Metal Oxide Semiconductor Field Effect Transistor

* npn-BJT

\[ I_C = \frac{V_{BE}}{V_{BE} + 0.7V} \exp \left( \frac{V_{BE}}{V_{TH}} \right) \]

\[ I_C \sim 0.7V \]

\[ I_C \sim 0.3V(SAT) \]

\[ VA : I_C = \beta I_B \]

* n-MOSFET

Transconductance and output IVs look similar to those of BJT

Zero DC gate current!
MOSFET structure

Let us consider this structure without gate.

When $V_{DS}$ is applied either (1) or (2) pn-junction is reversed biased and no current can flow between source and drain.

Equivalently, the pn-junction barrier prevents electron injection from source to channel, hence, no current.

NEED Gate to make channel conduct!
Voltage applied to gate controls the channel conductivity type and value.
Assume $V_{DS}$ is zero but $V_{GS}$ is applied.

Part of the applied voltage will drop across oxide layer but part of it will drop across semiconductor, i.e. electric field will penetrate inside Si and will change the conductivity of the Si near the interface between Si and SiO$_2$. (this is field effect in the name of transistor)

For $V_{GS} > V_T$

Inversion layer is formed at the interface, i.e. conducting channel is created between source and drain when gate-to-source voltage is near threshold.
Now, if \( V_{DS} \) is applied the current will flow between drain and source!

Plenty of electrons

Immobile charge of ionized acceptors

“Both pn-junctions are forward biased”

Voltage needed to create \( Q_B \) before \( Q_I \) can appear

\[
Q_I = -C_{ox} \left( V_G - V_T \right)
\]

Once \( Q_I \) is created – all extra voltage will go to oxide layer
MOSFET current through channel

W is design parameter so all charges in MOSFETs are calculated per unit area

\[
Q'_{\text{J}} = \frac{Q_s}{W \cdot L} = -C_{\text{ox}}'(U_{\text{GS}} - U_{\text{T}}), \text{ whence } C_{\text{ox}}' = \frac{\varepsilon_{\text{ox}} \cdot \varepsilon_0}{t_{\text{ox}}}
\]

For \( U_{\text{GS}} > U_{\text{T}} \) we got thin layer of mobile charge

Channel resistance

\[
R_{\text{eh}} = 9 \cdot \frac{L}{W \cdot d_{\text{eh}}}, \text{ or}
\]

Mobility of electrons in channel

\[
\mu = \frac{q \cdot n \cdot \mu}{(\varepsilon_0 \cdot \varepsilon_{\text{r}})^{-1}}
\]

Concentration of electrons in channel
MOSFET current through channel for small drain-to-source voltages

\[ Q_s' = -q \cdot n \cdot \frac{D}{L} \]

\[ R_{ch} = \frac{L}{w \cdot \frac{1}{q \cdot n \cdot \mu}} = \frac{L}{w \cdot \mu} \cdot \frac{1}{q \cdot n \cdot \mu} \]

\[ R_{ch} = \frac{L}{w \cdot \mu} - \frac{1}{Q_s'} = \frac{L}{w \cdot \mu} - \frac{1}{q \cdot n \cdot \mu} \cdot \cos \left( V_{gs} - V_T \right) \]

\[ I_D = \frac{V_{ds}}{R_{ch}} = \frac{W}{L} \cdot \mu \cdot \cos \left( V_{gs} - V_T \right) \cdot V_{ds} \]

We got voltage-controlled resistor, not transistor yet.
MOSFET Operation

We assumed so far that inversion layer charge is uniformly distributed over channel length, but it is not true. This is good approximation for very small drain-to-source voltages < 100 mV.

For positive $V_{DS}$ the inversion layer charge density per unit area decreases from source-to-drain

\[ Q'_x(x) = -e_\text{ex} (V_{GS} - V_T - V_{CS}(x)) \]

\[ Q'_x = -e_\text{ex} (V_{GD} - V_T) = -e_\text{ex} (V_{GS} - V_T - V_{DS}) \]

Hence channel conductivity decreases from source-to-drain

\[ dR = \frac{1}{q\mu_m(x) \mu W_0 deh} = \frac{dx}{W} \frac{1}{\mu_R} \left( -Q'_x(x) \right) \]
MOSFET Operation

\[
I_D = \frac{dV_{CS}}{dR}
\]

The same current for each \( dR \)

\[
I_D = W \cdot \mu \cdot \exp\left( \frac{V_{GS} - V_T - V_{CS}(x)}{2} \right) \frac{dV_{CS}}{dx} \int_{0}^{L} dx
\]

\[
I_D = W \cdot \mu \cdot \exp\left( \frac{V_{GS} - V_T}{2} \right) \left[ V_{CS} - \frac{V_{DS}^2}{2} \right]_{0}^{V_{DS}}
\]

\[
I_D = \frac{W}{L} \cdot \mu \cdot \exp\left( \frac{V_{GS} - V_T}{2} \right) \left[ (V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2} \right]
\]

Drain current saturates with drain-to-source voltage
MOSFET Operation in saturation

The drain current saturates to the value determined by $V_{GS}$. Current saturation occurs when channel is pinched-off near drain, i.e. inversion layer charge disappears near drain. After that – all extra $V_{DS}$ goes to pinch-off region and does not change field of concentration inside the channel.

Hence $V_{DS} = V_{GS} - V_T$, or $V_{GD} = V_T$, hence $Q_{DS} = 0$ near D

After $V_{DS} > V_{DS\text{SAT}} = V_{GS} - V_T$

$\begin{align*}
I_D &= I_{DS\text{SAT}} = \frac{W}{L} \kappa n \left( V_{GS} - V_T \right)^2 / 2 \\
I_D &= \frac{W}{L} \kappa n \left( V_{GS} - V_T \right) V_{DS} - \frac{V_{DS}^2}{2}
\end{align*}$

Drain current becomes independent of drain-to-source voltage
MOSFET input/output IV characteristics

* Dependence of the drain current on control voltage $V_{GS}$ is either linear or quadratic – slower than exponential like in BJT, hence we expect smaller transconductance from MOSFET as compared to BJT
MOSFET large signal equivalent circuit

Channel length modulation parameter
Enhancement and Depletion mode MOSFETs

Enhancement mode
“normally off”

Depletion mode
“normally on”
Enhancement p-MOSFET

Mobile charge in inversion layer - holes

\[ V_{GS} < V_T \text{ then } Q'_S = -C_0x(V_{GS} - V_T) \]

\[ I_{DS}^{\text{sat}} = -\frac{W}{L} \frac{V_p'}{2} (V_{GS} - V_1)^2 \]

\[ V_{p'} = C_0x - \mu_p \]
Body effect

So far we assumed $V_{BS} = 0$ and obtained:

$$Q_{SB}' = (-C_0'x) (V_{GS} - V_T)$$

Now consider assumed $V_{BS} < 0$, hence:

$$Q_{SB}' = (-C_0'x) (V_{GS} - V_T) + (-C'B') V_{BS}$$

$$V_{SB} = -V_{BS}$$

$$Q_{SB}' = (-C_0'x) \left[ V_{GS} - \left( V_T + \frac{C_V'}{C_0'x} V_{SB} \right) \right]$$

MOSFET threshold increases with $V_{SB}$

$$V_T(V_{SB}) = V_T(V_{SB} = 0) + \Delta V_T(V_{SB})$$
Body effect

\[ V_T(V_{SB}) = V_T(V_{SB} = 0) + \Delta V_T(V_{SB}) \]

Modern devices

\[ C_2 = \frac{\varepsilon_{sem} \cdot \varepsilon_0}{W_{max}} \]

\[ \Delta V_T(V_{SB}) \approx \frac{\varepsilon_{sem} \cdot \varepsilon_0}{\varepsilon_{ox} \cdot W_{max}} \cdot V_{SB} \]

Old devices with uniform doping

\[ \Delta V_T(V_{SB}) = \gamma \left( \sqrt{V_0 + V_{SB}} - \sqrt{V_0} \right) \]

\[ V_0 \sim 1 \text{V} \]

\[ \gamma = \frac{\sqrt{2q \varepsilon_{sem} \cdot \varepsilon_0 \cdot N_A}}{C_{ox}} \]

Body effect coefficient
Example

Hence, MOSFET can be on.

1. Assume saturation

\[ I_D^{\text{Sat}} = \frac{\kappa_n' W}{L} \cdot \frac{1}{2} (V_{GS} - V_T)^2 \]

\[ V_{GS} = V_G - I_D \cdot R_S = 5V - 6\kappa_n' W \cdot I_D \]

\[ I_D = 0.5 \frac{W}{V^2} \left(5V - 6\kappa_n' W \cdot I_D - 1V\right)^2 \]

\[ I_D \text{ in mA} \]
Example

1. Assume \( I_D = 0.188 \text{ mA} \)
   \[ V_S = 0.188 \text{ mA} \cdot 6 \Omega = 5.18 V > 5 V = V_G \]
   MOSFET is in cutoff, no current is flowing - wrong

2. Assume \( I_D = 0.15 \text{ mA} \)
   \[ V_S = 0.15 \text{ mA} \cdot 6 \Omega = 3 V \quad \text{good!} \]
   \[ V_{GS} = 5 V - 3 V = 2 V > 1 V \quad \text{OK, i.e. MOSFET is ON.} \]
   \[ V_D = 10 V - 6 \cdot 0.15 = 7 V \quad V_D = 7 V > 3 V = 5 V > V_{GS} - V_T = 1 V \]
   \[ V_{DS} = 1 V \quad 4 V = 10 - R_D \max \cdot 0.15 \text{ mA}, \quad \text{hence} \quad R_D \max = 12 \Omega \]