Last time: BJT CE low frequency response

Need FA

\[ V_{BE} \approx 0.7V \]

\[ V_{CE} > 0.3V \]

Equivalent circuit for low frequency small signal analysis

Coupling and bypass capacitors result into high pass filters (as could be expected).

\[
T_i(f) = \frac{j \cdot f / f_{Li}}{1 + j \cdot f / f_{Li}}
\]

\[
f_{L1} = \frac{1}{2 \cdot \pi \cdot C_1 \cdot (R_{in} + R_s)}
\]

\[
f_{L2} = \frac{1}{2 \cdot \pi \cdot C_2 \cdot (R_{out} + R_L)}
\]

\[
f_{L3} = \frac{1}{2 \cdot \pi \cdot \frac{C_E}{\beta + 1} \cdot (r_x + R_s \parallel R_B)}
\]
Role of the bypass cap $C_E$

3. Assume $C_E$ is finite while $C_1$ and $C_2$ are still infinite.
Bandwidth of Common Emitter amplifier

High frequency 3dB determines amplifier bandwidth.

Amplifier bandwidth is determined by BJT high frequency capabilities – determined by internal parasitic capacitances $C_{\pi}$ and $C_{\mu}$. 
**Frequency dependence of short circuit current gain**

1. \( C_u - C_M = 0 \) \( \frac{\tilde{I}_{out}}{\tilde{I}_{in}} = \frac{q_{in} V_{BE}}{I} = g_{m0} V_{CE} = \beta_0 \)

2. \( C_u \) & \( C_M \neq 0 \)

\( \tilde{I}_{out} = q_{in} V_{BE} - \tilde{I}_M = q_{in} V_{BE} - \frac{V_{BE}}{jwC_u} = (q_{in} - jwC_M) V_{BE} \)

\( \tilde{I}_{in} = \frac{V_{BE}}{R_A} + \frac{V_{BE}}{jwC_u} + \frac{V_{BE}}{jwC_M} \)

\( \frac{\tilde{I}_{out}(\omega)}{\tilde{I}_{in}(\omega)} = \frac{q_{in} - jwC_M}{\frac{1}{R_A} + jw(C_u + C_M)} = \frac{\beta_0 - j\frac{f}{D_{\beta}}}{1 + j\frac{f}{D_{\beta}}} \)

Common emitter current gain defined earlier.

Negligible since \( \ll \beta_0 \) for not extreme frequencies.

- \( C_u \sim 1 \text{pF} \)
- \( C_M \sim 0.1 \text{pF} \)
- \( f_{\beta} \sim 10 \text{MHz} \)

\( \delta_{\beta} = \frac{1}{2\pi R_A (C_u + C_M)} \)
Frequency dependence of short circuit current gain

Unity gain bandwidth \( f_T \):

\[
J_B (f_T) = 1 = \left| \frac{\beta_0}{1 + j \frac{f}{f_T}} \right| \approx \frac{\beta_0}{f_T}
\]

\[
f_T = \beta_0 \cdot \frac{1}{g_m} = \frac{\beta_0}{V_A} \cdot \frac{1}{2 \pi \left( \frac{C_b}{C_b+g_m} \right)} = \frac{g_m}{2 \pi \left( \frac{C_b}{C_b+g_m} \right)}
\]

Looks like it is supposed to improve with bias current because
Frequency dependence of short circuit current gain

Unity gain bandwidth $f_T$:

$$f_b = \frac{\beta_0}{2\pi v_i (e_i + e_p)}$$

$$f_T = \beta_0 \cdot f_b = \frac{\beta_0}{v_i} \cdot \frac{1}{2\pi (e_i + e_p)} = \frac{g_m}{\pi (e_i + e_p)}$$

Looks like it is supposed to improve with bias current because

However it does not. Why?
Frequency dependence of common base current gain

\[ \alpha(f) = \frac{\beta(f)}{\beta(f) + 1} \]

\[ \alpha'(f) = \frac{\beta_0}{1 + j\frac{f}{f_T}/\beta_0} = \frac{\alpha_0}{1 + j\frac{f}{f_T}/(f_\beta(1+\beta_0))} \approx \frac{\alpha_0}{1 + j\frac{f}{f_T}} \]

3dB frequency for \( \alpha \) is equal to \( f_T \).

Hence at \( f_T \) electrons from emitter can not reach collector.
Frequency dependence of common base current gain

\[ \alpha(f) = \frac{\beta(f)}{\alpha(f) + 1} \]

\[ \alpha'(f) = \frac{\beta_0}{1 + \frac{j f}{\beta_0}} = \frac{\alpha_0}{1 + j \frac{f}{f_T}} \]

3dB frequency for \( \alpha \) is equal to \( f_T \).

Hence at \( f_T \) electrons from emitter can not reach collector.

*There are also several parasitic caps associated with technology limitations*
**Base transport time and diffusion capacitance**

time of flight of electrons from emitter to collector.

\[
\tau_{TF} = \frac{w_B}{v_{diff}} \approx \frac{w_B^2}{2D_n}
\]

*Need thin base for high speed operation*

Effective velocity of diffusion electrons
**Base transport time and diffusion capacitance**

Time of flight of electrons from emitter to collector.

\[
\tau_{TF} = \frac{W_B}{V_{diff}} \approx \frac{W_B^2}{2 \Delta n}
\]

*Need thin base for high speed operation*

Electron charge stored in base when current IC is flowing

\[
Q_{TF} = I_c \cdot \tau_{TF} \approx q_v \frac{\Delta n}{2} \cdot W_B
\]

\[
C_{TF} = \frac{dQ_{TF}}{dV_{BE}} \bigg|_{I_c} = \tau_{TF} \cdot \frac{dI_c}{dV_{BE}} \bigg|_{I_c} - \tau_{TF} \cdot \frac{q}{\mu}
\]

\[
C_{TF} = C_{TF} + C_{BEj} \quad ; \quad C_{\mu} = C_{CBj}
\]

Charge storage capacitance  
Pn-junction depletion region capacitances and other parasitic caps
Unity gain bandwidth

\[ C_W = CT_F + C_{BEj} \quad \& \quad C_u = C_{CBj} \]

\[ \tau_T = \frac{C_u}{2 \mu \left( C_{BEj} + C_{CBj} \right) + 2 \mu \cdot g_m \cdot \tau_T} = \frac{1}{2 \mu \tau_T} \]

Total time delay

Minimum possible time delay

Ultimate limit for BJT speed
**Bandwidth of Common Emitter amplifier**

Amplifier bandwidth \( f_H \) is determined by BJT internal parasitic capacitances \( C_\pi \) and \( C_\mu \).

- **Charge storage capacitance**
- **Pn-junction depletion region capacitances and other parasitic caps**
Unity gain bandwidth

Unity gain bandwidth $f_T$.

\[ f_T = \frac{g_m}{2\pi (c_u + c_m)} \]

\[ f_T = \frac{f_0}{1 + j \frac{f}{f_B}} \]

\[ f_T = \beta_0 \cdot f_B \]

\[ \beta_0 = \frac{1}{2\pi \cdot r_n \cdot (c_u + c_m)} \]
Net voltage gain bandwidth of CE BJT amplifier

We are interested in high frequency cutoff, i.e. coupling and bypass capacitors can be replaced by short circuit for frequencies $>> f_L$.

We have got capacitive coupling between input and output.

$\text{Theorem: } \frac{v_{eq}}{v_s} = \frac{v_{eq}}{v_s} \frac{R_{eq}}{R_{eq} + \frac{1}{C_H}}$
Observe increase of $i_\mu$ with transconductance $g_m$. 

$$V_L = \hat{R}_L \left( i_\mu - g_m \hat{V}_{BE} \right) - \hat{V}_{BE} = \frac{\hat{z}_\mu}{jw C_\mu}, \text{ hence}$$

$$\hat{z}_\mu = \hat{V}_{BE} \frac{1 + g_m \hat{R}_L}{\hat{R}_L + \frac{1}{jw C_\mu}} = \hat{V}_{BE} \frac{(1 + g_m \hat{R}_L) \cdot jw C_\mu}{1 + jw C_\mu \cdot \hat{R}_L}$$
Net voltage gain bandwidth of CE BJT amplifier

\[ V_L = V_{BE} - \hat{I}_M \frac{1}{j \omega C_{BE}} = V_{BE} \left( \frac{j \omega C_{BE} \hat{R}_L - q_m \hat{R}_L}{1 + j \omega C_{BE} \hat{R}_L} \right) \]

\[ V_{ef} = V_{BE} + R_{ef} (\hat{I}_M + j \omega C_{BE} \hat{V}_{BE}) \]

\[ \frac{V_L}{V_{ef}} = \frac{j \omega C_{BE} \hat{R}_L - q_m \hat{R}_L}{(1 + j \omega C_{BE} R_{ef})(1 + j \omega C_{BE} \hat{R}_L) + (1 + q_m \hat{R}_L) j \omega C_{BE} \hat{R}_L} \]
Net voltage gain bandwidth of CE BJT amplifier

\[
\frac{\overline{V_L}}{\overline{V_{eq}}} = \frac{\frac{1}{\mu R_C} - \frac{1}{\mu R_L}}{(1 + jw C_{eq} R_C)(1 + jw C_{eq} R_L) + (1 + \frac{1}{\mu R_L}) jw C_{eq} R_C R_L}
\]
Net voltage gain bandwidth of CE BJT amplifier

\[
\frac{V_L}{V_{eq}} = \frac{j\omega C_\mu \hat{R}_L - j\mu \hat{R}_L}{(1 + j\omega C_{\mu \hat{R}_L})(1 + j\omega C_\mu \hat{R}_L) + (1 + j\mu \hat{R}_L)j\omega C_\mu \hat{R}_L}
\]

nominator

\[f_T \approx \frac{1}{2 \cdot \pi \cdot \tau_{TF}} \approx 1 \text{GHz}, \; C_\mu \approx 0.1 \text{pF}
\]

\[\omega \cdot C_\mu \leq 0.001 \Omega^{-1}
\]

\[g_m \approx 0.04
\]
Net voltage gain bandwidth of CE BJT amplifier

\[ \frac{V_L}{V_{eq}} = \frac{j\omega \mu R_C - \mu_m \hat{R}_L}{(1 + j\omega C_{\mu} R_{eq})(1 + j\omega C_{\mu} \hat{R}_L) + (1 + \mu_m \hat{R}_L) j\omega C_{\mu} R_{eq}} \]

nominator

\[ f_T \approx \frac{1}{2 \cdot \pi \cdot \tau_{TF}} \approx 1 \text{GHz}, \quad C_\mu \approx 0.1 \text{pF} \]

\[ \omega \cdot C_\mu \leq 0.001 \Omega^{-1} \]

\[ g_m \approx 0.04 \]
Net voltage gain bandwidth of CE BJT amplifier

\[
\frac{V_L}{V_{eq}} = \frac{jwC_\mu R_L - jwR_L}{(1+jwC_\mu R_L)(1+jwC_\mu \hat{R}_L) + (1+jw\hat{R}_L)jwC_\mu R_L}
\]

**nominator**

\[
f_T \approx \frac{1}{2 \cdot \pi \cdot \tau_{TF}} \approx 1 \text{GHz}, \quad C_\mu \approx 0.1 \text{pF}
\]

\[
\omega \cdot C_\mu << 0.001 \Omega^{-1}
\]

\[
g_m \approx 0.04 \Omega^{-1}
\]

**denominator**

for

\[
C_\mu \approx C_\pi / 10
\]

and \( f << f_T \)

\[
\omega \cdot C_\mu \cdot \hat{R}_L << 1
\]
Net voltage gain bandwidth of CE BJT amplifier

\[ \frac{V_L}{V_{eq}} = \frac{j\omega C_{\mu} R_L - g_m V_L}{(1 + j\omega C_{eq} R_L)(1 + j\omega C_{\mu} R_L) + (1 + g_m R_L) j\omega C_{\mu} R_L} \]

\( \text{nominator} \)

\[ f_T \approx \frac{1}{2 \cdot \pi \cdot \tau_{TF}} \approx 1 \text{GHz}, \quad C_{\mu} \approx 0.1 \text{pF} \]

\[ \omega \cdot C_{\mu} \ll 0.001 \Omega^{-1} \]

\[ g_m \approx 0.04 \Omega^{-1} \]

\( \text{denominator} \)

\[ \text{for} \]

\[ C_{\mu} \approx C_\pi /10 \]

\[ \text{and} \quad f \ll f_T \]

\[ \omega \cdot C_{\mu} \cdot \hat{R}_L \ll 1 \]
Net voltage gain bandwidth of CE BJT amplifier

\[ \frac{v_L}{v_{eq}} \approx \frac{-g_{m} \hat{R}_L}{1 + j\omega R_{eq} (C_A + (1 + g_{m} \hat{R}_L) \cdot C_M)} \]

\[ \frac{(\tilde{V}_L(f))}{(\tilde{V}_S(f))} = G_V(f) \propto \frac{-g_{m} \left( \hat{R}_L \parallel \hat{R}_c \parallel \hat{R}_o \right)}{1 + \frac{f}{f_M}} \]

\[ f_M = \frac{1}{2\pi R_{eq} (C_A + (1 + g_{m} \hat{R}_L) \cdot C_M)} \]

Resistor in series with input cap
equivalent input cap
Net voltage gain bandwidth of CE BJT amplifier

\[ G_{v0} = \frac{R_L}{R_{\text{in}} + R_S} \cdot A_{v0} \cdot \frac{R_L}{2 \pi f_{3dB}} \]

\[ A_{v0} = -g_{\mu m} (R_e \| R_L) \]

\[ f_M = \frac{1}{2 \pi R_{eq} (C_{\pi} + C_{\mu} (1 + g_{\mu m} (R_L \| R_e \| R_0)))} \]

\[ C_{\mu} (1 + A_{v0}) = C_{\mu} \]

“Miller” capacitor
Example

\[ V_{cc} = V_{ee} = 10 \, V \]  
\[ I_E = 1 \, mA \]  
\[ R_B = 100 \, k\Omega \]  
\[ \beta_n = 100 \]  
\[ V_A = 100 \, V \]  
\[ C_C = 1 \, pF \]  
\[ f_T = 800 \, MHz \]  
\[ R_S = R_L = 5 \, k\Omega \]

* confirm FA regime first ...

\[ 1. \quad q_m = \frac{I_E}{2q E_V} = 140 \, \mu A \quad \frac{V}{mA} \]

\[ R_T = \frac{100}{q_m} = 2.5 \, k\Omega \]

\[ R_o = \frac{100 \, V}{1 \, mA} = 100 \, k\Omega \]

\[ 2. \quad C_T + C_C = \frac{q_m}{\omega T} = \frac{4.0 \times 10^{-3}}{2\pi \times 800 \times 10^6} = 8 \, pF \]

\[ C_C = 1 \, pF \quad \text{hence} \quad C_T = 7 \, pF \]

\[ 3. \quad G_V = -\frac{R_{o} || R_{T} || V_{T}}{R_{o} || R_{T} + R_S} \times (-q_m (R_{o} || R_{T} || R_{o})) = -\frac{4.0 \times 10^{-3} \times 3 \times 10^3}{(100 \times 12.5 \Omega) + 5 \Omega} \times (100 \times 12.5 \Omega) \]

\[ G_V \approx -121 \frac{V}{V} \cdot \frac{1}{3} = -40 \frac{V}{V} \]

\[ q_m \cdot R_L = 121 \frac{V}{V} \]
Example – cont.

\[ V_{cc} = V_{EE} = 10 \, V \quad I_e = 1 \, mA \quad R_B = 100 \, \mu \Omega \]
\[ I_e = 100 \, \mu A \quad R_C = 8 \, \Omega \]
\[ V_A = 100 \, V \quad C_m = 1 \, pF \quad f_T = 800 \, MHz \quad R_S = R_L = 5 \, \Omega \]

\[ G_V \approx -121 \frac{V}{V} \cdot \frac{1}{2} \approx -40 \frac{V}{V} \]

\[ R_{eq} = (R_m \parallel R_S) \parallel R_S = 1 \, \Omega \]

\[ C_m \, \mu F = 7 \, pF + 121 \cdot 1 \, pF = 128 \, pF \gg C_m \]

\[ f_H = \frac{1}{2 \pi \cdot C_m \cdot R_s} = 750 \, \mu Hz \ll f_T \]

Even << \( f_B = 8 \, MHz \)

Reduced gain – improved BW

\[ R_L = 2 \, \Omega \quad \gamma_m \, R_L \approx 64 \frac{V}{V} \]

\[ f_H \approx 1.4 \, MHz \]
Miller Effect

\[ \tilde{V}_{\text{Z}} = \tilde{V}_{\text{in}} - \tilde{A}_V \cdot \tilde{I}_{\text{in}} = \tilde{V}_{\text{in}} (1 - \tilde{A}_V) \]

\[ \tilde{I}_{\text{Z}} = \frac{\tilde{V}_{\text{Z}}}{Z} = \frac{\tilde{V}_{\text{in}} (1 - \tilde{A}_V)}{Z} = \frac{\tilde{V}_{\text{in}}}{(1 - \tilde{A}_V)} \]

Miller transformation

\[ Z_{\text{in}} = \frac{Z}{1 - \tilde{A}_V} \]

\[ Z_{\text{out}} = \frac{Z - \tilde{A}_V}{\tilde{A}_V - 1} \]
If $i_\mu$ is not very high (small $C_\mu$), then

\[ \begin{align*}
Z_{in} &= \frac{1}{j\omega C_\mu (1 - A_v)} \\
Z_{out} &= \frac{1}{j\omega C_\mu A_v - 1}
\end{align*} \]

Low pass filter

Negligible, hence open circuit

Total equivalent input cap

\[
V_{out} = \left( -g_{m} R_L \right) \cdot V_{BE}
\]

\[
C_{in} = C_\mu \left( 1 + g_{m} R_L \right)
\]

\[
C_{out} = C_\mu
\]
CB amplifier does not suffer from Miller effect

Neglect $r_o$ and build equivalent circuit

Observe no capacitor coupling output and input – No Miller effect.

\[ V_L = \left(-\frac{q_m V_{BE}}{R_c \parallel R_L \parallel C_{\mu}}\right) \]

\[ V_{BE} = V_S + R_S \left( \frac{q_m V_{BE} + \frac{V_{BE}}{R_e \parallel C_P}}{V_c \parallel \frac{1}{j \omega C_P}} \right) \]

\[ \frac{V_L}{V_S} = \frac{q_m (R_c \parallel R_L \parallel C_{\mu})}{1 + R_S \left( \frac{q_m + \frac{V_c + \frac{1}{j \omega C_P}}{V_c - \frac{1}{j \omega C_P}}}{V_c \parallel \frac{1}{j \omega C_P}} \right)} \]
CB amplifier does not suffer from Miller effect

Neglect $r_o$ and build equivalent circuit

Observe no capacitor coupling output and input – No Miller effect.

\[
\frac{\mathcal{V}_L(f)}{\mathcal{V}_S(f)} = \frac{g_m (V_C || V_L)}{1 + 2 \mu (g_m + \frac{1}{r_o})} \left( \frac{1}{1 + j \omega (R_e || R_L) C_m} \right) \left( \frac{1}{1 + j \omega \frac{V_S}{1 + 2 \mu (g_m + \frac{1}{r_o})}} \right)
\]

\[
f_1 = \frac{1}{2 \pi (V_C || V_L) C_m} = \frac{1}{6.28 \cdot 3 \cdot 10^3 \cdot 10^{-12}} = 52 \text{ MHz}
\]

\[
f_2 = \frac{1}{2 \pi C_R (R_S || V_m || \frac{1}{g_m})} \approx \frac{g_m}{2 \pi C_m} = \frac{4 \cdot 10^{-3}}{6.28 \cdot 10^{-12}} \approx 910 \text{ MHz}
\]