{j\omega t}\}, \qquad \overline{V_o} = V_{om} e^{j\varphi}$$

$$\boxed{T(\omega) = \frac{\overline{V_o}(\omega)}{V_i}}$$

Amplitude response $|T(\omega)| = V_{om}(\omega)/V_{im}$

Phase response $\angle T(\omega) = \varphi(\omega)$

The Transfer Function of an *Integrating RC* Circuit



$$\overline{T}(\omega) = \frac{\overline{V}_o(\omega)}{V_i} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

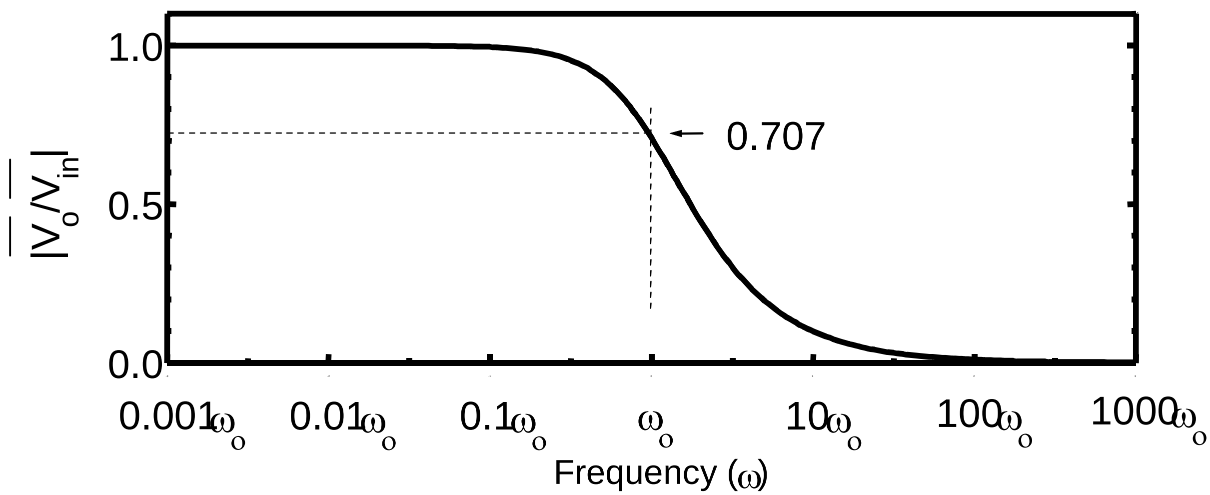
$$\boxed{\overline{T}(\omega) = \frac{1}{1 + j\omega\tau}} \qquad \tau = RC$$

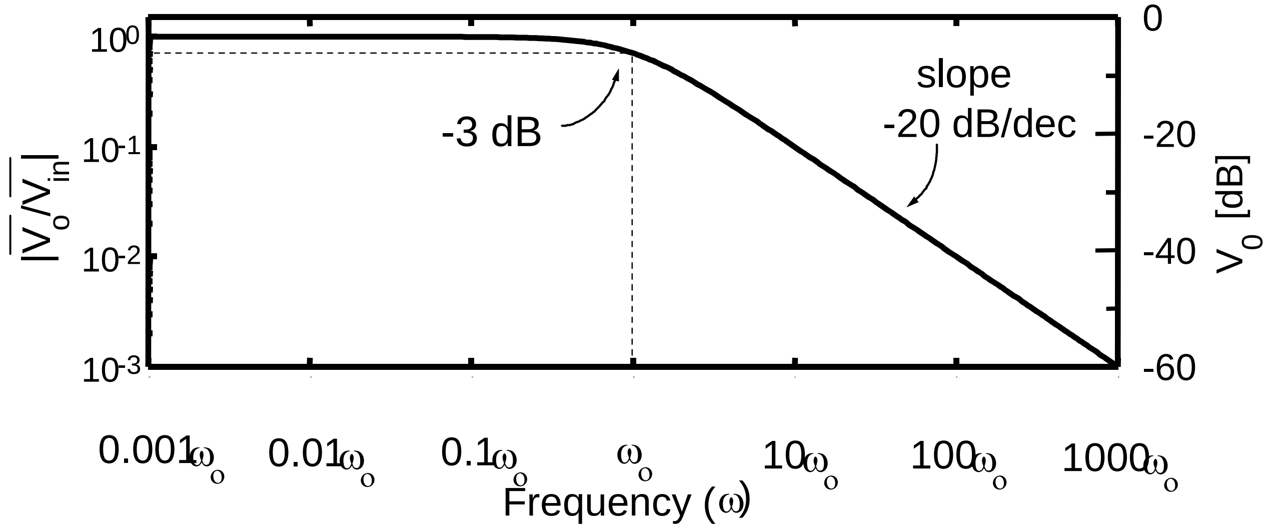
Amplitude Response of the Integrating *RC* Circuit

- The *magnitude* of the transfer function is

$$\left|\overline{T}(\omega)\right| = \frac{1}{\sqrt{1+\omega^2\tau^2}}$$

- The Integrating *RC* circuit is a LOW-PASS filter



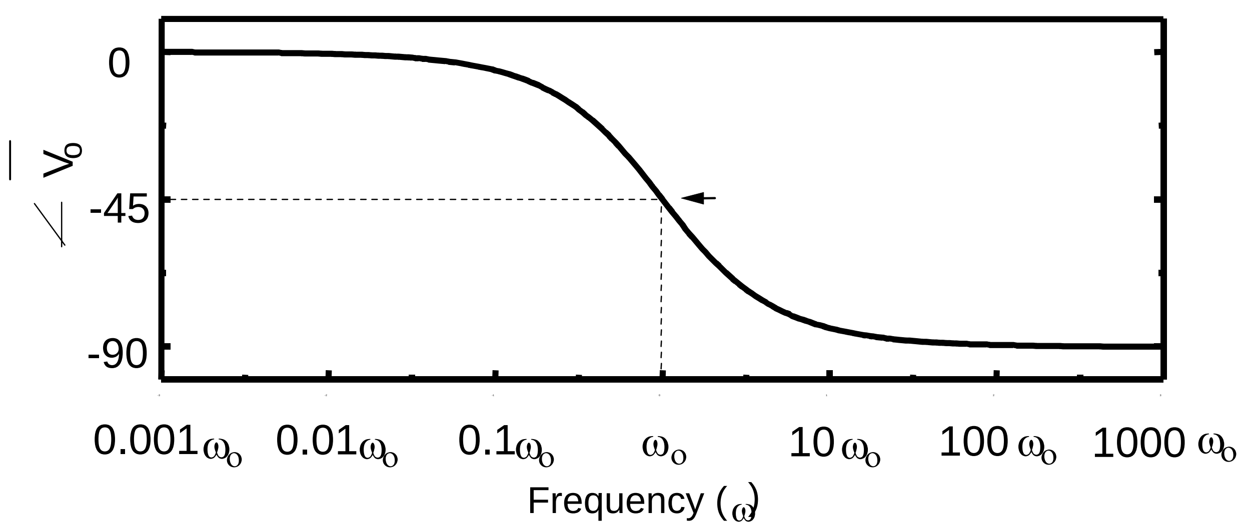


Phase Response of the Integrating *RC* Circuit

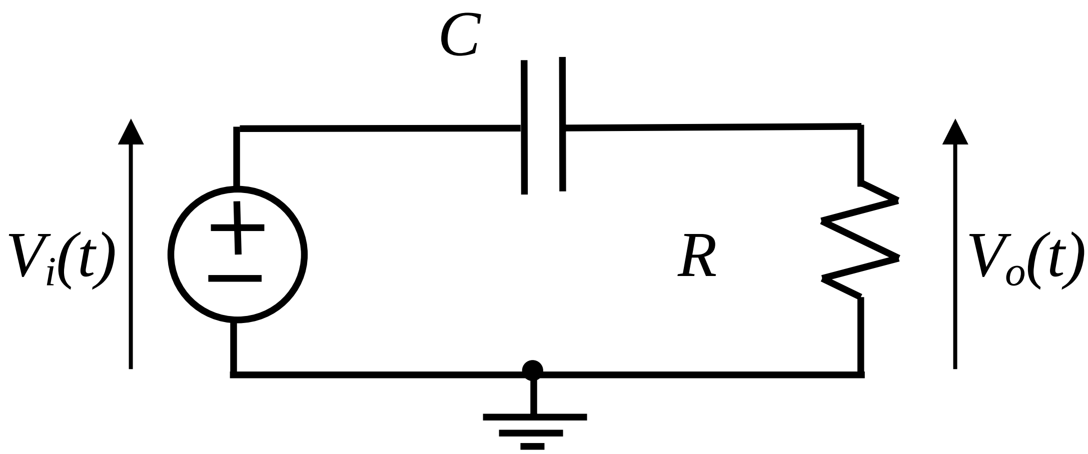
The *angle* of the transfer function is

$$\angle\overline{T}(\omega) = \arctan\frac{\operatorname{Im}\left[T(\omega)\right]}{\operatorname{Re}\left[T(\omega)\right]}$$

$$\angle\overline{T}(\omega) = \angle\frac{1 - j\omega\tau}{(1 + j\omega\tau)(1 - j\omega\tau)} = -\arctan(\omega\tau)$$



The Transfer Function of a *Differentiating* RC Circuit



$$\overline{T}(\omega) = \frac{\overline{V_o}(\omega)}{V_i} = \frac{\overline{I}(\omega)\cdot R}{\overline{I}(\omega)\cdot X_{RC}(\omega)}$$

$$\overline{T}(\omega) = \frac{R}{X_{RC}(\omega)} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}$$

$$\boxed{\overline{T}(\omega) = \frac{j\omega\tau(1 - j\omega\tau)}{1+\omega^2\tau^2} = \frac{\omega\tau(\omega\tau + j)}{1+\omega^2\tau^2}}$$

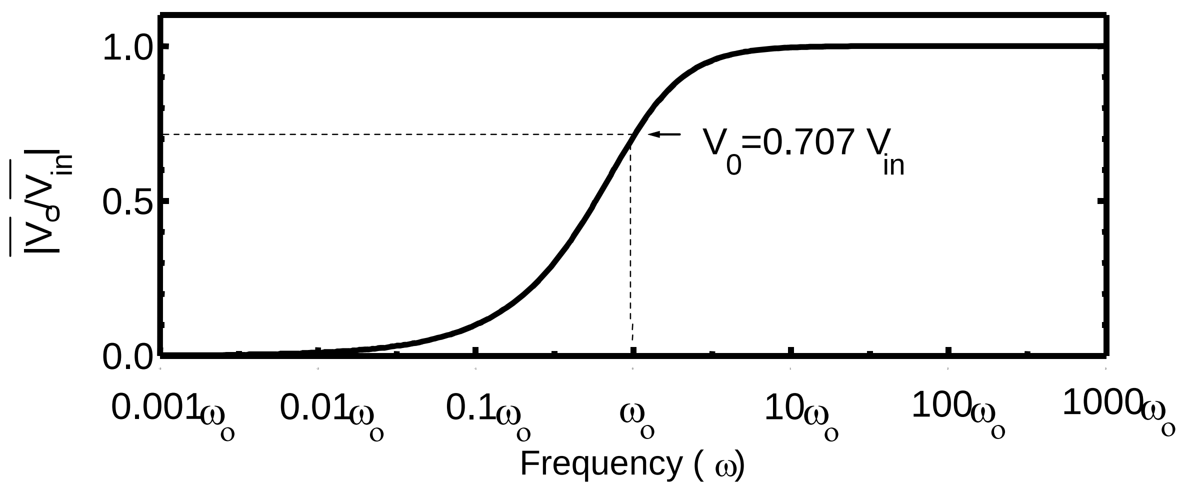
$$\tau = RC$$

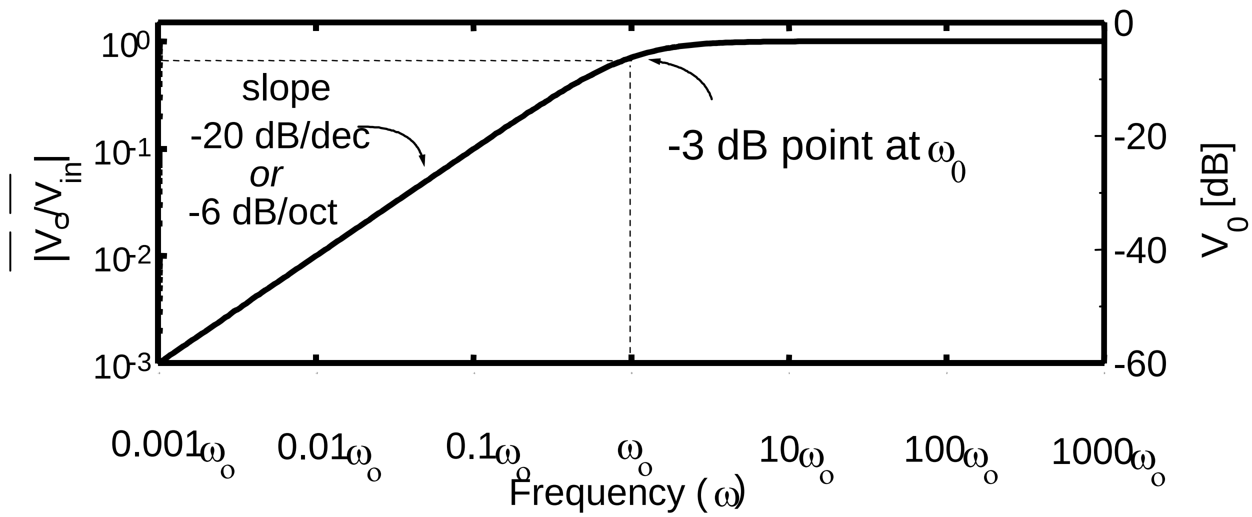
Amplitude response of the Differentiating RC Circuit

- *Amplitude* response of the RC circuit is the magnitude of $\overline{T}(\omega)$

$$\left|\overline{T}(\omega)\right| = \left|\frac{\omega\tau(\omega\tau + j)}{1+\omega^2\tau^2}\right| = \frac{\omega\tau}{\sqrt{1+\omega^2\tau^2}}$$

- The Differentiating RC circuit is a *HIGH-PASS* filter



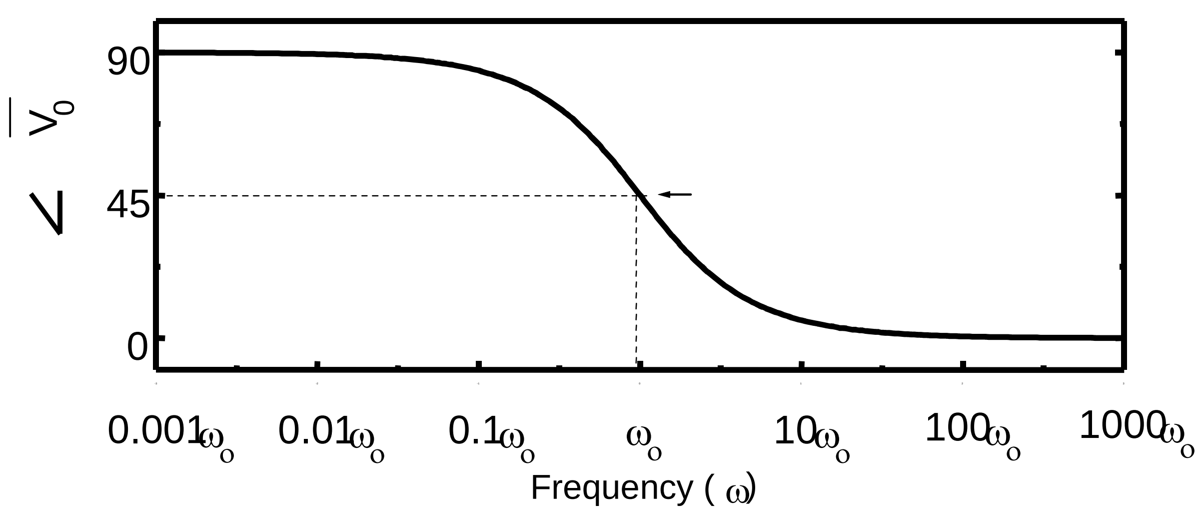


Phase response of the Differentiating RC Circuit

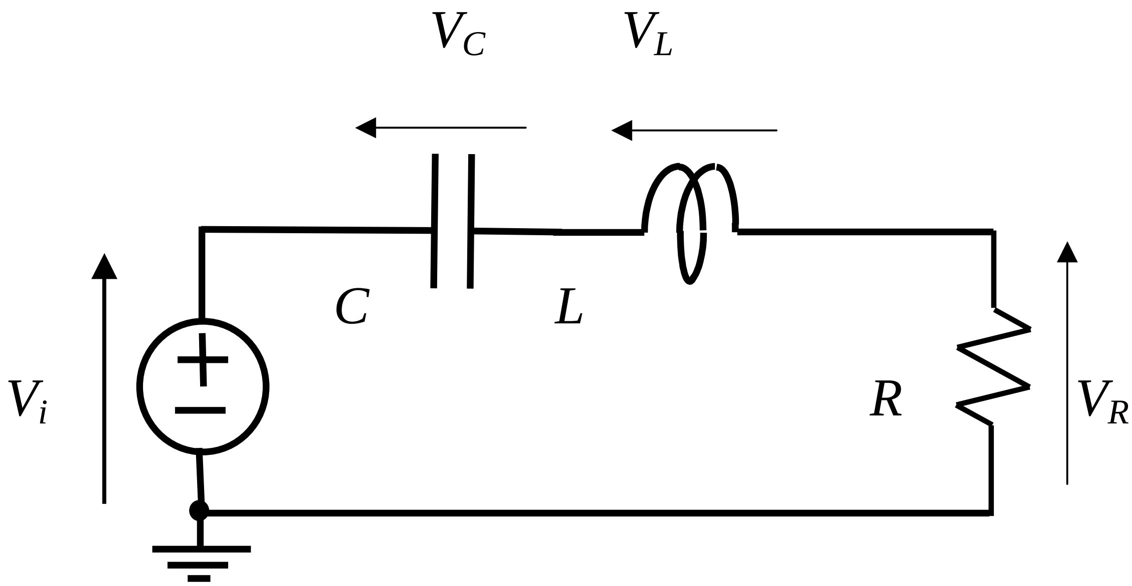
- *Phase* response of the *RC* circuit is the phase angle of $\overline{T}(\omega)$

$$\angle\overline{T}(\omega) = \angle\left\{\frac{\omega\tau(\omega\tau + j)}{1+\omega^2\tau^2}\right\}$$

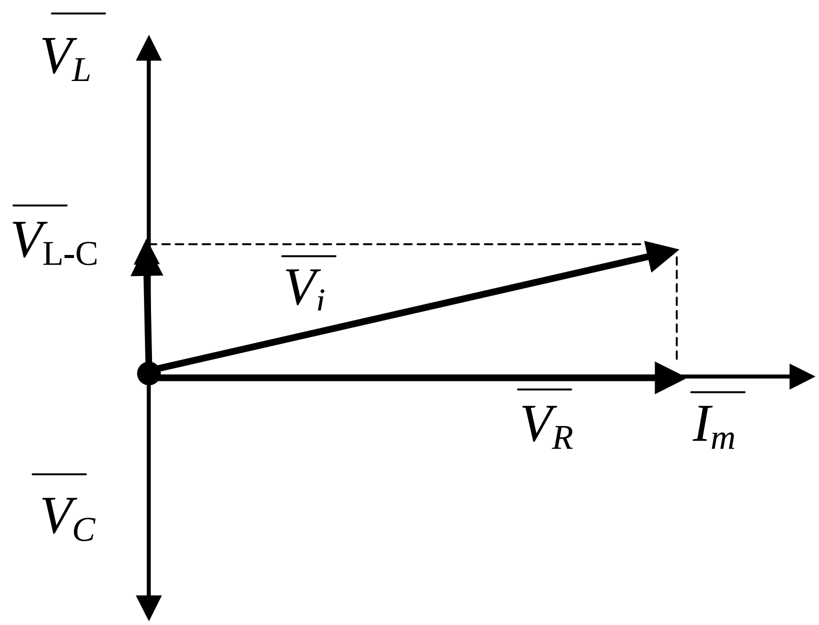
$$\boxed{\angle\overline{T}(\omega) = \arctan\frac{1}{\omega\tau}}$$



Frequency Response of a Series *RLC* Circuit



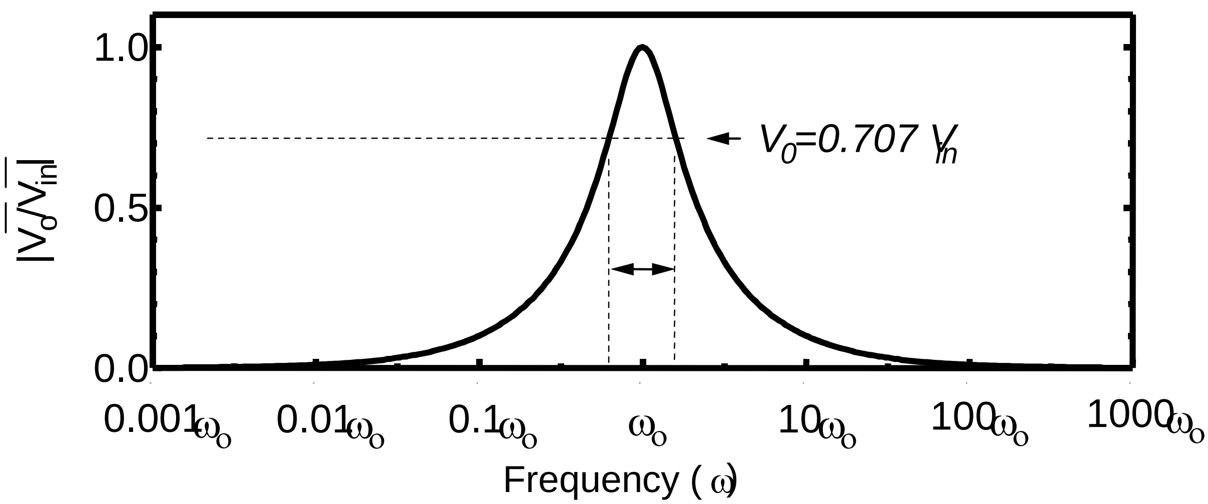
KVL: $\overline{V_i} = \overline{V_C} + \overline{V_L} + \overline{V_R}$

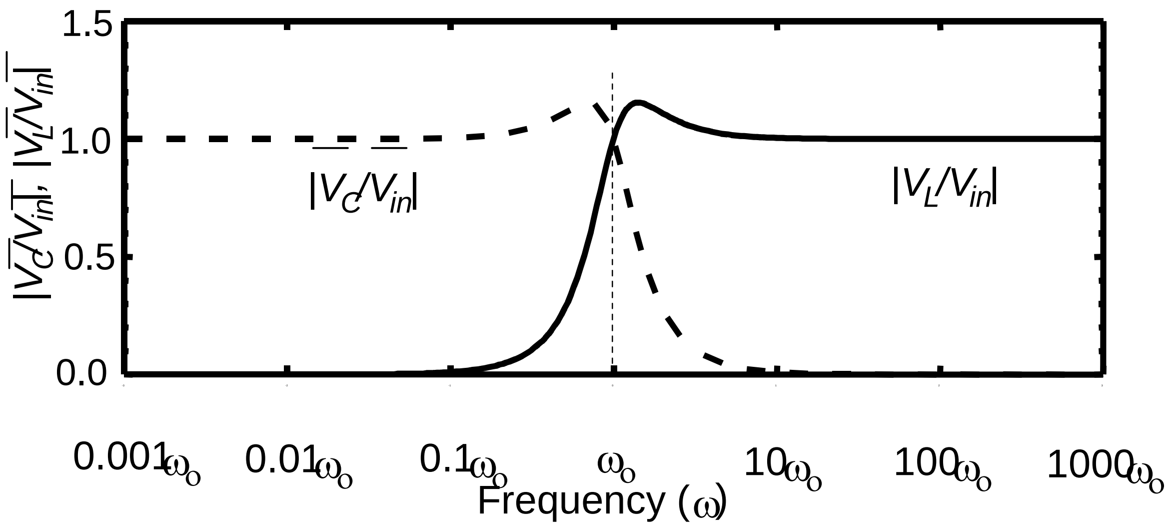


- Voltages across capacitor and inductor *compensate each other*

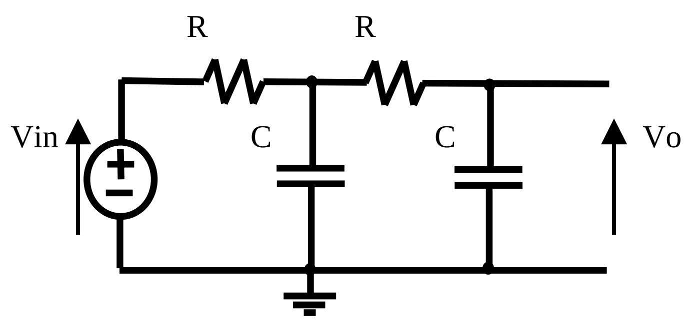
Amplitude Response of a Series *RLC* Circuit

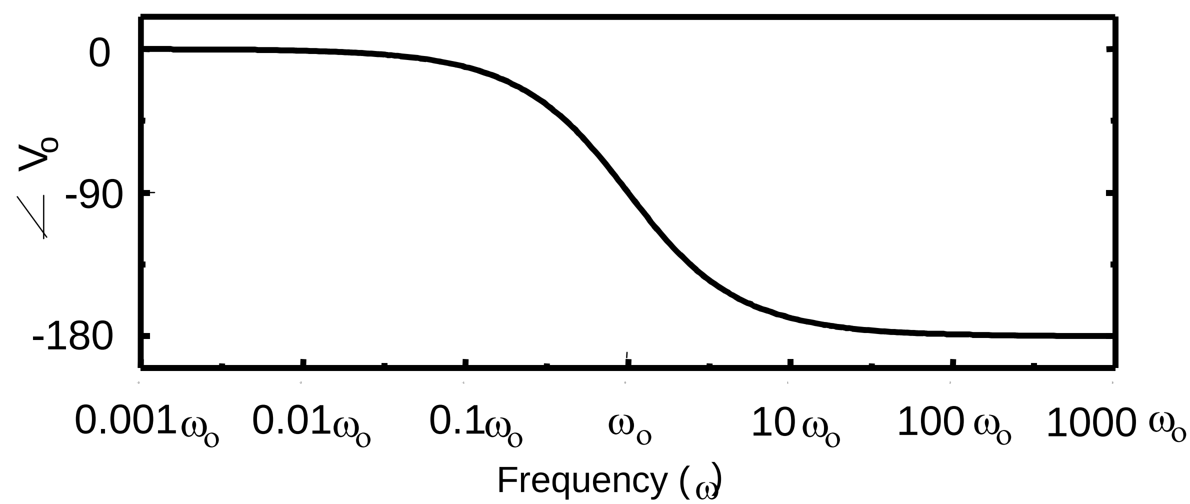
$$|\overline{V}_0/\overline{V}_{IN}| = \frac{R}{\left|j\omega L + \frac{1}{j\omega C} + R\right|} = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

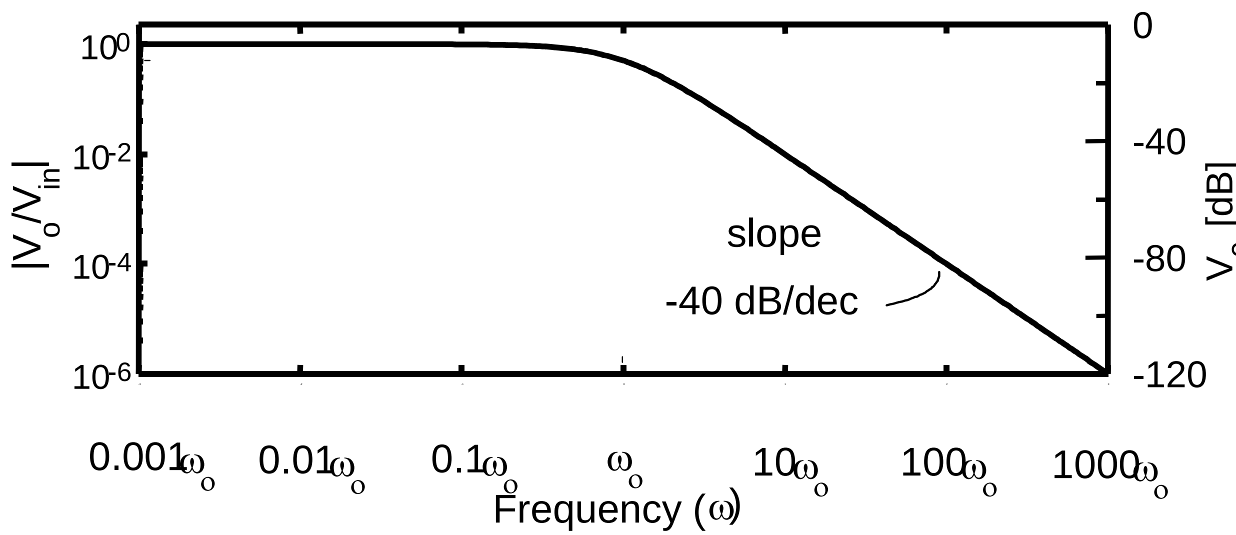




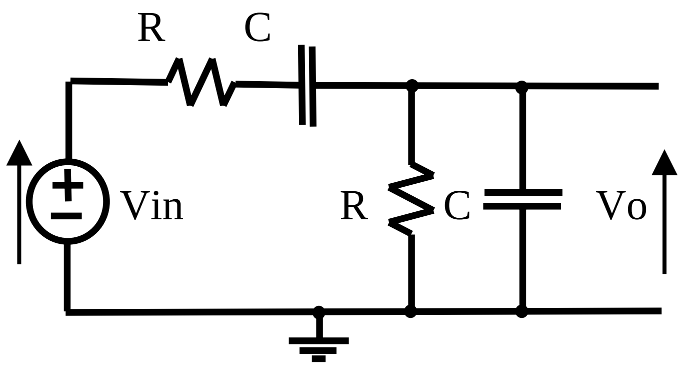
Amplitude and Phase Response of the Ladder *RC* Circuit







Amplitude and Phase Response of a Wien-Bridge Circuit



$$\overline{T}(\omega) = \frac{Z_{OUT}}{Z_{TOTAL}} = \frac{\left(\frac{1}{R} + j\omega C\right)^{-1}}{\left(\frac{1}{R} + j\omega C\right)^{-1} + R + \frac{1}{j\omega C}}$$