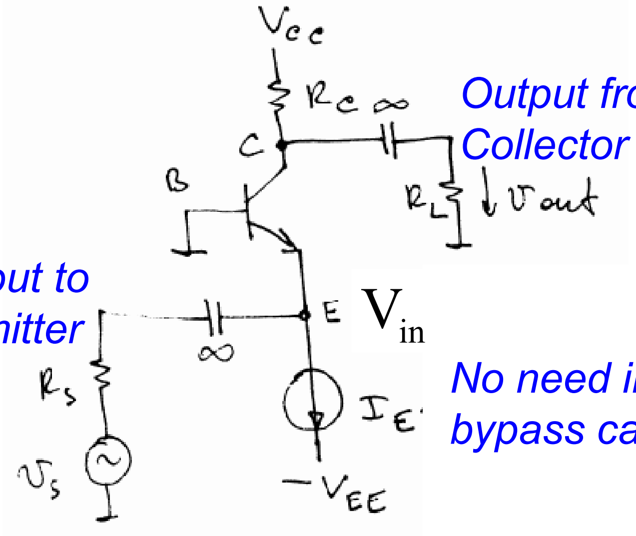
Common Base

Biased by current source



Output from Collector

Input to Emitter

No need in bypass cap

Start with bias DC analysis – make sure BJT is in FA, then calculate small signal parameters for AC analysis.

**ignore r_O for simplicity, then:*

$$A_v = \frac{v_{out}}{v_{in}} = g_m (R_L \| R_C)$$ *noninverting*

$$A_{vo} = A_v \Big|_{R_L=\infty} = g_m \cdot R_L$$

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{r_\pi}{\beta+1}$$ *Small !*

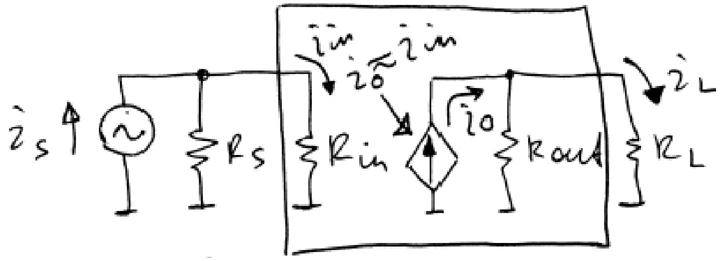
$$R_{out} = \frac{v_{out}}{i_{out}} \Big|_{v_s=0} = R_C$$

$$G_v = \frac{v_{out}}{v_s} = \frac{R_{in}}{R_{in}+R_s} \cdot A_v \approx \frac{R_L \| R_C}{R_s}$$

Short circuit current gain

$$\frac{i_{out}}{i_{in}} \Big|_{R_L=0} = \alpha \approx 1$$

The circuit is current buffer: delivers current from source to load

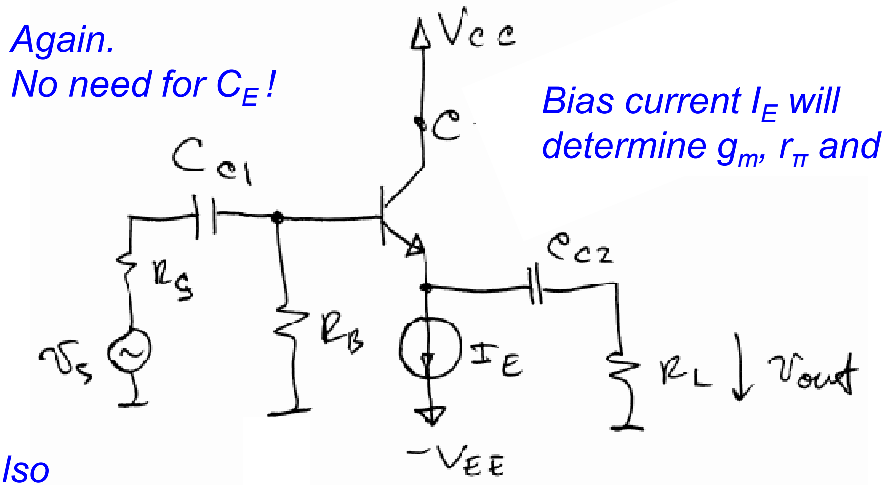


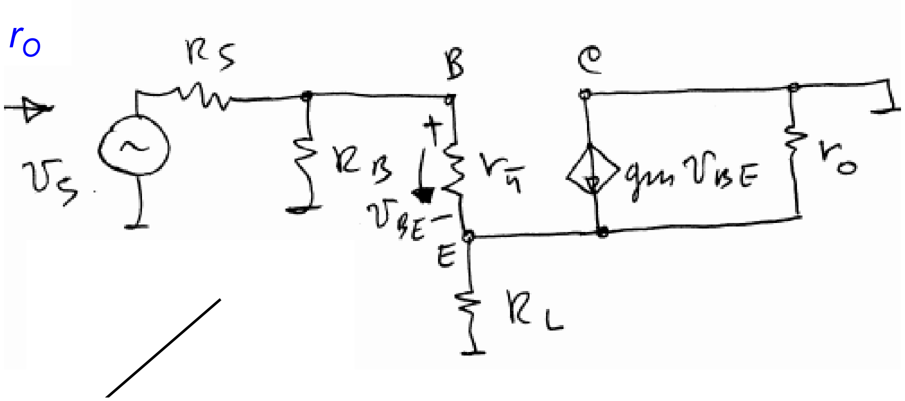
** when BJT output impedance r_O can not be neglected – the circuit is said to perform an impedance transformation.*

Common Collector amplifier

Again.
No need for C_E!

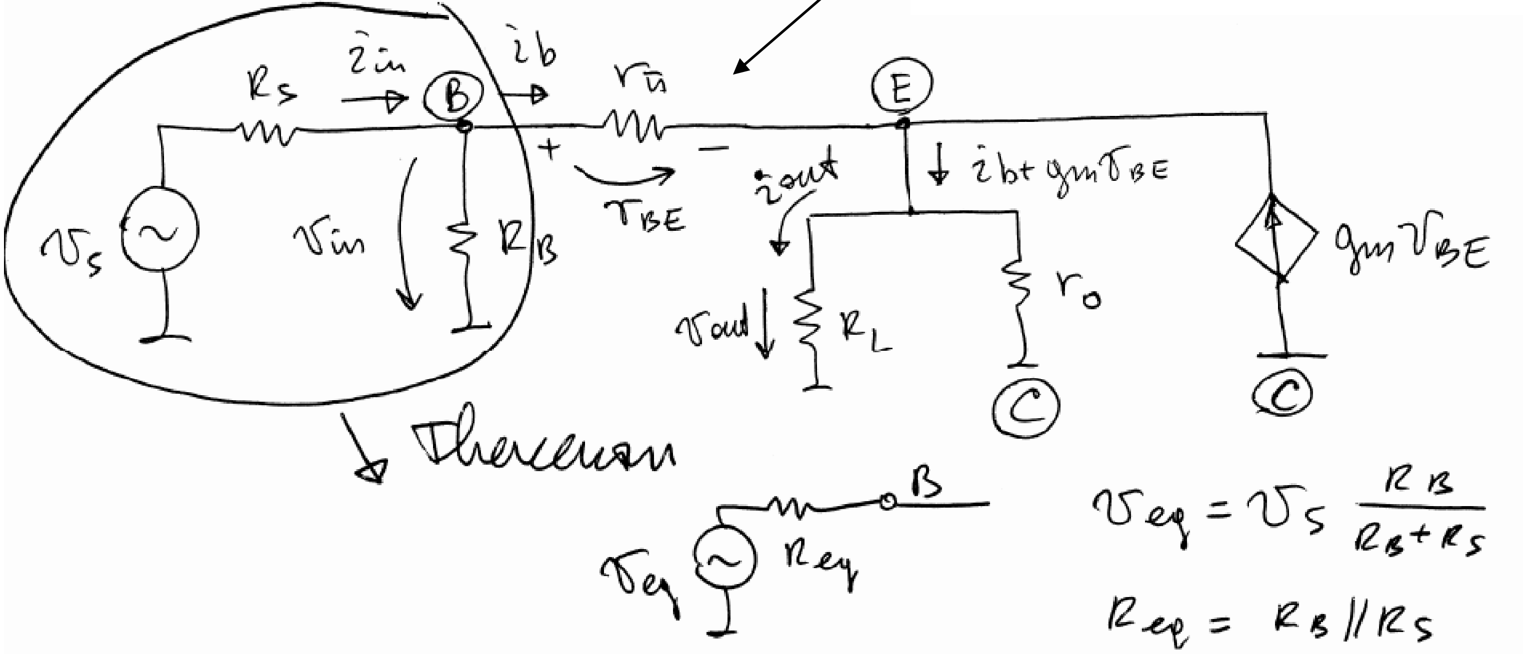
Bias current I_E will determine g_m, r_π and r_O





Also
R_B and C_{C1} can be eliminated

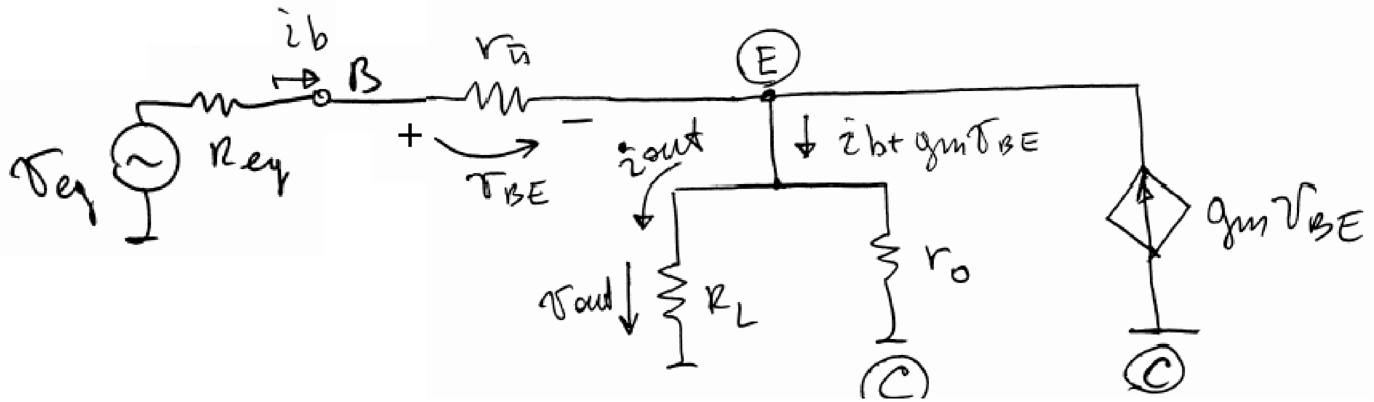
Redraw equivalent circuit in more convenient form



$$v_{eq} = v_S \frac{R_B}{R_B + R_S}$$

$$R_{eq} = R_B \parallel R_S$$

Common Collector amplifier



$$v_{eq} = v_s \frac{R_B}{R_B + R_S} = i_b(r_\pi + R_{eq}) + v_{out}$$

$$v_{out} = (i_b + g_m v_{BE}) \cdot (R_L \| r_o) = i_b(1 + \underset{g_m \cdot r_\pi}{\underset{\shortparallel}{\beta}})(R_L \| r_o)$$

$$G_V = \frac{v_{out}}{v_s} = \frac{v_{out}}{v_{eq}} \cdot \frac{v_{eq}}{v_s} = \frac{R_B}{\underbrace{R_S + R_B}} \cdot \frac{(\beta+1)(R_L \| r_o)}{\underbrace{(r_\pi + R_B \| R_S) + (\beta+1)(R_L \| r_o)}_{<1}} < 1$$

When

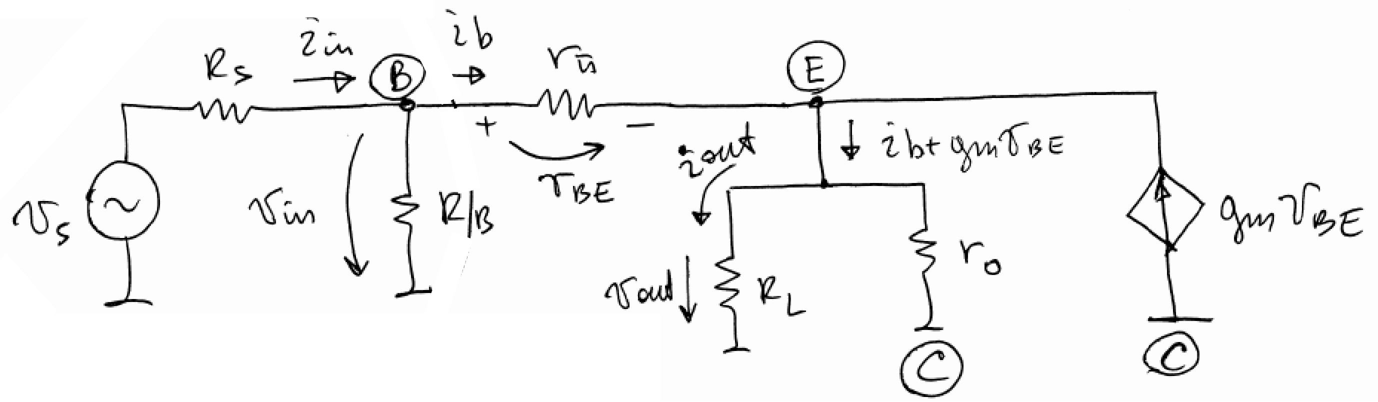
$R_B \gg R_S$

$r_o \gg R_L$

$$G_V = \frac{(\beta+1) R_L}{[\underbrace{R_S + r_\pi}_{k\Omega}] + \underset{\underset{k\Omega}{\downarrow}}{R_L}\underbrace{(\beta+1)}_{100}} \simeq 1$$

i.e. no voltage gain !

Common Collector amplifier



Input impedance

$$R_{in} = \frac{v_{in}}{i_{in}}$$

$$v_{in} = i_b \cdot r_\pi + \overbrace{(\beta+1)(R_L \| r_o)\cdot i_b}^{v_{out}}$$

$$i_{in} = v_{in}/R_B + i_b = i_b\left(\frac{r_\pi}{R_B} + 1 + \frac{\beta+1}{R_B}(R_L\|r_o)\right)$$

$$R_{in} = \frac{[r_\pi + (\beta+1)(R_L\|r_o)]\cdot R_B}{R_B + [r_\pi + (\beta+1)(R_L\|r_o)]} = \underset{\text{can be }\infty}{R_B} \| \left(\underset{k\Omega}{r_\pi} + \overset{100}{(\beta+1)}(\underset{k\Omega}{R_L} \| \underset{100k}{r_o})\right)$$

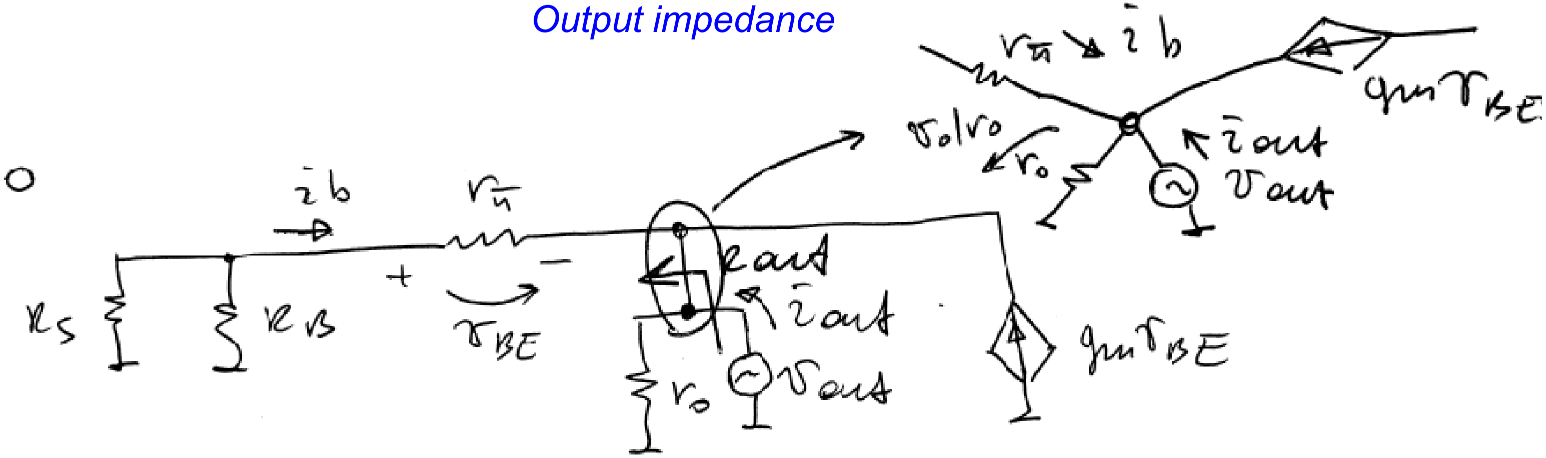
$$R_{in} \approx (\beta+1)\cdot R_L \quad \text{, for } R_L << r_o$$

Impedance transformation

Common Collector amplifier

Output impedance

$v_s = 0$



$$v_{out} = -i_b(R_S \| R_B) - v_{BE} = -i_b(R_S \| R_B + r_\pi)$$

$$i_{out} = \frac{v_{out}}{r_o} - i_b - g_m v_{BE} =$$

$$= -i_b \frac{R_S \| R_B + r_\pi}{r_o} - i_b - i_b \cdot \beta$$

$$R_{out} = \frac{r_\pi + R_S \| R_B}{(\beta+1) + \frac{r_\pi + R_S \| R_B}{r_o}} = \left(r_o \,\Big\|\, \frac{r_\pi + (R_S \| R_B)}{\beta+1} \right)$$

$R_B \gg R_S$ & $r_o \gg$ all then $R_{out} \simeq \frac{R_S}{\beta+1}$

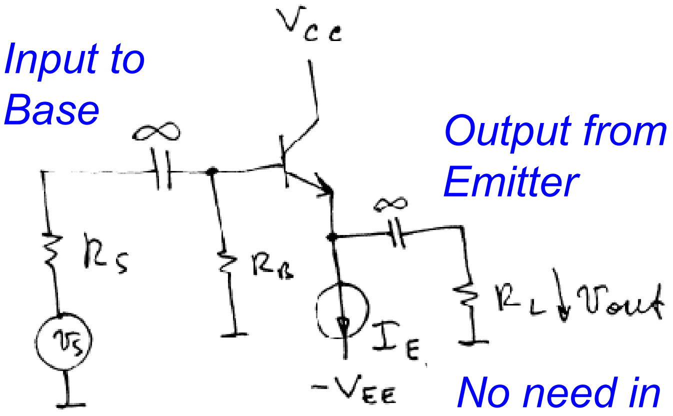
$R_S \gg r_\pi$

Impedance transformation

Common Collector (Emitter follower)

Biased by current source

Input to Base



Output from Emitter

No need in bypass cap

Short circuit current gain is almost the same as in case of CE amp, namely β+1.

Start with bias DC analysis – make sure BJT is in FA, then calculate small signal parameters for AC analysis.

$$G_v = \frac{v_{out}}{v_S} = \frac{R_B}{R_S + R_B} \cdot \frac{(\beta+1)(R_L \| r_o)}{\left[(R_S \| R_B) + r_\pi\right] + (\beta+1)(R_L + r_o)} < 1$$

Often $R_B >> R_S$ and $r_O >> R_L$, then

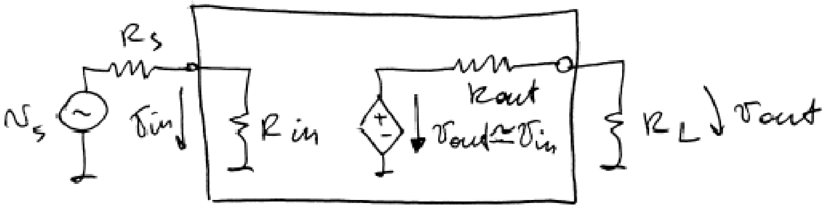
$$G_v \approx \frac{(\beta+1)\cdot R_L}{(R_S + r_\pi) + R_L(\beta+1)} \approx 1$$

$$R_{in} = \frac{v_{in}}{i_{in}} = \Big[\underset{\substack{\swarrow \\ \infty}}{R_B} \,\|\, (\underset{\substack{\downarrow \\ \sim k\Omega}}{r_\pi} + \underset{\substack{\downarrow \\ \sim 100}}{(\beta+1)}(\underset{\substack{\swarrow \\ \sim k\Omega}}{R_L} \| \underset{\substack{\searrow \\ \sim 100k\Omega}}{r_o})\Big] \approx (\beta+1)\cdot R_L$$

Impedances transformed

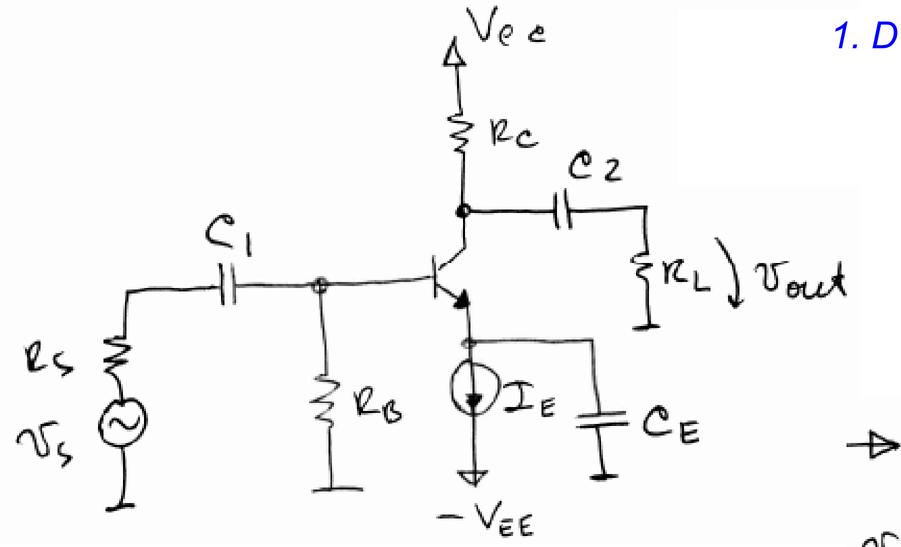
$$R_{out} = \left.\frac{v_{out}}{i_{out}}\right|_{v_S = 0} = r_o \,\|\, \frac{r_\pi + (R_S \| R_B)}{\beta+1} \approx \frac{R_S}{\beta+1}$$

The circuit is voltage buffer: delivers voltage from source to load



Frequency response of Common Emitter amplifier

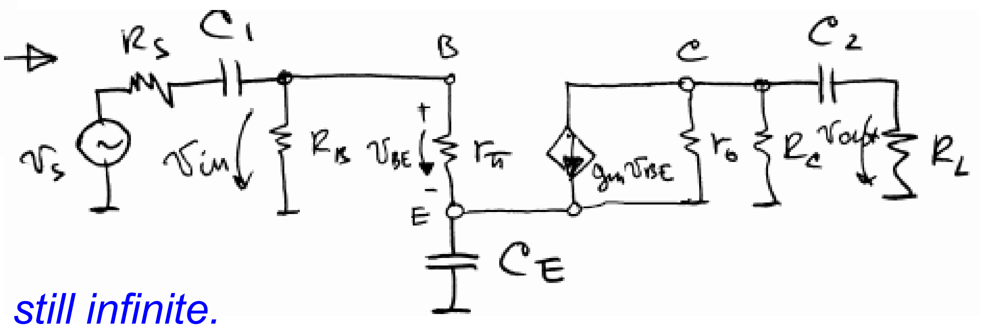
Low frequencies, i.e. BJT itself is fast enough



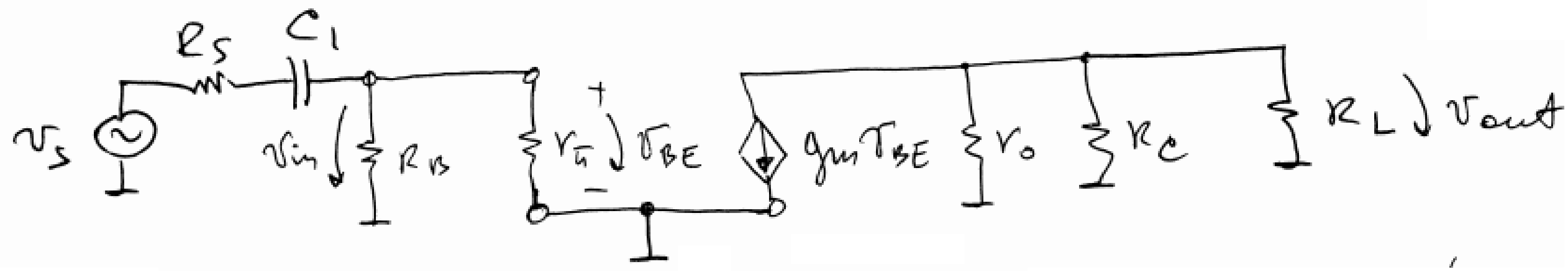
1. DC bias – make sure BJT is in FA - AC analysis.

***Before** – assumed coupling caps are big enough to act as a short circuit for any frequency of AC signal.*

***Now** – assume they have finite values.*



***1.** Assume C_1 is finite while C_2 and C_E are still infinite.*



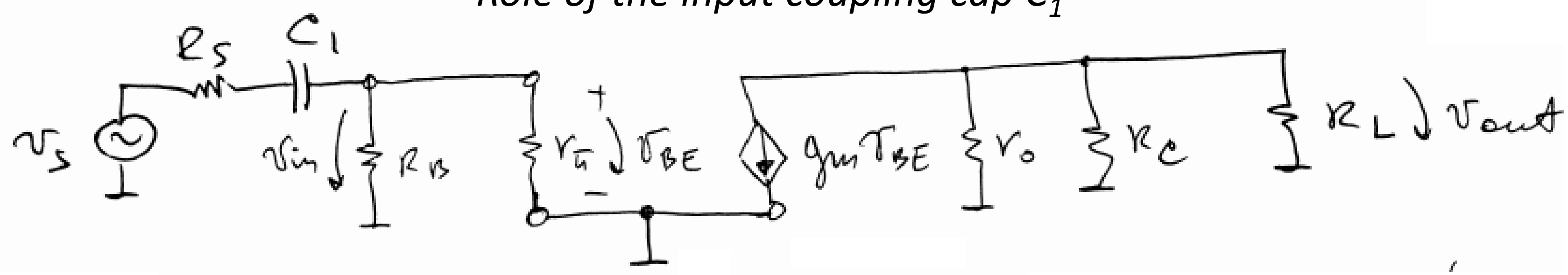
$$G_v(f) = \frac{v_{out}(f)}{v_s(f)} = \underbrace{\frac{v_{out}(f)}{v_{in}(f)}} \cdot \boxed{\frac{v_{in}(f)}{v_s(f)}} \rightarrow$$

Depends on frequency.

Frequency independent.

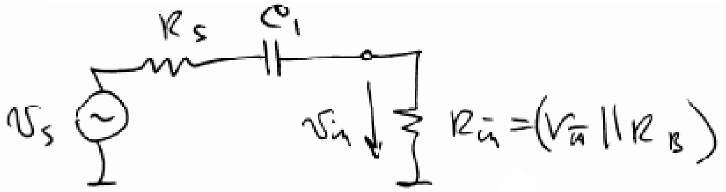
Frequency response of Common Emitter amplifier

Role of the input coupling cap C_1



$$G_v(f) = \frac{v_{out}(f)}{v_s(f)} = \underline{\frac{v_{out}(f)}{v_{in}(f)}} \cdot \boxed{\frac{v_{in}(f)}{v_s(f)}} \rightarrow$$

Depends on frequency.

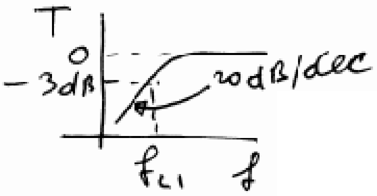


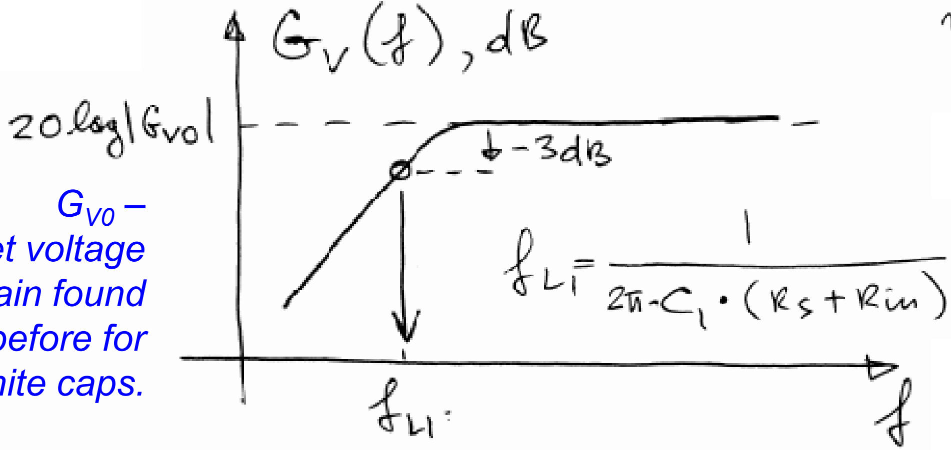
Voltage gain found before $(-g_m)(r_o \| R_C \| R_L) = A_V$

$$\frac{v_{in}}{v_s} = \frac{R_{in}}{R_s + R_{in} + \frac{1}{j\omega C_1}} = \underline{\frac{R_{in}}{R_s + R_{in}}} \cdot \frac{jf/f_{L1}}{1 + jf/f_{L1}}$$

Input voltage divider found before.

High Pass Filter





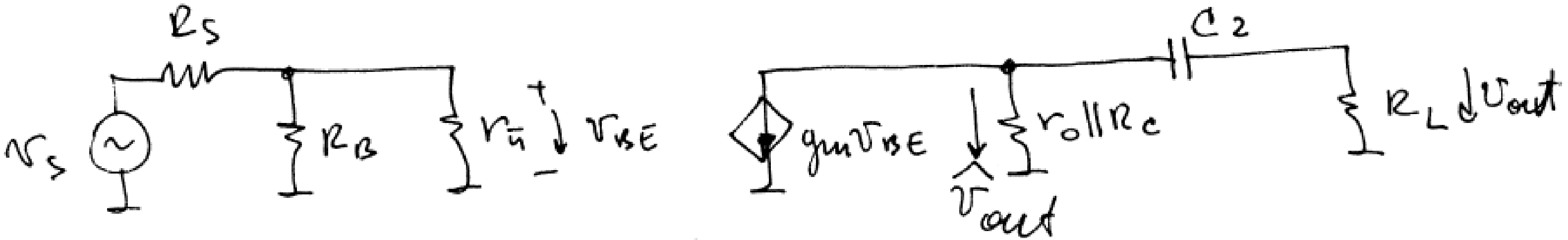
$$f_{L1} = \frac{1}{2\pi \cdot C_1 \cdot (R_s + R_{in})}$$

G_{V0} – net voltage gain found before for infinite caps.

Frequency response of Common Emitter amplifier

Role of the output coupling cap C_2

***2.** Assume C_2 is finite while C_1 and C_E are still infinite.*



$$G_V(f) = \frac{v_{out}(f)}{v_s(f)} = \frac{\hat{v}_{out}(f)}{v_s(f)} \cdot \frac{v_{out}(f)}{\hat{v}_{out}(f)}$$

$$G_V(f) = \frac{R_{in}}{R_{in}+R_s} \cdot A_{V0} \cdot \frac{R_L}{R_{out}+R_L+\frac{1}{j\omega C_2}}$$

$$G_V(f) = \underbrace{\frac{R_{in}}{R_{in}+R_s} \cdot A_{V0} \cdot \frac{R_L}{R_L+R_{out}}} \cdot \underbrace{\frac{R_L+R_{out}}{R_L+R_{out}+\frac{1}{j\omega C_2}}}$$

G_{V0} – net voltage gain found before for infinite caps.

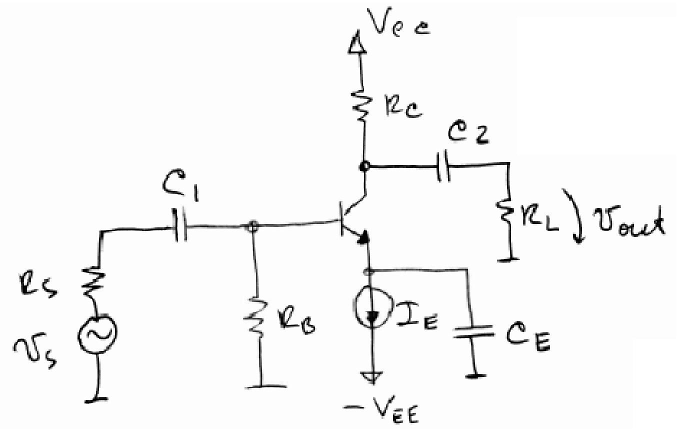
Again High Pass Filter but with 3dB frequency defined by C_2

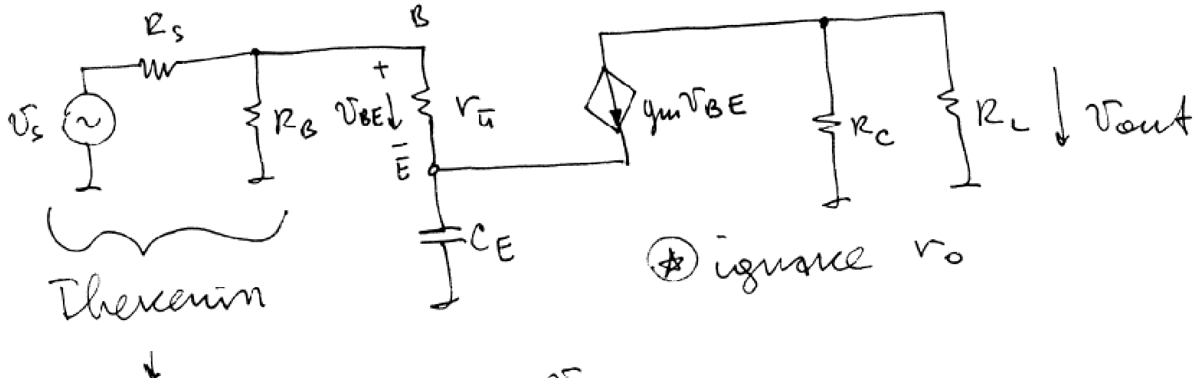
$$= \frac{jf/f_{L2}}{1+jf/f_{L2}}, \quad f_{L2} = \frac{1}{2\pi(R_L+R_{out})\cdot C_2}$$

Frequency response of Common Emitter amplifier

Role of the bypass cap C_E

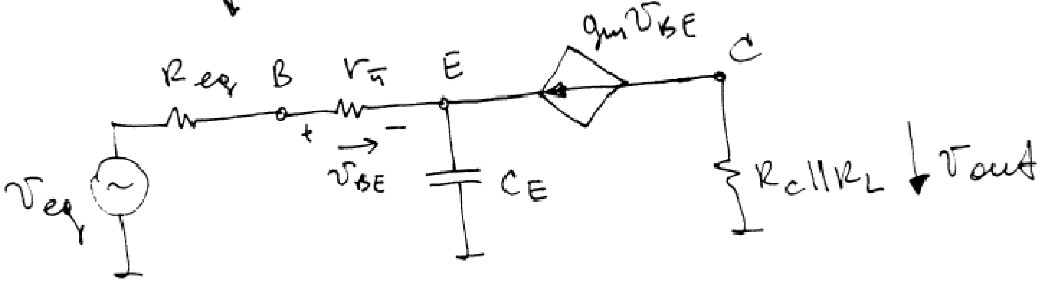
3. *Assume C_E is finite while C_1 and C_2 are still infinite.*





$$v_{eq} = v_s \frac{R_B}{R_S+R_B}$$

$$R_{eq} = \frac{R_B R_S}{R_S+R_B}$$



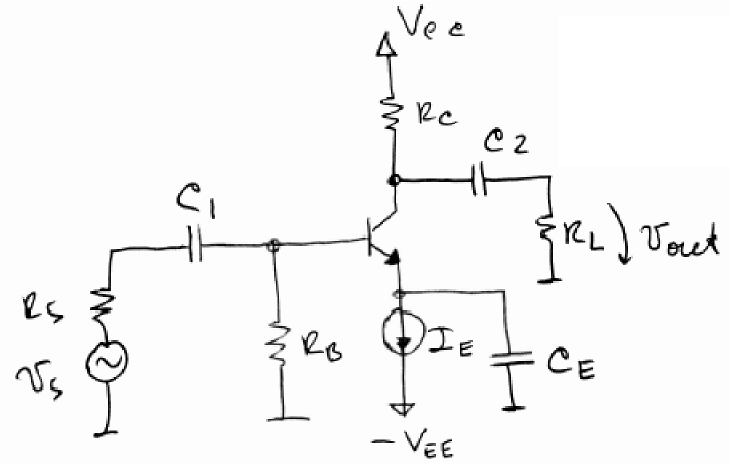
$$G_v(f) = \frac{v_{out}}{v_s} = \frac{-g_m v_{BE}(R_C \| R_L)}{\frac{R_S+R_B}{R_B}\cdot v_{eq}} \propto \frac{v_{BE}}{v_{eq}}.$$

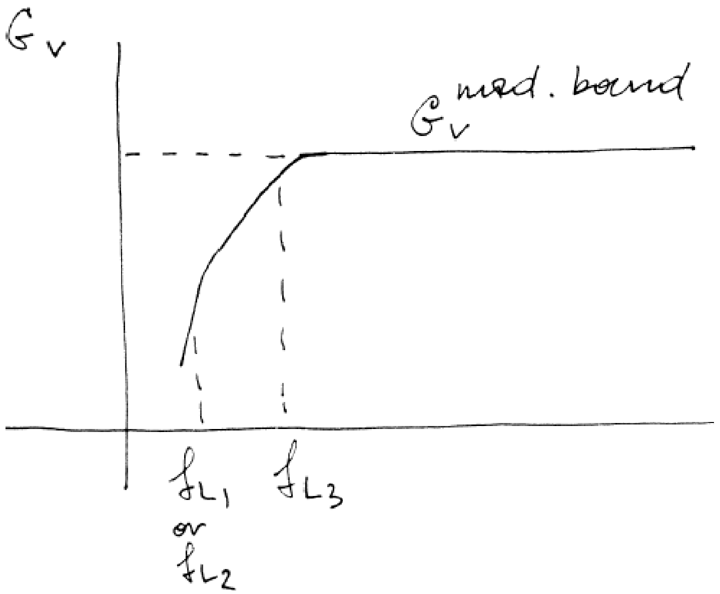
$$\frac{v_{BE}}{v_{eq}} = \frac{\cancel{i_b}\cdot r_\pi}{(r_\pi+R_{eq})\cancel{i_b} + \cancel{i_b}(\beta+1)\frac{1}{j\omega C_E}} = \frac{r_\pi}{r_\pi+R_{eq}}\cdot\frac{j\omega(r_\pi+R_{eq})\frac{C_E}{\beta+1}}{j\omega(r_\pi+R_{eq})\frac{C_E}{\beta+1}+1}$$

Hence: $G_v = G_{v0}\cdot\frac{jf/f_{L3}}{1+jf/f_{L3}}$; $f_{L3} = \frac{1}{2\pi\underbrace{(r_\pi+R_{eq})}\frac{C_E}{\beta+1}}$

Frequency response of Common Emitter amplifier

Low frequency





We have identified three HPF.

$$T_i(f) = \frac{j \cdot f/f_{Li}}{1 + j \cdot f/f_{Li}}$$

$$f_{L1} = \frac{1}{2 \cdot \pi \cdot C_1 \cdot (R_{in} + R_S)}$$

$$f_{L2} = \frac{1}{2 \cdot \pi \cdot C_2 \cdot (R_{out} + R_L)}$$

$$f_{L3} = \frac{1}{2 \cdot \pi \cdot \frac{C_E}{\beta + 1} \cdot (r_\pi + R_S \,||\, R_B)}$$

$$C_1 = C_2 = C_E = 1\,\mu$$

$$R_{in} + R_S \sim kOhm \rightarrow f_{L1} < 200Hz$$

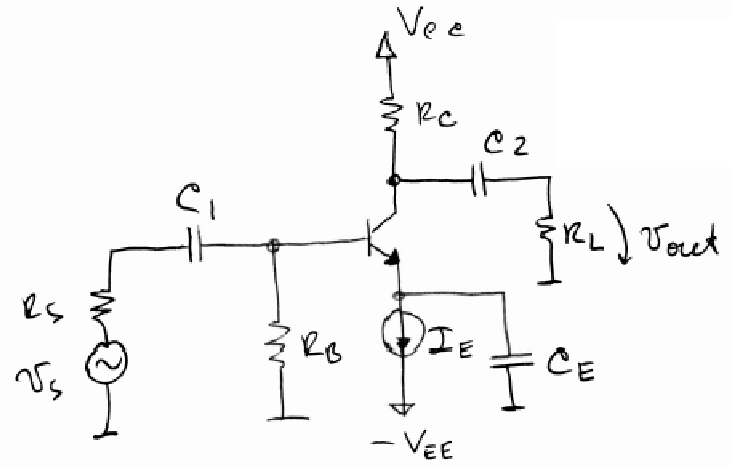
$$R_{out} + R_L \sim 10kOhm \rightarrow f_{L2} < 20Hz$$

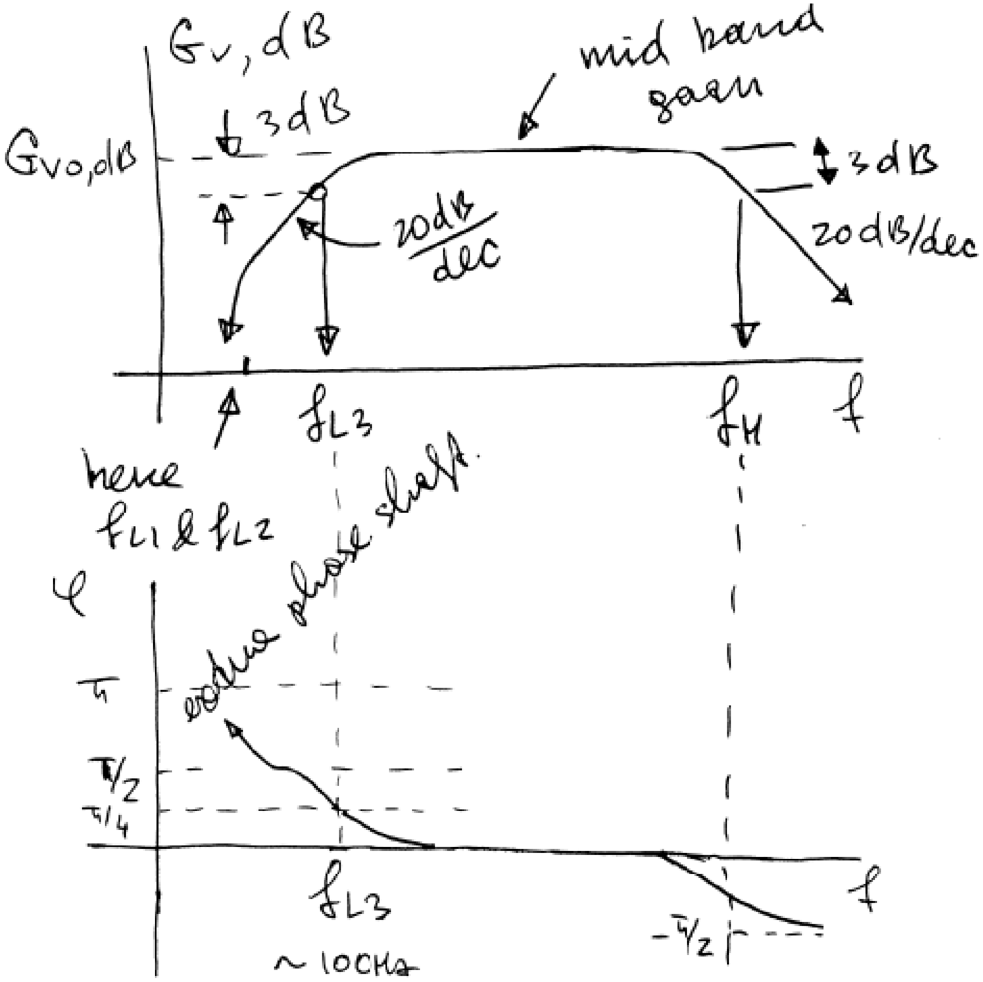
$$r_\pi + R_S \,//\, R_B \sim kOhm \rightarrow f_{L3} > kHz$$

Low frequency cutoff is determined by C_E

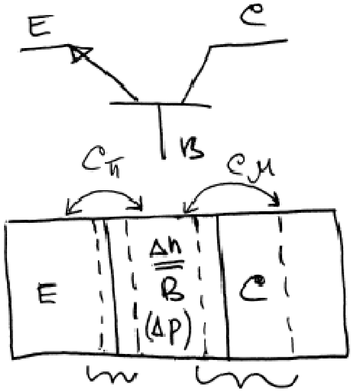
Frequency response of Common Emitter amplifier

Bandwidth





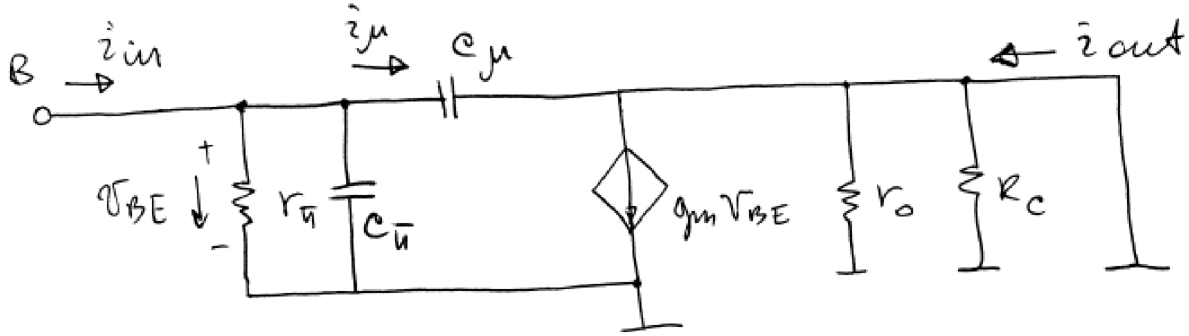
High frequency 3dB determines amplifier bandwidth.



Amplifier bandwidth is determined by BJT high frequency capabilities – determined by internal parasitic capacitances C_π and C_μ.

Frequency response of Common Emitter amplifier

Short circuit current gain at high frequencies



$$\frac{i_{out}}{i_{in}} = ?$$

① $C_\pi = C_\mu = 0$ $\quad \left(i_{out}/i_{in}\right) = \frac{g_m v_{BE}}{i_b} = g_m \cdot r_\pi = \beta_0$

Common emitter current gain defined earlier.

② C_π & $C_\mu \neq 0$

$$i_{out} = g_m v_{BE} - i_\mu = g_m v_{BE} - \frac{v_{BE}}{\frac{1}{j\omega C_\mu}} = \left(g_m - j\omega C_\mu\right)\cdot v_{BE}$$

$$i_{in} = \frac{v_{BE}}{r_\pi} + \frac{v_{BE}}{\frac{1}{j\omega C_\pi}} + \frac{v_{BE}}{\frac{1}{j\omega C_\mu}}$$

Negligible since << β_0 for not extreme frequencies.

✪ $C_\pi \sim 1\,pF$

$C_\mu \sim 0{,}1\,pF$

$f_\beta \sim 10\,MHz$

$$\frac{i_{out}(\omega)}{i_{in}(\omega)} = \frac{g_m - j\omega C_\mu}{\frac{1}{r_\pi} + j\omega\left(C_\pi + C_\mu\right)} = \frac{\beta_0 - j f\left(2\pi r_\pi \cdot C_\mu\right)}{1 + j f/f_\beta}$$

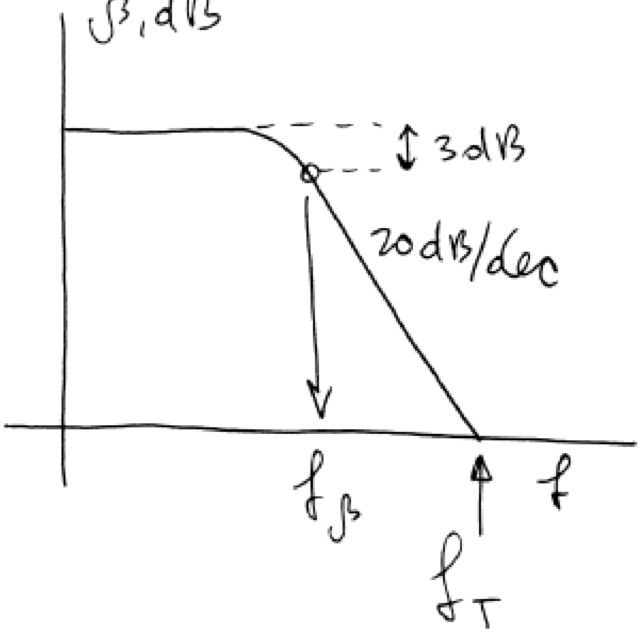
$$f_\beta = \frac{1}{2\pi r_\pi \left(C_\pi + C_\mu\right)}$$

Frequency response of Common Emitter amplifier

Short circuit current gain at high frequencies

$$\beta(f) \simeq \frac{\beta_0}{1 + j f / f_\beta}$$

$$f_\beta = \frac{1}{2\pi r_\pi (C_\pi + C_\mu)}$$

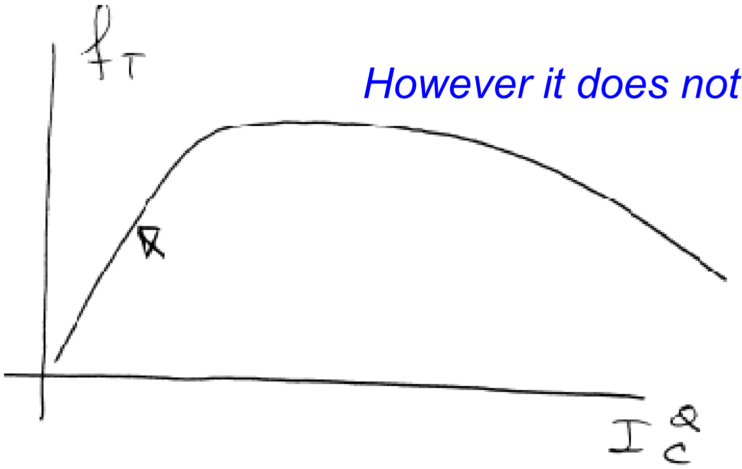


Unity gain bandwidth f_T.

$$\beta(f_T) = 1 = \left| \frac{\beta_0}{1 + j f_T / f_\beta} \right| \approx \frac{\beta_0 \cdot f_\beta}{f_T}$$

$$f_T = \beta_0 \cdot f_\beta = \frac{\beta_0}{r_\pi} \cdot \frac{1}{2\pi (C_\pi + C_\mu)} = \frac{g_m}{2\pi (C_\pi + C_\mu)}$$

Looks like it is supposed to improve with bias current because $g_m = \frac{I_C^Q}{V_{th}}$



However it does not. Why?

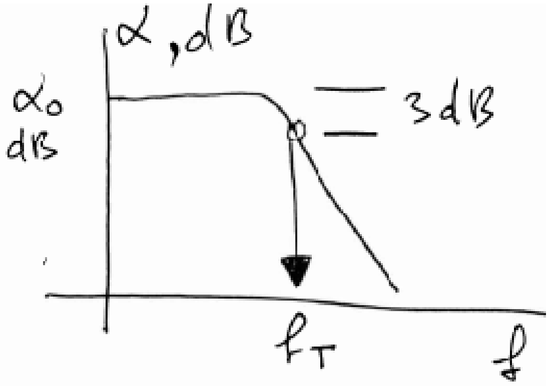
Frequency response of Common Emitter amplifier

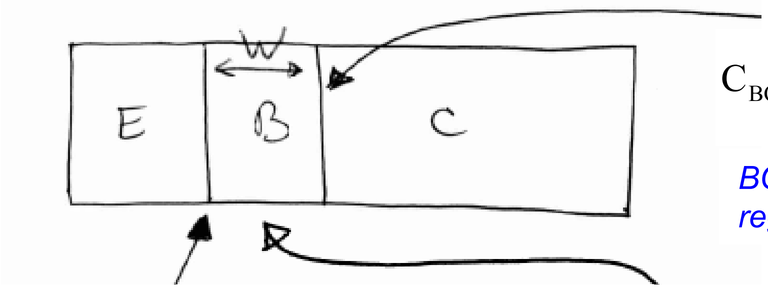
Frequency dependence of common base current gain

$$\alpha(f) = \frac{\beta(f)}{\beta(f)+1}$$

$$\alpha(f) = \frac{\beta_0}{1+\beta_0+jf/f_\beta} = \frac{\alpha_0}{1+jf/(f_\beta(1+\beta_0))} \simeq \frac{\alpha_0}{1+jf/f_T}$$

3dB frequency for α is equal to f_T.





$$C_{BC}(V_{CB}) = \frac{C_{BC0}}{\left(1+V_{CB}/V_{bi}\right)^{1/2}}$$

BC-junction depletion region capacitance

$$C_{BE}(V_{BE}) = \frac{C_{BE0}}{\left(1-V_{BE}/V_{bi}\right)^{1/2}}$$

EB-junction depletion region capacitance

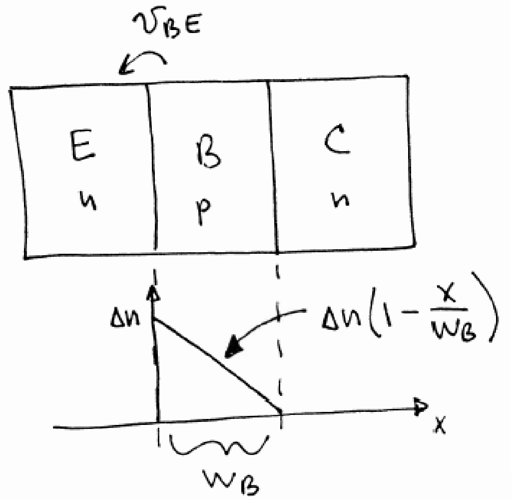
Base transport time – time of flight of electrons from emitter to collector.

Hence at f_T electrons from emitter can not reach collector.

**There are also several parasitic caps associated with technology limitations*

Frequency response of Common Emitter amplifier

Base transport time and associated diffusion capacitance



$$J_{diff} = q \cdot D_n \cdot \frac{dn}{dx} = q \cdot n \cdot \underbrace{\left(\frac{D_n}{n} \cdot \frac{dn}{dx}\right)}$$

Effective velocity of diffusion electrons → v_{dif}.

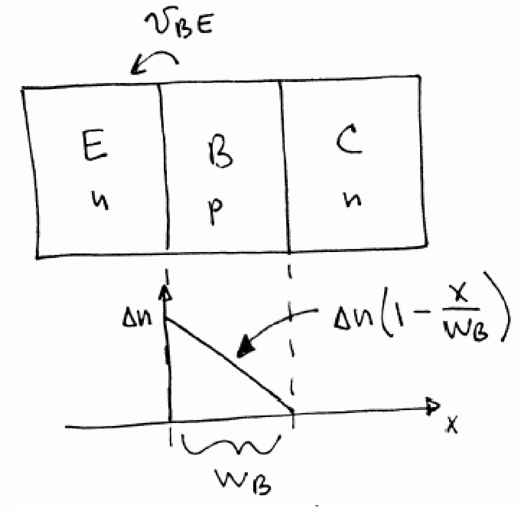
time of flight of electrons from emitter to collector.

$$\tau_{TF} = \frac{W_B}{v_{diff}} \simeq \frac{W_B^2}{2D_n}$$

*Need thin base for high speed operation

Frequency response of Common Emitter amplifier

Base transport time and associated diffusion capacitance



time of flight of electrons from emitter to collector.

$$\tau_{TF} = \frac{W_B}{v_{diff}} \approx \frac{W_B^2}{2D_n}$$

**Need thin base for high speed operation*

Electron charge stored in base when current IC is flowing

$$Q_{TF} = I_C \cdot \tau_{TF} \approx q\,\frac{\Delta n}{2}\cdot W_B$$

$$C_{TF} = \left.\frac{dQ_{TF}}{dV_{BE}}\right|_{I_C^Q} = \tau_{TF}\cdot\left.\frac{dI_C}{dV_{BE}}\right|_{I_C^Q} = \tau_{TF}\cdot g_m$$

$$C_\pi = C_{TF} + C_{BEj} \quad ; \quad C_\mu = C_{CBj}$$

Charge storage capacitance

Pn-junction depletion region capacitances and other parasitic caps

Frequency response of Common Emitter amplifier

Unity gain bandwidth

$$C_\pi = \underbrace{C_{TF}}_{= \tau_{TF}\cdot g_m} + C_{BEj} \quad \& \quad C_\mu = C_{CBj}$$

$$f_T = \frac{g_m}{2\pi\left(C_{BEj} + C_{CBj}\right) + 2\pi \cdot g_m \cdot \tau_{TF}} = \frac{1}{2\pi\,\tau_T}$$

Total time delay

$$\tau_T = \tau_{TF} + \frac{C_{BEj} + C_{CBj}}{g_m} \quad \& \quad g_m \sim I_C^Q$$

Minimum possible time delay

Ultimate limit for BJT speed