# Lab 6: Active low pass filters.

## 1. OBJECTIVES

Understand the operation of active RC filters:

Characterize the "Miller capacitor" circuit based on 741 operational amplifiers;

Characterize the first order active RC low pass filter;

Characterize second order Butterworth low pass filter.

### 2. INTRODUCTION

### 2.1. Passive RC low pass filter.

Filters are used to pass energy in certain frequency bands while attenuating energy in other complementary bands. Contemporary circuits can rely on digital signal processing to perform filtering function with unprecedented precision. There are still plenty of applications for analog filters, for instance, basic low pass filtering often can be more efficiently performed using analog circuits. Moreover, digital circuit design often requires use of analog filters (actually, there are a lot of analog issues to be addressed in design of digital circuit).

In it's the most basic form the low pass filter is simple first order passive RC integrator. In RC integrator input signal is applied to series connection of R and C and output is taken across C. As we know, RC integrator

has maximum gain of 0 dB (1 V/V), and 3dB cut-off frequency  $f_{3dB} = \frac{1}{2 \cdot \pi \cdot R \cdot C}$  above which the frequency response decays with the rate of 20 dB/decade (6 dB/octave). This is a magnitude frequency response of

unloaded passive RC integrator fed with signal generator having zero impedance.

Figure 1 below shows more realistic case (source and load can have complex impedances, but simple resistive case is sufficient for illustration). Presence of Rs and RL modifies the frequency response and changes its 3 dB cut-off. Active elements can be used to eliminate this loading effect. For instance voltage buffer could be introduced between output of RC integrator and load.





Passive-only filter design approach would necessitate use of inductors to realize filter functions required for certain applications. For low frequency operation the required inductors are bulky and have nonlinear characteristics. Additionally, an inductor is extremely unfriendly element for circuit integration. This fact was one of the important motivations behind development of the active filters when integrated circuits became available. RC circuits with gain elements can be designed to act like inductors (this circuit is called gyrators). Current and voltage buffers can address impedance issues. There is variety of circuit solutions to realize low-,

(1)

(2)

 $\mathbf{f}_{3\mathrm{dB}} = \frac{1}{2 \cdot \boldsymbol{\pi} \cdot \mathbf{R}_{\mathrm{F}} \cdot \mathbf{C}} \,.$ 

high-, band-pass, notch or tunable analog filters. We will explore OpAmp-based low pass filters only.

#### 2.2. Miller integrator.

Figure 2a (Figure 2.39 from 5<sup>th</sup> edition Sedra/Smith) shows the OpAmp-based inverting amplifier with capacitor in negative feedback loop. The corresponding magnitude response is shown in Figure 2b.





The circuit is nearly perfect integrator called Miller integrator thanks to presence of capacitive coupling between input and output. Product RC determines an intercept point of magnitude response with frequency axis - integrator frequency. As you know, absence of the feedback at DC in combination with nonidealities of OpAmp (DC offset voltage and current) can drive the circuit output to power supply voltage. Often, it is required to add an additional feedback resistor RF in parallel with C to stabilize the circuit (Figure 2c). Circuit  $\frac{V_{O}}{V_{I}} = \frac{-\frac{R_{F}}{R}}{1 + j \cdot 2 \cdot \pi \cdot f \cdot R_{F} \cdot C},$ 

from Figure 2c will have transfer characteristics of the low pass filter:

with 3dB cut-off:

Thanks to the properties of the OpAmp amplifier 3 dB point is much less sensitive to source/load impedances. The rate of decrease of the magnitude response above 3 dB cut-off is 20 dB/decade.

#### 2.3. Butterworth filters.

Slope of the magnitude frequency response often has to be increased. We will characterize the performance of second order Butterworth filter with slope of 40 dB/decade. Family of Butterworth filters is characterized by flat response in pass band (sometimes they are called maximum flat response filters). The degree of flatness increases with order. Our circuit would require one OpAmp, two resistors and two capacitors (Figure 3).



Figure 3.

For the special case of R1 = R2 = R, C1 = 2\*C and C2 = C the magnitude response will be equal to:

$$\left|\frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}}\right| = \frac{1}{\sqrt{1 + 2^2 \cdot \left(\boldsymbol{\omega}^2 \cdot (\mathbf{R} \cdot \mathbf{C})^2\right)^2}} \,. \tag{3}$$

Clearly, it is classical Butterworth magnitude response of the second order with 3dB cut-off:

$$f_{_{3dB}} = \frac{1}{2 \cdot \pi \cdot R \cdot C \cdot \sqrt{2}} \,. \tag{4}$$

#### 3. PRELIMINARY LAB

**3.1.** Plot magnitude and phase responses of the circuit from Figure 1 for  $R = 2.5 \text{ k}\Omega$  and C = 1.5 nF. (a) Assume Rs = 0 and  $RL = \infty$ ; (b) Assume  $Rs = RL = 10 \text{ k}\Omega$ .

**3.2.** Plot phase response for Figure 2a circuit. Assume ideal OpAmp.

**3.3.** Assume ideal OpAmp. Derive (1) and (2).

**3.4.** For Figure 2c circuit with  $R = 2.5 \text{ k}\Omega$  and C = 1.5 nF find RF to obtain 3 dB cutoff frequency of 1 kHz. Plot corresponding magnitude and phase frequency responses. How would magnitude frequency response change if R is increased from 2.5 to 10 k $\Omega$ .

**3.5.** Assume ideal OpAmp. Derive (3) and (4).

**3.6**. For Figure 3 circuit with C1 = 20 nF and C2 = 10 nF find R1 = R2 = R to obtain 3 dB cutoff of 1 kHz.

## 4. EXPERIMENT.

**4.1.** Build Miller integrator circuit (Figure 2a). Do not forget about power supply -  $\pm 15$  V for 741 OpAmp. Use R = 2.5 k $\Omega$  and C = 0.22  $\mu$ F. Perform measurements of the magnitude and phase frequency response.

Present the measurement results in the form of table and on Bode plot. Determine integrator frequency from experimental data. Compare with results of calculation.

**4.2.** Apply square wave input (1 Vpp and 1 kHz) to the circuit from part 4.1. Obtain input and output waveforms on oscilloscope screen.

Sketch the input and output waveforms. Determine input and output peak-to-peak voltages. Compare with results of calculation.

**4.3.** Build circuit from Figure 2c. Use  $R = 2.5 \text{ k}\Omega$  and C = 1.5 nF. Select RF to obtain 3dB cutoff of 1kHz. Perform measurements of the magnitude and phase frequency response.

Present the measurement results in the form of table and on Bode plot. Determine 3 dB frequency and slope of the low pass filter from experimental data. Compare with results of calculation. What would be integrator frequency of the Miller integrator made of R and C if  $R_F$  is removed? Can you determine this value from Bode plot of magnitude frequency response of the circuit with  $R_F$ ?

**4.4.** Build circuit from Figure 3. C1 = 20 nF and C2 = 10 nF. Select R1 = R2 = R to obtain 3 dB cutoff of 1 kHz. Perform measurements of the magnitude frequency response.

Present the measurement results in the form of table and on Bode plot. Determine 3 dB frequency and slope of the low pass filter from experimental data. Compare with results of calculation.

**4.5.** Apply saw wave (ramp mode of waveform generator) input (100 % symmetry, 0 offset, 1 Vpp and 500 Hz) to the circuit from part 4.4. Obtain input waveform on oscilloscope screen. Switch Oscilloscope to FFT mode. Adjust frequency span (~20 kHz) and center frequency (~ 10 kHz) to visualize the frequency spectrum of saw wave input. Using oscilloscope memory function (save/recall) save the FFT of input waveform. Obtain FFT of output waveform on oscilloscope screen. Recall FFT of input from oscilloscope memory to visualize both input and output signal FFTs together. Observe action of second order low pass filter in frequency domain.

Sketch frequency spectrum of the input and output signals. Estimate 3dB cutoff and slope of the active filter from your this experiment.

# 5. REPORT

The report should include the lab goals, short description of the work, the experimental and simulated data presented in plots, the data analysis and comparison followed by conclusions. Please follow the steps in the experimental part and clearly present all the results of measurements. Be creative; try to find something interesting to comment on.