Common Base

Biased by current source

Start with bias DC analysis – make sure BJT is in FA, then calculate small signal parameters for AC analysis.

*ignore \( r_O \) for simplicity, then:

\[ A_V = \left. \frac{V_{out}}{V_{in}} \right|_{V_{in} \to 0} = g_m (R_L || R_C) \]

noninverting

\[ A_{V_{o}} = \left. A_V \right|_{R_L = \infty} = g_m R_L \]

\[ R_{in} = \left. \frac{V_{in}}{i_{in}} \right|_{V_{in} \to 0} = \frac{R_C}{s+1} \]

Small!

\[ R_{out} = \left. \frac{V_{out}}{i_{out}} \right|_{v_{S} \to 0} = R_C \]

\[ G_v = \frac{V_{out}}{V_{S}} = \frac{R_{in}}{R_{in}+R_S} \cdot A_V \approx \frac{R_L || R_C}{R_S} \]

The circuit is current buffer: delivers current from source to load

* when BJT output impedance \( r_O \) can not be neglected – the circuit is said to perform an impedance transformation.
Common Collector amplifier

Again.
No need for $C_E$!

Bias current $I_E$ will determine $g_m$, $r_n$, and $r_o$

Also
$R_B$ and $C_{C1}$ can be eliminated

Redraw equivalent circuit in more convenient form

Therefor
$\eta_{eq} = \frac{R_{eq}}{R_{eq} + R_S}$

$R_{eq} = \frac{R_B}{1 + \frac{R_B}{R_S}}$
Common Collector amplifier

\[ V_{eq} = \frac{V_S}{R_S + R_S} = \beta b (V_T + R_{eq}) + V_{out} \]

\[ V_{out} = (\beta b + q_m V_{BE}) \cdot (R_L || V_0) = \beta b \left( 1 + \beta \right) \left( R_L || V_0 \right) \]

\[ G_V = \frac{V_{out}}{V_{eq}} = \frac{V_{out}}{V_{eq}} \cdot \frac{T_{eq}}{V_S} = \frac{R_B}{R_S + R_B} \cdot \frac{(\beta + 1) (R_L || V_0)}{(R_T + R_B || R_S) + (\beta + 1) (R_L || V_0)} \leq 1 \]

When

\[ R_B \gg R_S \]
\[ R_0 \gg R_L \]

\[ G_V = \frac{(\beta + 1) R_L}{[R_S + R_T] + R_L \left( \frac{\beta + 1}{R_T} \right)} \leq 1 \]

\[ \beta \approx 1 \]

\[ i.e. \ no\ voltage\ gain! \]
Common Collector amplifier

Input impedance

\[ R_{i_{in}} = \frac{\overline{V}_{i_{in}}}{\overline{i}_{i_{in}}} \]

\[ \overline{V}_{i_{in}} = \overline{i}_{b} \cdot r_{a} + (\beta + 1)(R_{L} \ || V_{o}) \cdot \overline{i}_{b} \]

\[ \overline{i}_{i_{in}} = \frac{\overline{V}_{i_{in}}}{R_{b}} + 2 \overline{i}_{b} = \overline{i}_{b} \left( \frac{V_{T}}{R_{b}} + 1 + \frac{\beta + 1}{R_{b}} \right) \]

\[ R_{i_{in}} = \frac{\left( V_{T} + (\beta + 1)(R_{L} \ || V_{o}) \right) \cdot R_{b}}{R_{b} + \left( V_{T} + (\beta + 1)(R_{L} \ || V_{o}) \right)} = R_{b} \ || \left( \frac{V_{T} + (\beta + 1)(R_{L} \ || V_{o})}{\sum R_{b}} \right) \]

\[ R_{i_{in}} \approx (\beta + 1) \cdot R_{L} \quad \text{for} \ R_{L} \ll V_{o} \]

Impedance transformation
Common Collector amplifier

Output impedance

\[ V_S = 0 \]

\[ i_b \quad V_{BE} \quad i_{out} \]

\[ V_{out} = -i_b (R_s || R_B) - \eta V_{BE} = -i_b (R_s || R_B + V_o) \]

\[ i_{out} = \frac{V_{out}}{V_o} - i_b - \eta V_{BE} = \]

\[ = -i_b \frac{R_s || R_B + V_o}{V_o} - i_b - \frac{V_o}{V_o} \]

\[ R_{out} = \frac{V_{out} + R_s || R_B}{(j\omega + 1) + \frac{V_{out} + R_s || R_B}{V_o}} = \left( \frac{R_o || \frac{V_{out} + (R_s || R_B)}{j\omega + 1}} \right) \]

\[ R_B \gg R_s \quad R_v \gg \eta \quad \text{then} \quad R_{out} \approx \frac{R_s}{V_o + 1} \]
Common Collector (Emitter follower)  
*Biased by current source*

Start with bias DC analysis – make sure BJT is in FA, then calculate small signal parameters for AC analysis.

\[
\frac{V_{out}}{V_{in}} = \frac{R_S}{R_S + R_B} \frac{(\beta + 1)(R_L || R_o)}{[(R_S || R_B) + R_o] + (\beta + 1)(R_L + V_o)} < 1
\]

Often \( R_B \gg R_S \) and \( r_o \gg R_L \), then

\[
\frac{V_{out}}{V_{in}} \approx \frac{(\beta + 1) \cdot R_L}{(R_S + R_o) + R_L (\beta + 1)} \approx 1
\]

\[
R_{in} = \frac{V_{in}}{I_{in}} = \frac{R_L}{(R_S || R_B) + R_o} \approx (\beta + 1) \cdot R_L
\]

\[
R_{out} = \frac{V_{out}}{I_{out}} |_{V_o = 0} = \frac{R_L}{(R_S || R_B) + R_o} \approx \frac{R_S}{\beta + 1}
\]

The circuit is voltage buffer: delivers voltage from source to load.
Frequency response of Common Emitter amplifier

*Low frequencies, i.e. BJT itself is fast enough*

1. DC bias – make sure BJT is in FA - AC analysis.

   **Before** – assumed coupling caps are big enough to act as a short circuit for any frequency of AC signal.

   **Now** – assume they have finite values.

1. Assume $C_1$ is finite while $C_2$ and $C_E$ are still infinite.

   $G_V(t) = \frac{V_{out}(t)}{V_s(t)} = \frac{V_{out}(t)}{\sin(t)} \cdot \frac{\sin(t)}{R_L} = \frac{V_{out}(t)}{V_s(t)}$

   Depends on frequency.

   Frequency independent.
Frequency response of Common Emitter amplifier

**Role of the input coupling cap \( C_1 \)**

\[
G_V(f) = \frac{V_{out}(f)}{V_S(f)} = \frac{V_{out}(f)}{V_{in}(f)} \cdot \frac{T_{in}(f)}{T_S(f)}
\]

- Depends on frequency.
- Voltage gain found before

\[
G_{V0} \quad \text{net voltage gain found before for infinite caps.}
\]

\[
T_{in} = \frac{R_{in}}{R_C + R_{in} + \frac{1}{j \omega C_{1}}} = \frac{R_{in}}{R_C + R_{in}} \cdot \frac{j \omega C_{1}}{1 + j \omega C_{1}}
\]

- Input voltage divider found before.
- High Pass Filter

\[
T_0 = -3 \text{dB} \quad \frac{f_0}{f_{l1}} \quad 20 \text{dB/dec}
\]
Frequency response of Common Emitter amplifier

Role of the output coupling cap $C_2$

2. Assume $C_2$ is finite while $C_1$ and $C_E$ are still infinite.

$G_V(f) = \frac{\hat{V}_{out}(f)}{\hat{V}_s(f)} = \frac{\hat{V}_{out}(f)}{\hat{V}_s(f)} \cdot \frac{\hat{V}_{out}(f)}{\hat{V}_{out}(f)}$

$G_V(f) = \frac{R_L}{R_L + \frac{1}{j\omega C_2}} \cdot A_{v0} \cdot \frac{R_L}{R_L + \frac{1}{j\omega C_2}}$

$G_{V0} = \text{net voltage gain found before for infinite caps.}$

Again High Pass Filter but with 3dB frequency defined by $C_2$
Frequency response of Common Emitter amplifier

Role of the bypass cap $C_E$

3. Assume $C_E$ is finite while $C_1$ and $C_2$ are still infinite.
We have identified three HPF.

\[ T_i(f) = \frac{j \cdot f / f_{Li}}{1 + j \cdot f / f_{Li}} \]

\[ f_{L1} = \frac{1}{2 \pi \cdot C_1 \cdot (R_{in} + R_S)} \]

\[ f_{L2} = \frac{1}{2 \pi \cdot C_2 \cdot (R_{out} + R_L)} \]

\[ f_{L3} = \frac{1}{2 \pi \cdot \frac{C_E}{\beta+1} \cdot (r_\pi + R_S \parallel R_B)} \]

\[ C_1 = C_2 = C_E = 1 \mu \]

\[ R_{in} + R_S \sim k\text{Ohm} \rightarrow f_{L1} < 200\text{Hz} \]

\[ R_{out} + R_L \sim 10k\text{Ohm} \rightarrow f_{L2} < 20\text{Hz} \]

\[ r_\pi + R_S \parallel R_B \sim k\text{Ohm} \rightarrow f_{L3} > kHz \]

Low frequency cutoff is determined by \( C_E \)
Frequency response of Common Emitter amplifier

**Bandwidth**

High frequency 3dB determines amplifier bandwidth.

Amplifier bandwidth is determined by BJT high frequency capabilities – determined by internal parasitic capacitances $C_{\pi}$ and $C_{\mu}$. 
Frequency response of Common Emitter amplifier

*Short circuit current gain at high frequencies*

1. $C_u - C_M = 0 \quad (\frac{\tilde{I}_{out}}{\tilde{I}_{in}}) = \frac{g_{m} V_{BE}}{\tilde{V}_{b}} = g_{m} \cdot R_o = \beta_0$

2. $C_u \neq 0$

\[
\tilde{I}_{out} = g_{m} V_{BE} \quad \tilde{I}_{in} = g_{m} V_{BE} - \frac{V_{BE}}{j \omega C_M} = (g_{m} - j \omega C_M) V_{BE}
\]

\[
\tilde{I}_{in} = \frac{V_{BE}}{R_o} + \frac{V_{BE}}{\frac{1}{j \omega C_u}} + \frac{V_{BE}}{\frac{1}{j \omega C_M}}
\]

\[
\frac{\tilde{I}_{out} (\omega)}{\tilde{I}_{in} (\omega)} = \frac{g_{m} - j \omega C_M}{\frac{1}{R_o} + j \omega (C_u + C_M)} = \frac{\beta_0 - j \frac{\omega}{Q} (2\pi R_o \cdot C_M)}{1 + j \frac{Q}{\beta_0}}
\]

Negligible since $\ll \beta_0$ for not extreme frequencies.

Common emitter current gain defined earlier.

$C_u \sim 1 \text{pF}$

$C_M \sim 0.1 \text{pF}$

$R_o \sim 10 \text{M} \Omega$
Frequency response of Common Emitter amplifier

*Short circuit current gain at high frequencies*

\[
\beta(t) \approx \frac{\beta_0}{1 + j \frac{f}{f_T}}
\]

\[
\beta = \frac{1}{2 \pi \nu_m (\epsilon_a + \epsilon_p)}
\]

**Unity gain bandwidth** \(f_T\):

\[
\beta(f_T) = 1 = \left| \frac{\beta_0}{1 + j \frac{f_T}{f_A}} \right| \approx \frac{\beta_0}{f_T}
\]

\[
f_T = \beta_0 \cdot \frac{1}{\nu_m} \cdot \frac{1}{2 \pi \nu_m (\epsilon_a + \epsilon_p)} = \frac{g_m \frac{V_1}{I}}{2 \pi (\epsilon_a + \epsilon_p)}
\]

**Looks like it is supposed to improve with bias current because**

\[
g_m = \frac{J}{V}
\]

However it does not. Why?
Frequency response of Common Emitter amplifier

**Frequency dependence of common base current gain**

\[ \alpha(f) = \frac{\beta(f)}{\beta(f) + 1} \]

\[ \alpha'(f) = \frac{\beta_0}{1 + \frac{j}{\beta_0} \cdot \frac{f}{f_{3dB}}} = \frac{\alpha_0}{1 + \frac{j}{f_{3dB}(1 + \alpha_0)}} \approx \frac{\alpha_0}{1 + j \cdot f / f_T} \]

3dB frequency for \( \alpha \) is equal to \( f_T \).

There are also several parasitic caps associated with technology limitations.

Hence at \( f_T \) electrons from emitter can not reach collector.
Frequency response of Common Emitter amplifier

Base transport time and associated diffusion capacitance

\[
\tau_T = \frac{W_B}{v_{diff}} \approx \frac{W_B^2}{2D_n}
\]

*Need thin base for high speed operation

**Effective velocity of diffusion electrons**

*Time of flight of electrons from emitter to collector.*
Frequency response of Common Emitter amplifier

*Base transport time and associated diffusion capacitance*

\[ \tau_{TF} = \frac{W_B}{V_{diff}} \approx \frac{W_B^2}{2 \cdot D_n} \]

*Need thin base for high speed operation*

**Electron charge stored in base when current IC is flowing**

\[ Q_{TF} = I_c \cdot \tau_{TF} \approx q \cdot \frac{\Delta n}{\pi} \cdot W_B \]

\[ C_{TF} = \frac{dQ_{TF}}{dV_{BE}} \bigg|_{I_c} = \tau_{TF} \cdot \frac{dI_c}{dV_{BE}} \bigg|_{I_c} - \tau_{TF} \cdot \varnothing \cdot m \]

\[ C_n = C_{TF} + C_{BEB} \quad ; \quad C_{\mu} = C_{CBE} \]

**Charge storage capacitance**

**Pn-junction depletion region capacitances and other parasitic caps**
Frequency response of Common Emitter amplifier

*Unity gain bandwidth*

\[
\begin{align*}
C_W &= C_{TF} + C_{BE} \quad \text{and} \quad C_U = C_{CB}
\end{align*}
\]

\[
L_W = C_{TF} \cdot q_m
\]

\[
\tau_T = \frac{q_m}{2 \pi \left( C_{BE} + C_{CB} \right) + 2 \pi \cdot q_m \cdot \tau_{TF}} = \frac{1}{2 \pi \tau_{TF}}
\]

*Total time delay*

\[
\tau_T = \tau_{TF} + \frac{C_{BE} + C_{CB}}{q_m} \quad \text{and} \quad q_m \sim I_C
\]

*Minimum possible time delay*

\[
\text{log } \tau
\]

*Ultimate limit for BJT speed*