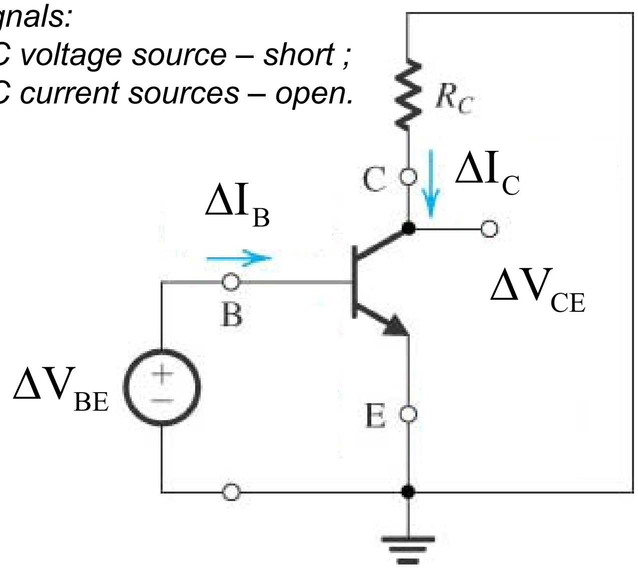
Small signal equivalent circuit

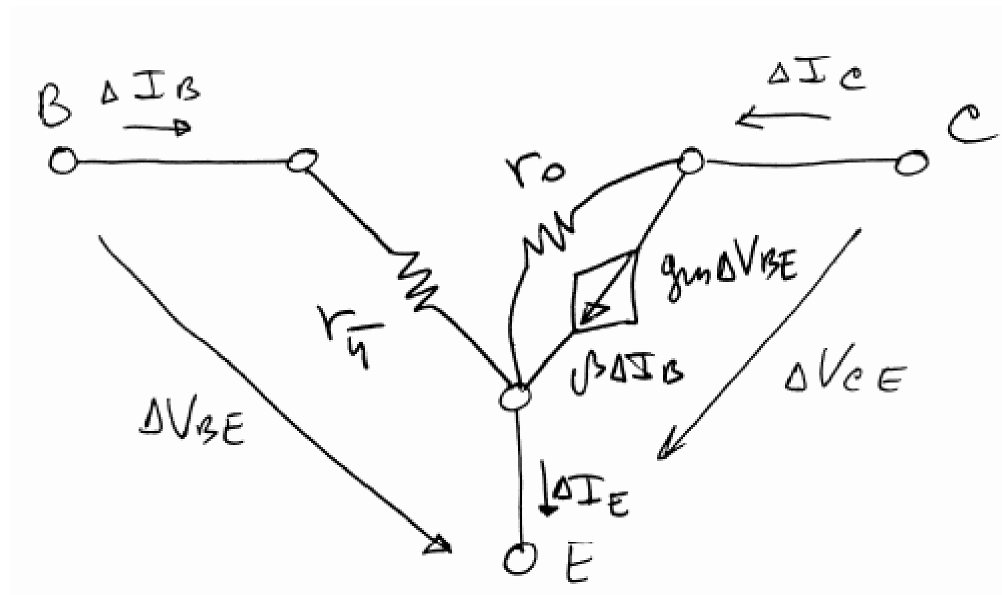
After bias is figured out we care only about circuit for signals:
DC voltage source – short ;
DC current sources – open.



$$\Delta I_C = g_m \cdot \Delta V_{BE} = \frac{I_C^Q}{V_{th}} \cdot \Delta V_{BE} \qquad g_m = \frac{I_C^Q}{V_{th}}$$

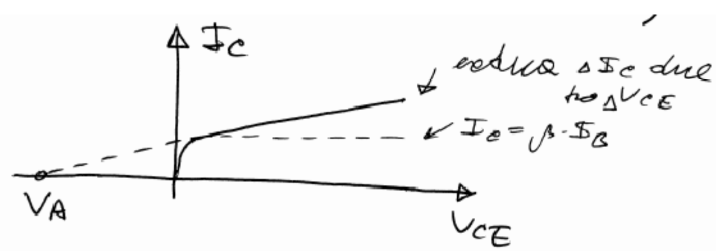
$$\Delta I_B = \frac{\Delta I_C}{\beta} = \frac{g_m}{\beta} \cdot \Delta V_{BE} = \frac{\Delta V_{BE}}{r_\pi} \qquad r_\pi = \frac{\beta}{g_m}$$

Hybrid-π model



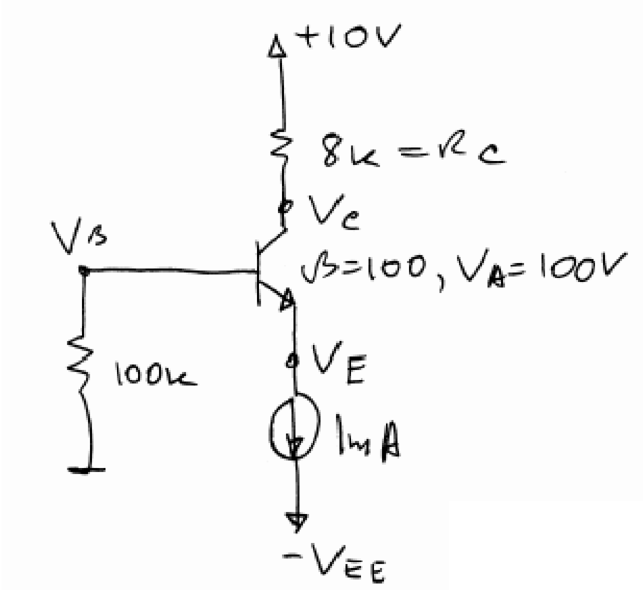
* Bias current I_C^Q determines g_m, r_π and r_0

Early effect – finite output resistance



$$r_o = \frac{\Delta V_{CE}}{\Delta I_C} \simeq \frac{V_A}{I_C^Q}$$

Example



Find bias point and determine small signal parameters

1. Bias point V_C

$$I_B = \frac{I_E}{\beta+1}, \text{ hence } I_B = \frac{1mA}{101} \simeq 10\mu A$$

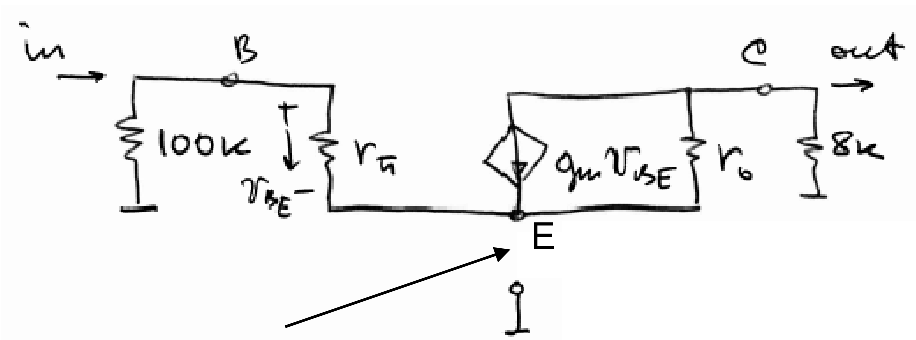
$$V_B = -10\mu A \cdot 100k\Omega = -1V$$

$$V_E = V_B - 0{,}7V = -1{,}7V$$

$$V_C = 10V - 8k \cdot 1mA = +2V$$

** Observe (a) V_{CE} > 0.2-0.3V, hence indeed FA mode;*
(b) R_C determines output voltage swing

Hybrid-π model



** Observe open circuit for signal without bypass capacitor*

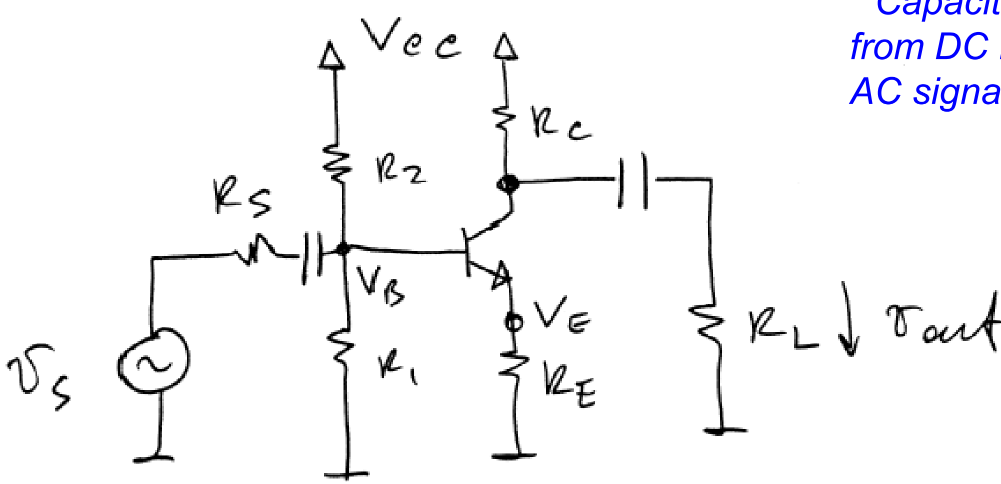
2. Small signal parameters

$$g_m = \frac{I_C^Q}{V_{th}} = \frac{1mA}{25mV} = 40\frac{mA}{V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40\,mA/V} = 2.5k\Omega$$

$$r_O = \frac{V_A}{I_C^Q} = \frac{100V}{1mA} = 100k\Omega$$

Common Emitter Amplifier with resistive bias (“classic” bias)



** Capacitors are required to decouple signal and load from DC bias circuit. Assume they are short circuits for AC signals but open circuits for DC.*

Bias: $V_B \simeq V_{cc}\,\dfrac{R_1}{R_1+R_2}$

$V_E = V_B - 0{,}7\,V$

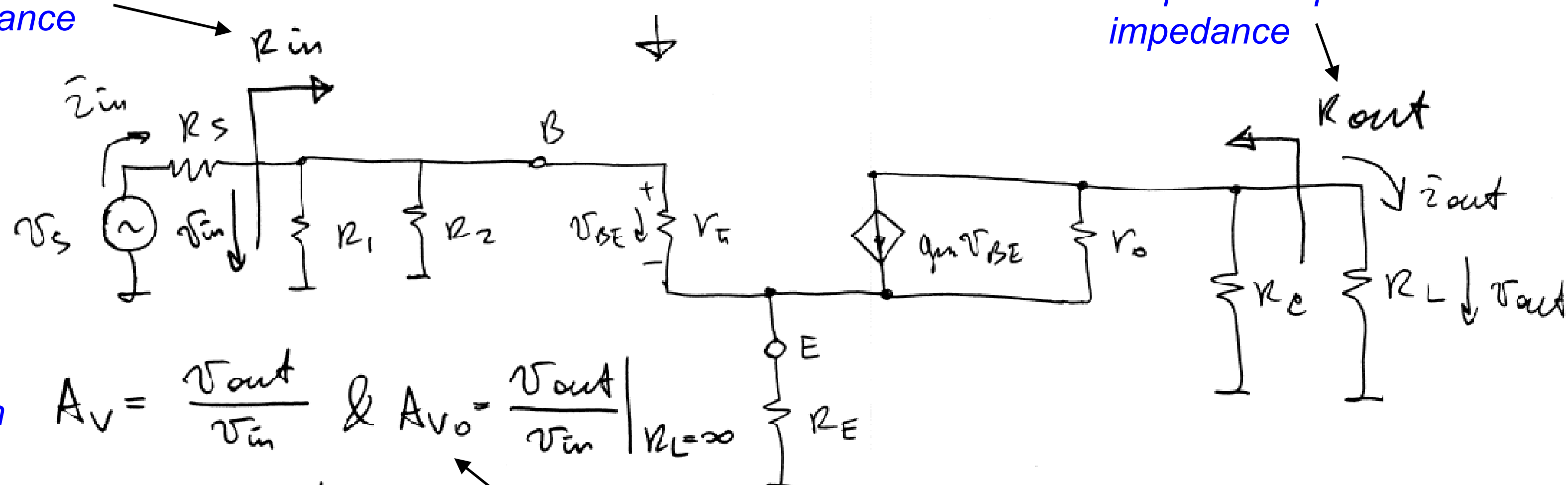
$I_C \simeq V_E/R_E$

& find g_m, r_π & r_o !

Equivalent circuit for signals

Amplifier input impedance

Amplifier output impedance



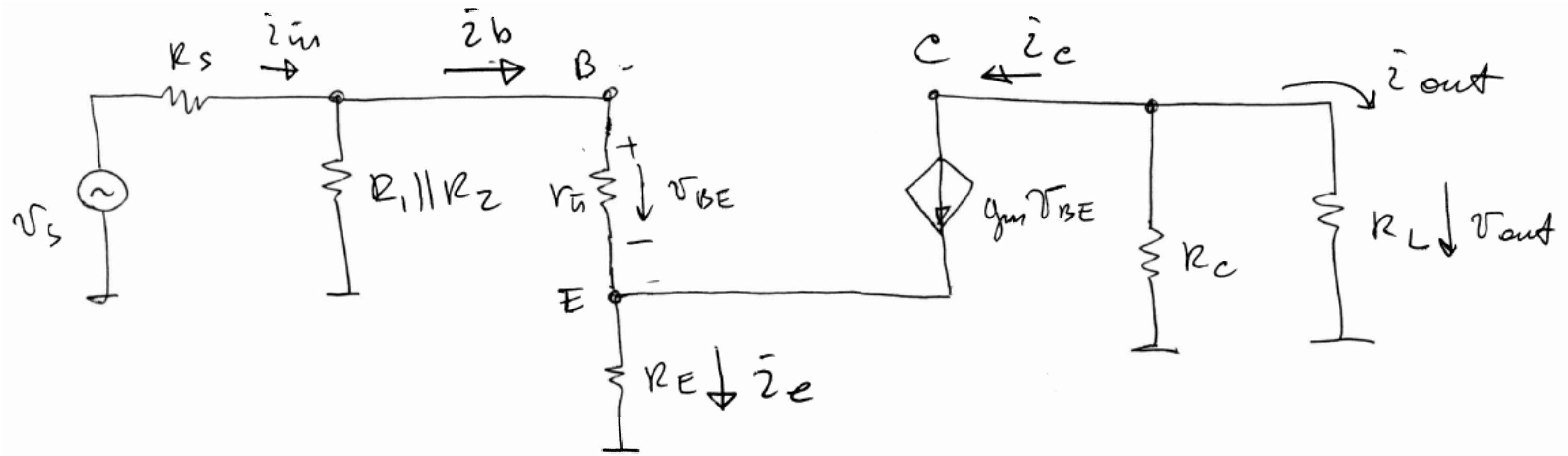
Voltage gain $A_V = \dfrac{v_{out}}{v_{in}}$ & $A_{Vo} = \left.\dfrac{v_{out}}{v_{in}}\right|_{R_L=\infty}$

Open circuit voltage gain

Net voltage gain $G_V = \dfrac{v_{out}}{v_s}$

Common Emitter Amplifier with resistive bias ("classic" bias): ignore r_o.

** Exact analysis is possible but more complicated.*



Voltage gains

$$v_{in} = v_{BE} + i_e \cdot R_E = i_b \left(r_\pi + (\beta+1) R_E \right)$$

$$v_{out} = -i_c \left(R_C \| R_L \right) = -\beta \cdot i_b \left(R_C \| R_L \right)$$

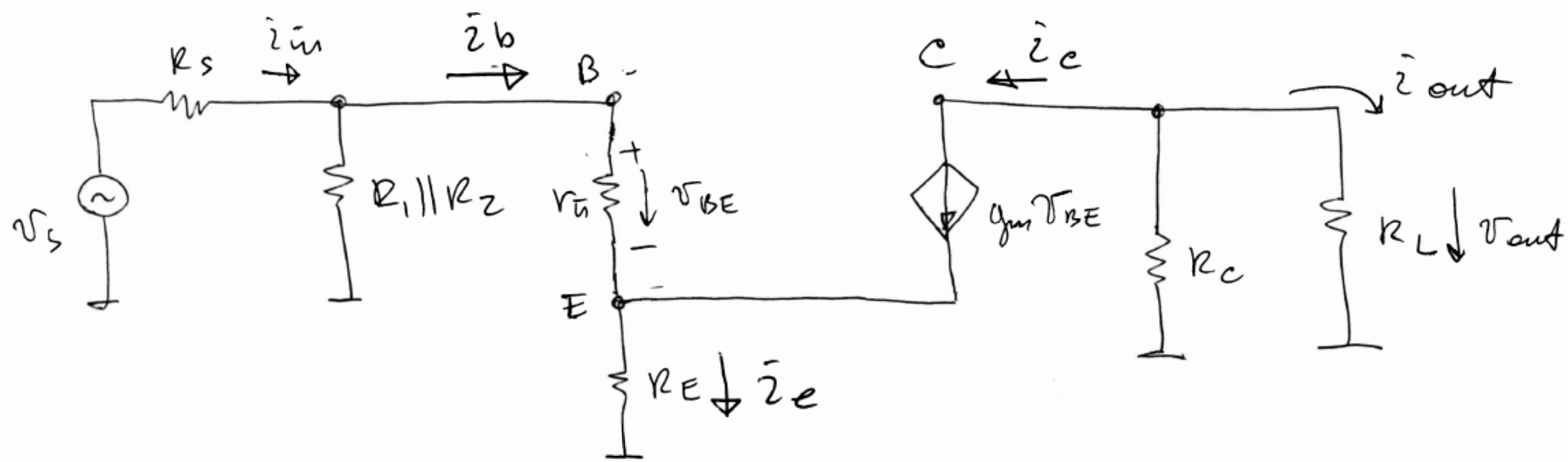
$$A_V = -\frac{\beta \left(R_C \| R_L \right)}{r_\pi + (\beta+1) R_E} \quad ; \quad A_{V0} = -\frac{\beta \cdot R_C}{r_\pi + (\beta+1) R_E}$$

$$R_E = 0 \rightarrow A_{V0} = -\frac{\beta}{r_\pi} \cdot R_C = -g_m \cdot R_C$$

$$R_E >> \frac{1}{g_m} \rightarrow A_{V0} \approx -\frac{R_C}{R_E}$$

Common Emitter Amplifier with resistive bias (“classic” bias): ignore r_o.

** Exact analysis is possible but more complicated.*



Input impedance

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{\frac{i_b}{v_{in}} + \frac{1}{(R_1 \| R_2)}} = \frac{1}{\frac{1}{r_\pi + (\beta+1)R_E} + \frac{1}{R_1 \| R_2}}$$

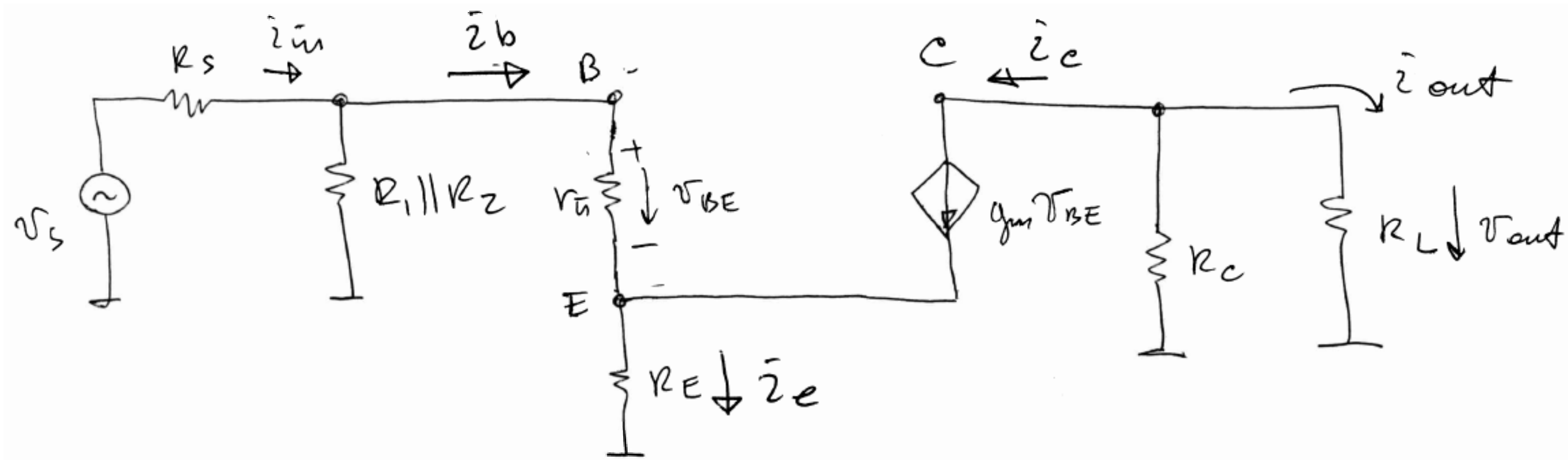
$$i_{in} = i_b + \frac{v_{in}}{(R_1 \| R_2)} \; ; \qquad R_{in} = \left(R_1 \| R_2 \| (r_\pi + (\beta+1)R_E\right)$$

$$R_E = 0 \rightarrow R_{in} = R_1 \| R_2 \| r_\pi \approx r_\pi$$

$$R_E >> \frac{1}{g_m} \rightarrow R_{in} >> r_\pi$$

Common Emitter Amplifier with resistive bias ("classic" bias): ignore r_o.

** Exact analysis is possible but more complicated.*



Output impedance and net voltage gain

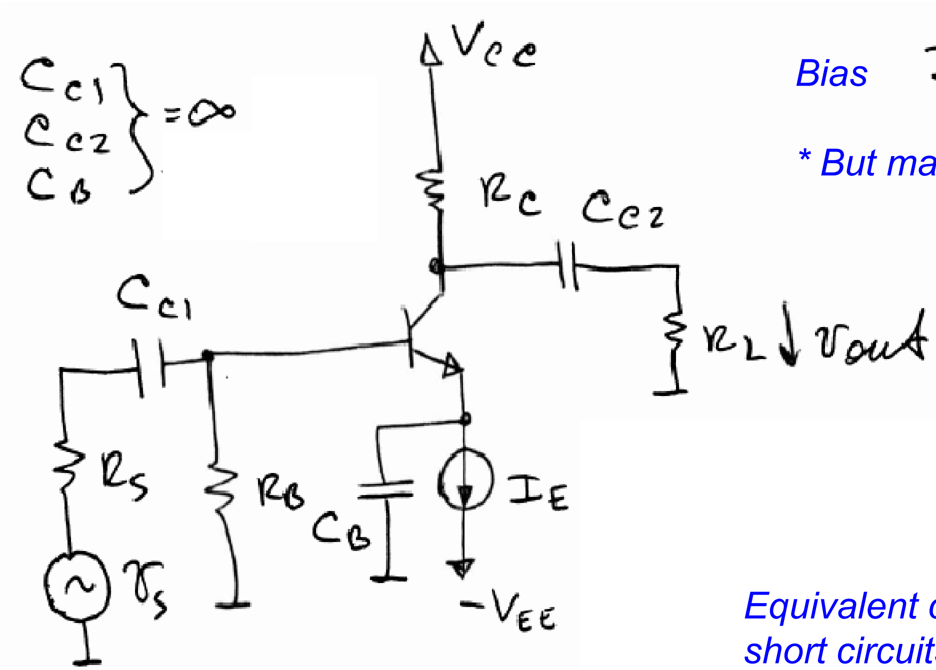
$$R_{out} = -\frac{v_{out}}{i_{out}} \text{ for } v_s = 0 \text{, hence } R_{out} = R_C$$

$$G_V = A_V \cdot \frac{v_{in}}{v_S} = A_V \cdot \frac{R_{in}}{R_{in} + R_S} = \frac{R_{in}}{R_{in} + R_S} \cdot A_{VO} \cdot \frac{R_C // R_L}{R_C}$$

$$\frac{R_C // R_L}{R_C} = \frac{R_L}{R_{out} + R_L}$$

Summary:
CE amp is inverting amplifier with current and voltage gains.
Input resistance increases and voltage gain decreases when R_E is used to stabilize DC bias.

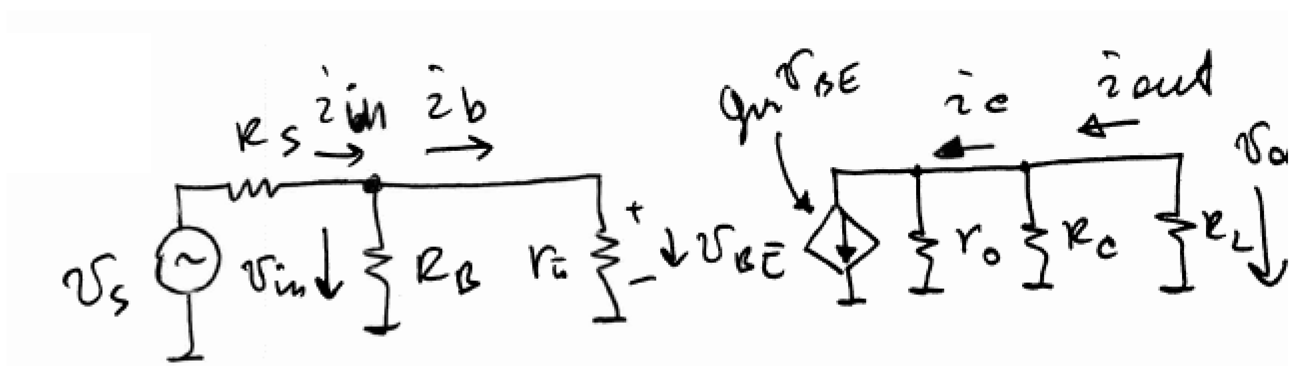
Common Emitter Amplifier biased by current source.



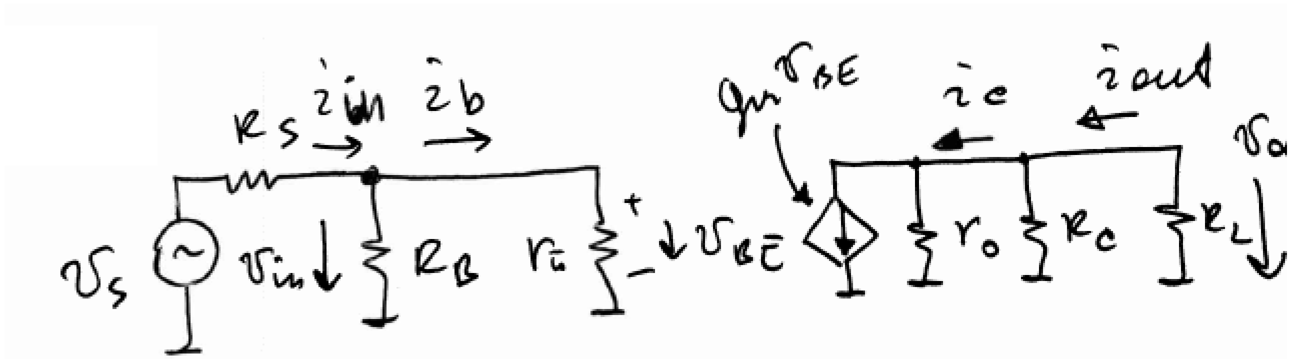
Bias $I_E \rightarrow I_C^Q = \alpha \cdot I_E \rightarrow g_m, r_\pi, r_o$

** But make sure that BJT is in FA, i.e. $V_{CE}^Q > 0.2\text{-}0.3V$*

Equivalent circuit based on Hybrid-π model, caps are short circuits for signals



Common Emitter Amplifier biased by current source.



Voltage gains

$$A_v = \frac{v_{out}}{v_{in}} = \frac{(-g_m v_{BE})(r_o \| R_C \| R_L)}{v_{in} = v_{BE}} = -g_m (r_o \| R_C \| R_L)$$

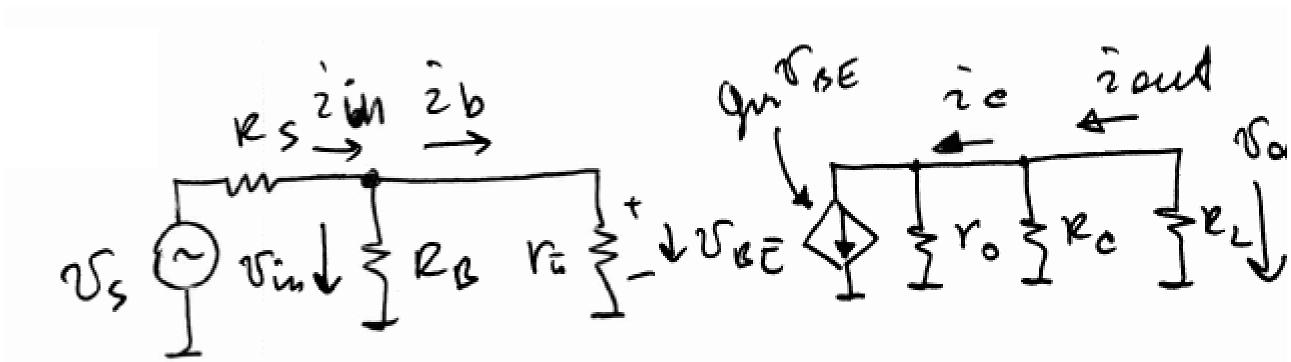
since $R_E = 0$!

$$A_{vo} = -g_m (r_o \| R_C)$$

$$A_{V0}^{MAX} = -g_m \cdot r_O$$

Finite output resistance puts upper limit on amplifier voltage gain

Common Emitter Amplifier biased by current source.



Input/output impedances

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{v_{BE}}{\frac{v_{BE}}{R_B} + i_b} = \frac{1}{\frac{1}{R_B} + \frac{1}{r_\pi}} = (R_B \| r_\pi) \simeq r_\pi$$

Not high enough - problem

$$R_{out} = \left.\frac{v_{out}}{i_{out}}\right|_{v_s=0} = (R_c \| r_o) \simeq R_c$$

Net voltage gain $G_v = A_{vo} \cdot \frac{R_{in}}{R_{in}+R_s} \cdot \frac{R_L}{R_L + R_{out}}$

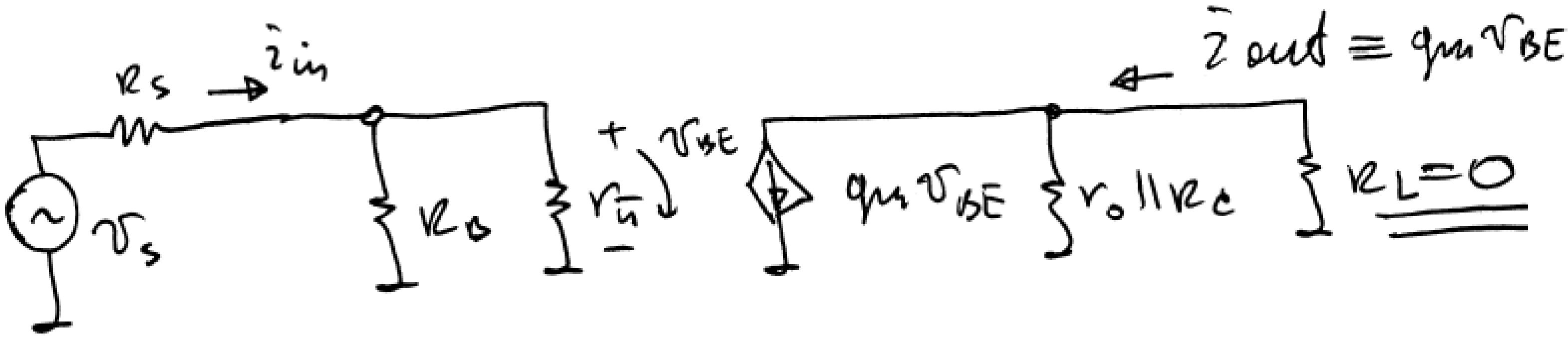
$$G_v = A_v \frac{R_{in}}{R_{in}+R_s} \simeq \left| \begin{matrix} R_B \gg r_\pi \\ R_{in} \simeq r_\pi \end{matrix} \right| \simeq -\overbrace{\left(\beta / r_\pi\right)}^{g_m} \cdot (r_o \| R_c \| R_L) \cdot \frac{r_\pi}{r_\pi + R_s}$$

$$G_v \simeq \beta \cdot \frac{r_o \| R_c \| R_L}{R_s} \quad \text{for} \quad R_s \gg r_\pi$$

Depends on β, hence unstable

Common Emitter Amplifier biased by current source.

Short circuit current gain

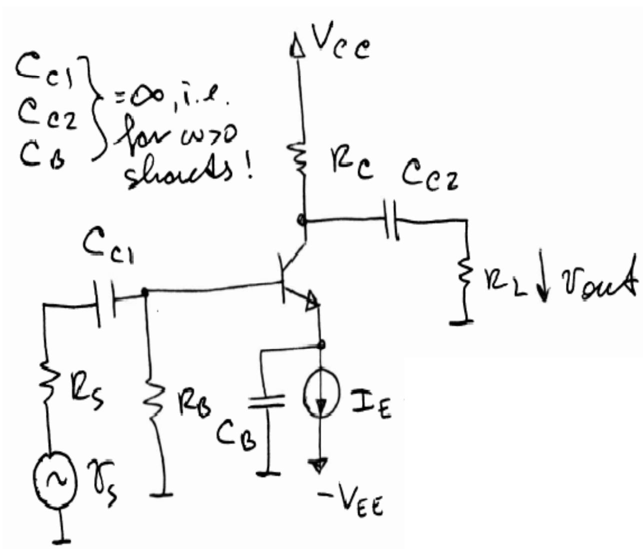


$$i_{in} = v_{BE}/R_{in}$$

$$A_{I\,sh.c.} = \frac{g_m \cdot v_{BE}}{v_{BE}/R_{in}} = g_m \cdot R_{in} \underset{\text{for } R_B \gg r_\pi}{\simeq} g_m \cdot r_\pi \equiv \beta$$

Common emitter current gain

Example



npn - BJT with $\beta = 100$ and $V_A = 100\text{V}$.

Bias current $\quad I_E = 1\text{mA}, \quad V_{CC} = -V_{EE} = 10\text{V}$

$R_B = 100\text{k}, \quad R_C = 8\text{k}, \quad R_S = 5\text{k}, \quad R_L = 5\text{k}$

1. Check if BJT is in FA regime

$$I_B = \frac{I_E}{\beta + 1} = \frac{1\text{mA}}{101} \approx 10\mu$$

$$V_B = -10\mu \cdot 100\text{k} = -1\text{V} \rightarrow V_E = -1\text{V} - 0.7\text{V} = -1.7\text{V} > -10\text{V}.$$

$$V_C \approx 10\text{V} - 1\text{m} \cdot 8\text{k} = 2\text{V} \rightarrow V_{CE} \approx 2\text{V} - (-1.7\text{V}) = 3.7\text{V} > 0.3\text{V}$$

2. Calculate small signal parameters

$$g_m = \frac{I_C^Q}{V_{th}} \approx \frac{1\text{mA}}{25\text{mV}} = 40\frac{\text{mA}}{\text{V}}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40\,\text{mA}/\text{V}} = 2.5\text{k}\Omega$$

$$r_O = \frac{V_A}{I_C^Q} \approx \frac{100\text{V}}{1\text{mA}} = 100\text{k}\Omega$$

3. Find voltage gain

$$G_V = \frac{R_{in}}{R_{in} + R_S} \cdot A_{V0} \cdot \frac{R_L}{R_L + R_{out}}$$

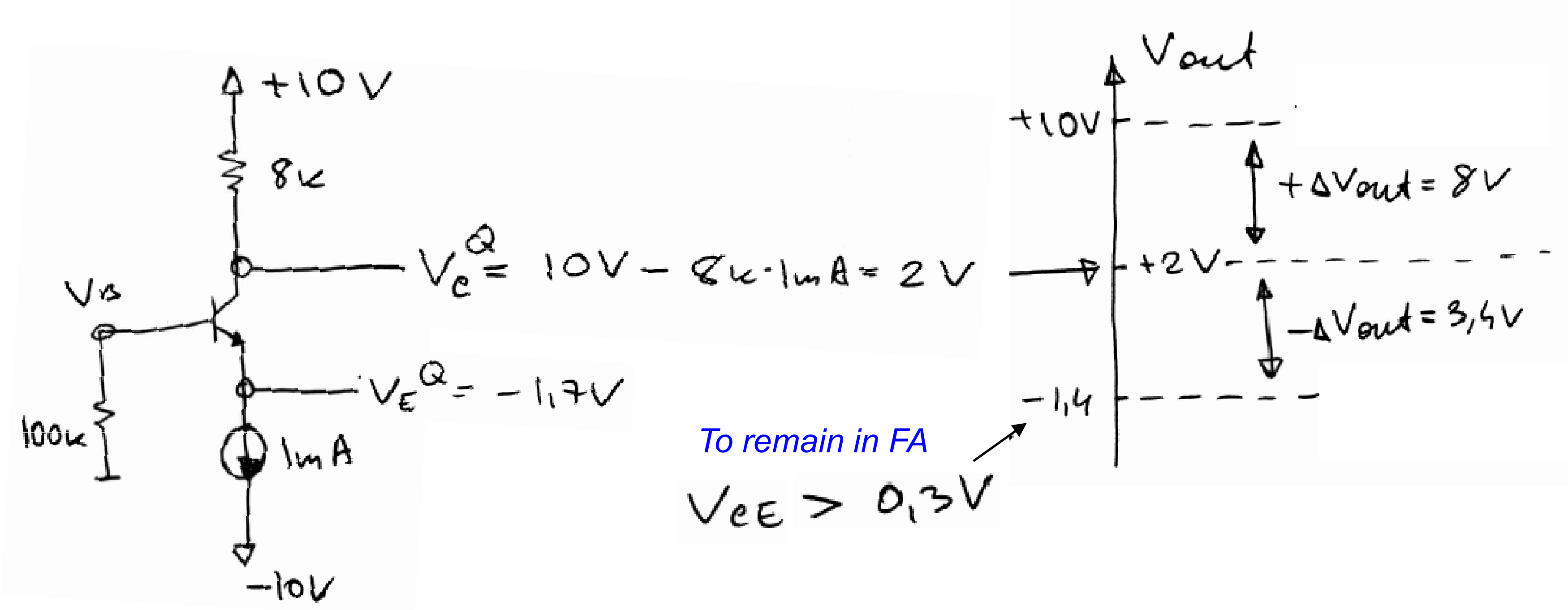
$$A_{V0} = -g_m \cdot (R_C \parallel r_O) \approx -296\frac{\text{V}}{\text{V}}$$

$$R_{in} = (R_B \parallel r_\pi) \approx 2.4\text{k}, \quad R_{out} = (R_C \parallel r_O) \approx 7.4\text{k}$$

$$G_V \approx 0.32 \cdot \left(-296\frac{\text{V}}{\text{V}}\right) \cdot 0.4 \approx -38\frac{\text{V}}{\text{V}}$$

30mV p-p signal corresponds to about 10mV p-p variation of V_{BE}.
This produces about 1.2V p-p across load.

Example – output voltage swing

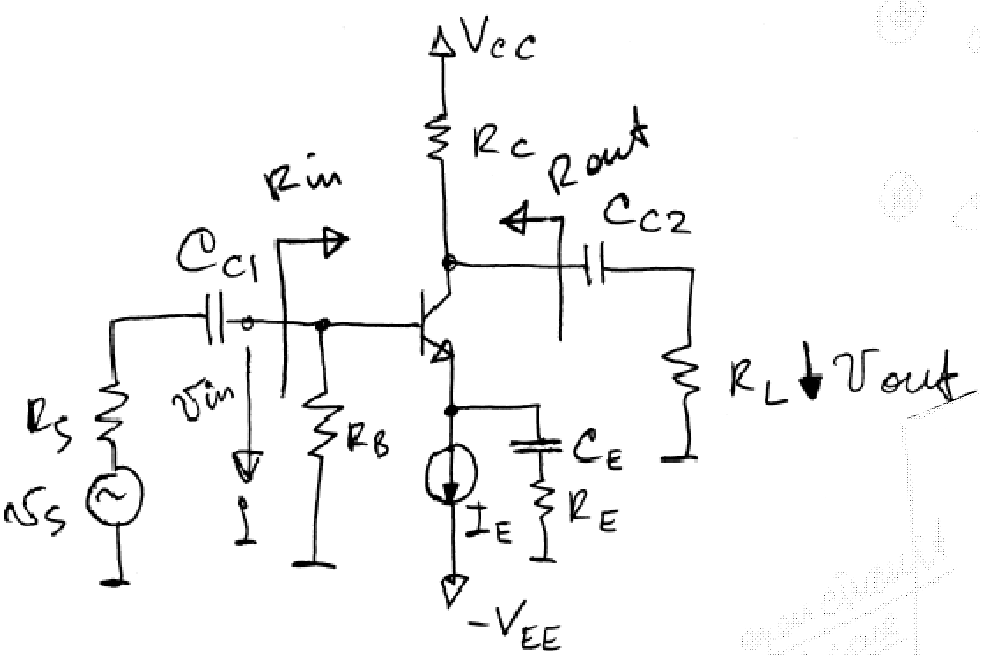


Maximum amplitude of the undistorted sine wave at the output is limited by maximum negative voltage swing, i.e. by 3.4 V.

For 1mA bias current what could we do to increase the maximum output voltage amplitude?

What is the maximum value of this amplitude?

CE amp biased by current source and with R_E.



Case 1

$$R_E = 0$$

$$R_{in} = (R_B \parallel r_\pi) \approx r_\pi$$

$$A_{V0} = -g_m \cdot (R_C \parallel r_O)$$

Case 2 (neglect r_O)

$$R_E \neq 0$$

$$R_{in} = (R_B \parallel (r_\pi + R_E \cdot [\beta + 1])) > r_\pi$$

$$A_{V0} = -\frac{\beta \cdot R_C}{r_\pi + R_E \cdot [\beta + 1]}$$

Negative feedback resistor R_E improves input impedance for the expense of gain