Last time: BJT CE frequency response

**Need FA**

\[ V_{BE} \approx 0.7V \]

\[ V_{CE} > 0.3V \]

**Equivalent circuit for low frequency small signal analysis**

\[ T_i(f) = \frac{j \cdot f}{1 + j \cdot f / f_{L1}} \]

\[ f_{L1} = \frac{1}{2 \cdot \pi \cdot C_1 \cdot (R_{in} + R_s)} \]

\[ f_{L2} = \frac{1}{2 \cdot \pi \cdot C_2 \cdot (R_{out} + R_L)} \]

\[ f_{L3} = \frac{1}{2 \cdot \pi \cdot \frac{C_E}{\beta + 1} \cdot (r_x + R_s \parallel R_B)} \]

Coupling and bypass capacitors result into high pass filters (as could be expected).
Frequency response of Common Emitter amplifier

Bandwidth

High frequency 3dB determines amplifier bandwidth.

Amplifier bandwidth is determined by BJT high frequency capabilities – determined by internal parasitic capacitances $C_\pi$ and $C_\mu$. 
Frequency response of Common Emitter amplifier

*Short circuit current gain at high frequencies*

1. $C_u = C_m = 0$

\[
\frac{\tilde{i}_{out}}{\tilde{i}_{in}} = \frac{g_m V_{BE}}{\tilde{z}_b} = g_m R_a = \beta_0
\]

Common emitter current gain defined earlier.

2. $C_u$ & $C_m \neq 0$

\[
\tilde{i}_{out} = g_m V_{BE} - \tilde{i}_m = g_m V_{BE} - \frac{V_{BE}}{\frac{1}{j \omega C_m}} = (g_m - j \omega C_m) V_{BE}
\]

Negligible since $<< \beta_0$ for not extreme frequencies.

\[
\tilde{i}_m = \frac{V_{BE}}{R_a} + \frac{V_{BE}}{\frac{1}{j \omega C_m}} + \frac{V_{BE}}{\frac{1}{j \omega C_m}}
\]

\[
\frac{\tilde{i}_{out} \left( \omega \right)}{\tilde{i}_{in} \left( \omega \right)} = \frac{g_m - j \omega C_m}{\frac{1}{R_a} + j \omega (C_u + C_m)} = \frac{\beta_0 - j \omega (2\pi f \beta)}{1 + j \omega \frac{f}{f_\beta}}
\]

For $f_\beta \approx 10\text{ MHz}$

$C_u \approx 1\text{ pF}$

$C_m \approx 0.1\text{ pF}$
Frequency response of Common Emitter amplifier

*Short circuit current gain at high frequencies*

\[ f_{\beta} = \frac{\beta_0}{2\pi r_n (c_n + e_m)} \]

\[ f_T = \beta_0 \cdot f_{\beta} = \frac{\beta_0}{\sqrt{2}} \cdot \frac{1}{\pi \left(c_n + e_m\right)} = \frac{g_m}{2\pi \left(c_n + e_m\right)} \]

Unity gain bandwidth \( f_T \).

Looks like it is supposed to improve with bias current because
Frequency response of Common Emitter amplifier

*Short circuit current gain at high frequencies*

Unity gain bandwidth $f_T$:

$$f_T = \beta_0 \cdot \frac{1}{g_{m}} = \frac{1}{\frac{1}{\beta_0} + \frac{1}{g_{m}}}$$

Looks like it is supposed to improve with bias current because

$$g_m = \frac{I_C}{V_{be}}$$

However it does not. Why?
Frequency response of Common Emitter amplifier

Frequency dependence of common base current gain

\[ \alpha(f) = \frac{\beta_0}{1 + j\frac{f}{f_T}} \]

\[ \alpha(f) = \frac{\beta_0}{1 + j\frac{f}{f_T}} \approx \frac{\alpha_0}{1 + j\frac{f}{f_T}} \]

3dB frequency for \( \alpha \) is equal to \( f_T \).

Hence at \( f_T \) electrons from emitter can not reach collector.
Frequency response of Common Emitter amplifier

**Frequency dependence of common base current gain**

\[
\alpha(f) = \frac{\beta_0}{1 + \beta_0 + j \frac{f}{f_T}} = \frac{\alpha_0}{1 + j \frac{f}{f_T}} \sim \frac{\alpha_0}{1 + j \frac{f}{f_T}}
\]

3dB frequency for \(\alpha\) is equal to \(f_T\).

\[
C_{BC}(V_{CB}) = \frac{C_{BC0}}{(1 + V_{CB}/V_{bi})^{1/2}}
\]

BC-junction depletion region capacitance

\[
C_{BE}(V_{BE}) = \frac{C_{BE0}}{(1 - V_{BE}/V_{bi})^{1/2}}
\]

EB-junction depletion region capacitance

Base transport time – time of flight of electrons from emitter to collector.

Hence at \(f_T\) electrons from emitter can not reach collector.

*There are also several parasitic caps associated with technology limitations*
Frequency response of Common Emitter amplifier

*Base transport time and associated diffusion capacitance*

**Effective velocity of diffusion electrons**

\[ \tau_{TF} = \frac{W_B}{v_{diff}} \approx \frac{W_B^2}{2D_n} \]

*Need thin base for high speed operation*

**Time of flight of electrons from emitter to collector**
Frequency response of Common Emitter amplifier

*Base transport time and associated diffusion capacitance*

**Electron charge stored in base when current IC is flowing**

\[ Q_{TF} = I_c \cdot \tau_{TF} \approx q \cdot \frac{\Delta n}{2} \cdot W_B \]

\[ C_{TF} = \left. \frac{dQ_{TF}}{dV_{BE}} \right|_{I_c} = \tau_{TF} \cdot \left. \frac{dI_c}{dV_{BE}} \right|_{I_c} = \tau_{TF} \cdot q \cdot n \]

**Charge storage capacitance**

**Pn-junction depletion region capacitances and other parasitic caps**

\[ C_{\mu} = C_{TF} + C_{BE} \]

\[ C_{\mu} = \]

*Need thin base for high speed operation*

time of flight of electrons from emitter to collector.
Frequency response of Common Emitter amplifier

Unity gain bandwidth

\[ \tau_T = \frac{g_m}{2\mu (C_{BEJ} + C_{CBJ}) + 2\mu \cdot g_m \cdot \tau_{TF}} = \frac{1}{2\mu \mu \tau_T} \]

Total time delay

Minimum possible time delay

Ultimate limit for BJT speed
Frequency response of Common Emitter amplifier

*Bandwidth*

Amplifier bandwidth \( f_H \) is determined by BJT internal parasitic capacitances \( C_\pi \) and \( C_\mu \).

Charge storage capacitance

Pn-junction depletion region capacitances and other parasitic caps
Unity gain bandwidth of BJT

\[ \frac{\text{\dot{i}_{out}}}{\text{\dot{i}_{in}}} = ? \]

Unity gain bandwidth \( f_T \)

\[ \frac{1}{2\pi f_T} = \frac{g_m}{2\pi \left( C_u + e_{\mu} \right)} = \frac{I_c e^q}{V_{th}} \frac{e^q}{V_{th}} \frac{e^q}{V_{th}} \]

\[ j_{\beta} \]

\[ j_{\beta} \]

\[ j_{\beta} \]

\[ j_{\beta} \]

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\[ j_{\beta} \]

\[ j_{\beta} \]

\[ j_{\beta} \]
We are interested in high frequency cutoff, i.e. coupling and bypass capacitors can be replaced by short circuit for frequencies $\gg f_L$.

We have got capacitive coupling between input and output.
Net voltage gain bandwidth of CE BJT amplifier

\[ V_L = R_L \left( \hat{I}_M - g_m V_{BE} \right) = V_{BE} - \frac{\hat{I}_M}{\frac{1}{j\omega C_m} + R_L} \]

\[ \hat{I}_M = V_{BE} \left( 1 + \frac{g_m R_L}{R_L + \frac{1}{j\omega C_m}} \right) = V_{BE} \frac{(1 + g_m R_L) \cdot j\omega C_m}{1 + j\omega C_m \cdot R_L} \]

Observe increase of \( i_M \) with transconductance \( g_m \).
Net voltage gain bandwidth of CE BJT amplifier

\[ V_L = V_{BE} - i_m \frac{1}{jwC_m} = V_{BE} \frac{jwC_m \hat{R}_L - q_m \hat{R}_L}{1 + jwC_m \hat{R}_L} \]

\[ V_{eq} = V_{BE} + R_{eq} (\hat{i}_m + jwC_m V_{BE}) \]

\[ \frac{V_L}{V_{eq}} = \frac{jwC_m \hat{R}_L - q_m \hat{R}_L}{(1 + jwC_m R_{eq})(1 + jwC_m \hat{R}_L) + (1 + q_m \hat{R}_L)jwC_m R_{eq}} \]
Net voltage gain bandwidth of CE BJT amplifier

\[
\frac{\tilde{V}_L}{\tilde{V}_{eq}} = \frac{jw C_m \hat{R}_L - q_m \hat{R}_L}{(1 + jw C_m R_{eq})(1 + jw C_m \hat{R}_L) + (1 + q_m \hat{R}_L)jw C_m R_{eq}}
\]
Net voltage gain bandwidth of CE BJT amplifier

\[ \frac{\hat{V}_L}{\hat{V}_{eq}} = \frac{j\omega C_\mu \hat{R}_L - g_m \hat{R}_L}{(1 + j\omega C_\mu \hat{R}_{eq})(1 + j\omega C_\mu \hat{R}_L) + (1 + g_m \hat{R}_L)j\omega C_\mu \hat{R}_{eq}} \]

**nominator**

\[ f_T \approx \frac{1}{2 \cdot \pi \cdot \tau_{TF}} \approx 1\,GHz, \quad C_\mu \approx 0.1\,pF \]

\[ \omega \cdot C_\mu \leq 0.001\,\Omega^{-1} \]

\[ g_m \approx 0.04 \]
Net voltage gain bandwidth of CE BJT amplifier

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Net voltage gain bandwidth of CE BJT amplifier

\[ \frac{\hat{V}_L}{\hat{V}_{eq}} = \frac{j\omega C_{\mu} R_L - g_m \hat{R}_L}{(1 + j\omega C_{\mu} R_{eq})(1 + j\omega C_{\mu} \hat{R}_L) + (1 + g_m \hat{R}_L) j\omega C_{\mu} R_{eq}} \]

\text{nominator}

\[ f_T \approx \frac{1}{2 \cdot \pi \cdot \tau_{TF}} \approx 1 \text{GHz}, \quad C_{\mu} \approx 0.1 \text{pF} \]

\[ \omega \cdot C_{\mu} << 0.001 \Omega^{-1} \]

\[ g_m \approx 0.04 \Omega^{-1} \]

\text{denominator}

\[ \text{for} \]

\[ C_{\mu} \approx C_{\pi}/10 \]

\[ \text{and} \quad f << f_T \]

\[ \omega \cdot C_{\mu} \cdot \hat{R}_L << 1 \]
Net voltage gain bandwidth of CE BJT amplifier

\[
\frac{\mathcal{V}_L}{\mathcal{V}_{eq}} = \frac{j\omega C_\mu R_L - q_m \hat{R}_L}{(1 + j\omega C_\mu R_{eq})(1 + j\omega q_m \hat{R}_L) + (1 + q_m \hat{R}_L) j\omega C_\mu R_{eq}}
\]

**nominator**

\[
f_T \approx \frac{1}{2 \cdot \pi \cdot \tau_{TF}} \approx 1 \text{GHz}, \quad C_\mu \approx 0.1 \text{pF}
\]

\[
\omega \cdot C_\mu << 0.001 \Omega^{-1}
\]

\[
g_m \approx 0.04 \Omega^{-1}
\]

**denominator**

for

\[
C_\mu \approx C_\pi / 10
\]

and \( f << f_T \)

\[
\omega \cdot C_\mu \cdot \hat{R}_L << 1
\]
Net voltage gain bandwidth of CE BJT amplifier

\[ \frac{V_L}{V_{eq}} = \frac{-\mu_m R_L}{1 + j\omega R_{eq} \left( C_a + (1 + \mu_m \hat{R}_L)C_m \right)} \]

\[ \frac{V_L(t)}{V_{eq}(t)} = G_V(t) \propto \frac{-\mu_m (R_L || R_c || R_o)}{1 + \frac{f}{f_H}} \]

\[ f_H = \frac{1}{2\pi R_{eq} \left( C_a + (1 + \mu_m \hat{R}_L)C_m \right)} \]

Resistor in series with input cap

Equivalent input cap
Net voltage gain bandwidth of CE BJT amplifier

\[ G_{V_0} = \frac{R_{\text{in}} \cdot A_{V_0} \cdot R_L}{R_{\text{out}} + R_L} \]

\[ A_{V_0} = -g_m (R_L||R_v) \]

\[ f_M = \frac{1}{2\pi R_{\text{eq}} (C_C + C_{\mu} (1 + g_m (R_L||R_v)))} \]

\[ C_{\mu} (1 + A_{V_0}) = C_M \]

“Miller” capacitor
Example

\[ V_{cc} = V_{EE} = 10\, V \quad \therefore I_E = 1\, mA \quad R_E = 100\, k\Omega \]
\[ V_{BE} = 0.6 \quad \therefore I_B = 1\, mA \quad R_E = 8\, k\Omega \]
\[ V_A = 100\, V \quad \therefore C_{\mu} = 1\, pF \quad f_T = 800\, MHz \quad R_S = R_L = 5\, k\Omega \]

*confirm FA regime first …*

1. \[ g_m = \frac{I_m}{2eV} = \frac{1\, mA}{2eV} = 40\, \mu A/V \]
2. \[ C_T + C_{\mu} = \frac{g_m}{\omega T} = \frac{40 \cdot 10^{-3}}{2\pi \cdot 800 \cdot 10^6} = 8\, pF \]
   \[ C_{\mu} = 1\, pF \text{ hence } C_T = 7\, pF \]
3. \[ R_L || R_E || R_0 = 3\, k\Omega \]
4. \[ G_V = \frac{R_E || R_T}{R_E || R_T + R_S} \cdot (-g_m(R_L || R_E || R_0)) = -\frac{40 \cdot 10^{-3} \cdot 3 \cdot 10^3}{(100 \times 12.5k) + 5k} \cdot (100 \times 12.5k) \]
   \[ G_V \approx -121 \frac{V}{V} \cdot \frac{1}{3} \approx -40 \frac{V}{V} \quad \therefore g_m \cdot R_L = 121 \frac{V}{V} \]
Example

\[ \begin{align*}
V_{cc} &= V_{EE} = 10 \text{ V} \quad ; \quad I_E = 1 \text{ mA} \quad ; \quad R_B = 100 \text{ k}\Omega \\
V_{be} &= 100 \text{ mV} \quad ; \quad C_m = 1 \text{ pF} \quad ; \quad f_T = 800 \text{ MHz} \\
V_A &= 100 \text{ V} \quad ; \quad R_S = R_L = 5 \text{ k}\Omega
\end{align*} \]

\[ G_V \approx -121 \frac{V}{V} \cdot \frac{1}{3} = -40 \frac{V}{V} \]

\[ R_{eq} = \left( R_m || R_B \right) || R_S = 1.7 \text{ k}\Omega \]

\[ C_m = 7 \text{ pF} + 121 \cdot 1 \text{ pF} = 128 \text{ pF} >> C_m \]

\[ f_H = \frac{1}{2 \pi \cdot C_m \cdot R_{eq}} = 750 \text{ kHz} \ll f_T \]

Even \ll f_\beta = 8\text{MHz}

Reduced gain – improved BW

\[ R_L = 2 \text{ k}\Omega \quad ; \quad \gamma_m \cdot R_L \approx 64 \frac{V}{V} \]

\[ f_H \approx 1.4 \text{ MHz} \]
Miller Effect

\[ \tilde{V}_Z = \tilde{V}_{in} - \tilde{V}_{out} = \tilde{V}_{in} (1 - AV) \]

\[ \tilde{I}_Z = \frac{\tilde{V}_Z}{Z} = \frac{\tilde{V}_{in} (1 - AV)}{Z} = \tilde{I}_{in} \frac{1}{1 - AV} \]

**Miller transformation**

\[ \tilde{Z}_{in} = \frac{Z}{1 - AV} \]

\[ \tilde{Z}_{out} = \frac{Z - AV}{AV - 1} \]
Miller Effect in CE amplifier

If $i_\mu$ is not very high (small $C_\mu$), then

\[ Z_{in} = \frac{1}{j\omega C_\mu (1-AV)} \]
\[ Z_{out} = \frac{1}{j\omega C_\mu AV-1} \]

Negligible, hence open circuit

Low pass filter

Total equivalent input cap
CB amplifier does not suffer from Miller effect

Neglect $r_o$ and build equivalent circuit

Observe no capacitor coupling output and input – No Miller effect.
CB amplifier does not suffer from Miller effect

Neglect $r_o$ and build equivalent circuit

Observe no capacitor coupling output and input – No Miller effect.

\[
\frac{\tilde{\xi}_L(f)}{\tilde{\xi}_s(f)} = \frac{g_m (R_c || R_L)}{1 + R_s (g_m + \frac{1}{r_o})} \quad \text{and} \quad \frac{1}{1 + jw (R_c || R_L) C_m} \quad \text{and} \quad \frac{1}{1 + jw C_a} \frac{r_s}{1 + R_s (g_m + \frac{1}{r_o})} \quad \frac{1}{\tau_1} \quad \frac{1}{\tau_2}
\]

\[
f_1 = \frac{1}{2 \pi (R_c || R_L) C_m} = \frac{1}{6.28 \cdot 3 \cdot 10^3 \cdot 10^{-12}} = 52 \text{ MHz}
\]

\[
f_2 = \frac{1}{2 \pi C_a (R_s || V_a || \frac{1}{jw})} \approx \frac{g_m}{2 \pi C_a} = \frac{90 \cdot 10^{-3}}{6.28 \cdot 10^{-12}} \approx 510 \text{ MHz}
\]