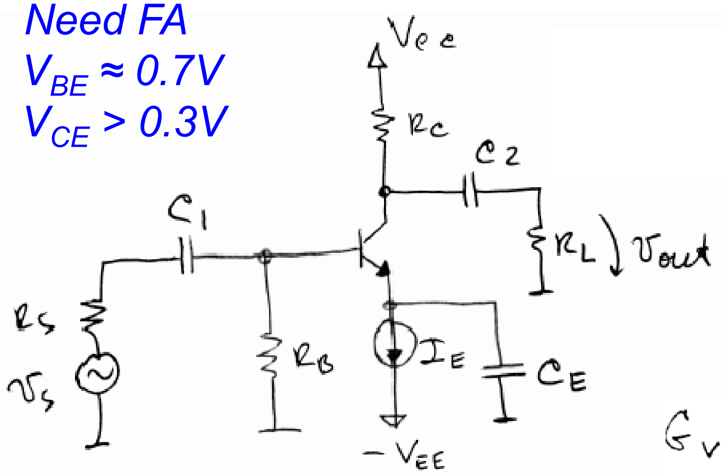
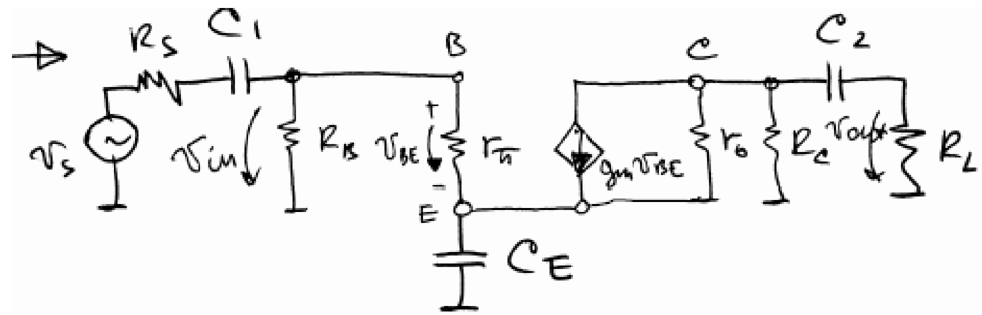
Last time: BJT CE low frequency response

Need FA
$V_{BE} \approx 0.7V$
$V_{CE} > 0.3V$



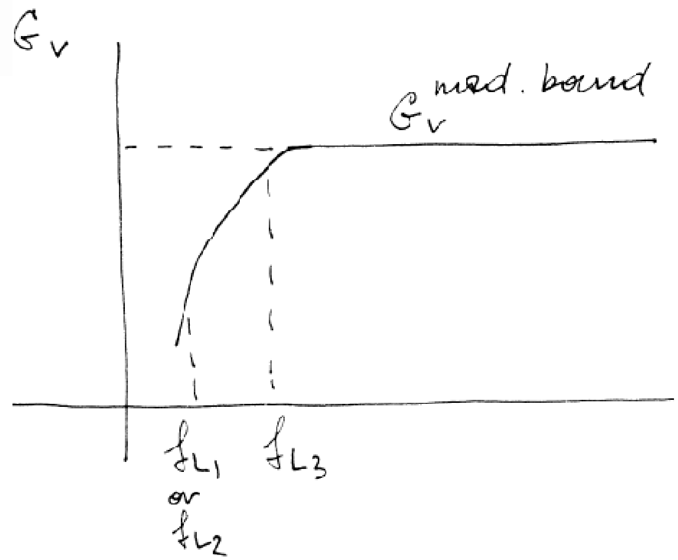
Equivalent circuit for low frequency small signal analysis



$$g_m = \left(\frac{dI_c}{dV_{BE}}\right)\Bigg|_{I_c^Q} = \frac{I_c^Q}{V_{th}}$$

$$r_\pi = \frac{dV_{BE}}{dI_B}\Bigg|_{I_B^Q} = \frac{\beta}{g_m}$$

$$r_o = \frac{dV_{CE}}{dI_c}\Bigg|_{I_c^Q} = \frac{V_A}{I_c^Q}$$



$$T_i(f) = \frac{j \cdot f/f_{Li}}{1 + j \cdot f/f_{Li}}$$

$$f_{L1} = \frac{1}{2 \cdot \pi \cdot C_1 \cdot (R_{in} + R_S)}$$

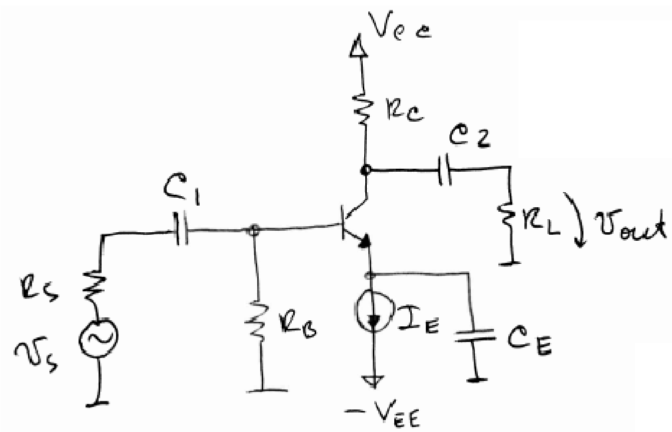
$$f_{L2} = \frac{1}{2 \cdot \pi \cdot C_2 \cdot (R_{out} + R_L)}$$

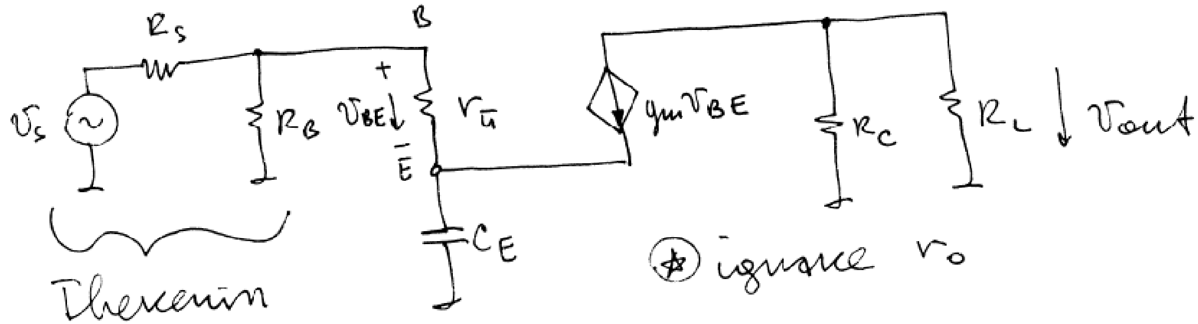
$$f_{L3} = \frac{1}{2 \cdot \pi \cdot \frac{C_E}{\beta + 1} \cdot (r_\pi + R_S \,||\, R_B)}$$

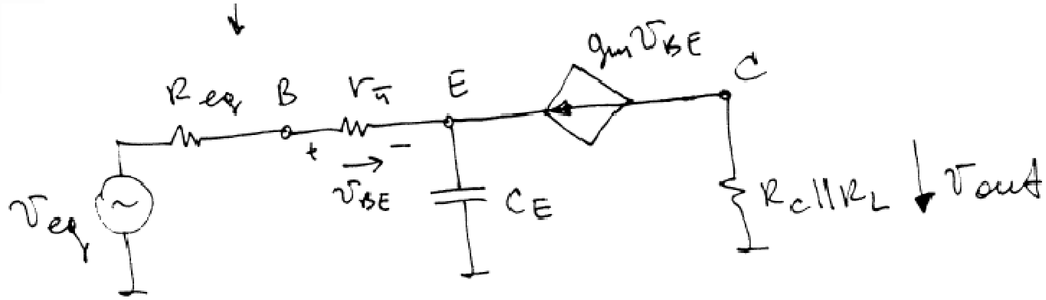
Coupling and bypass capacitors result into high pass filters (as could be expected).

Role of the bypass cap C_E

3. *Assume C_E is finite while C_1 and C_2 are still infinite.*







$$v_{eq} = v_s \frac{R_B}{R_S + R_B}$$

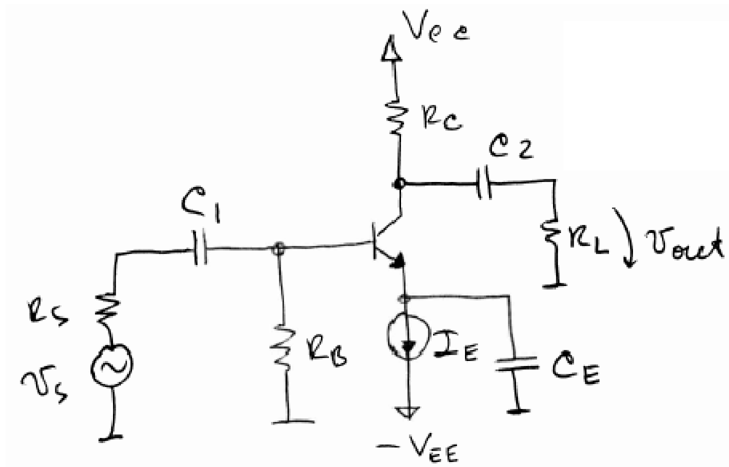
$$R_{eq} = \frac{R_B R_S}{R_S + R_B}$$

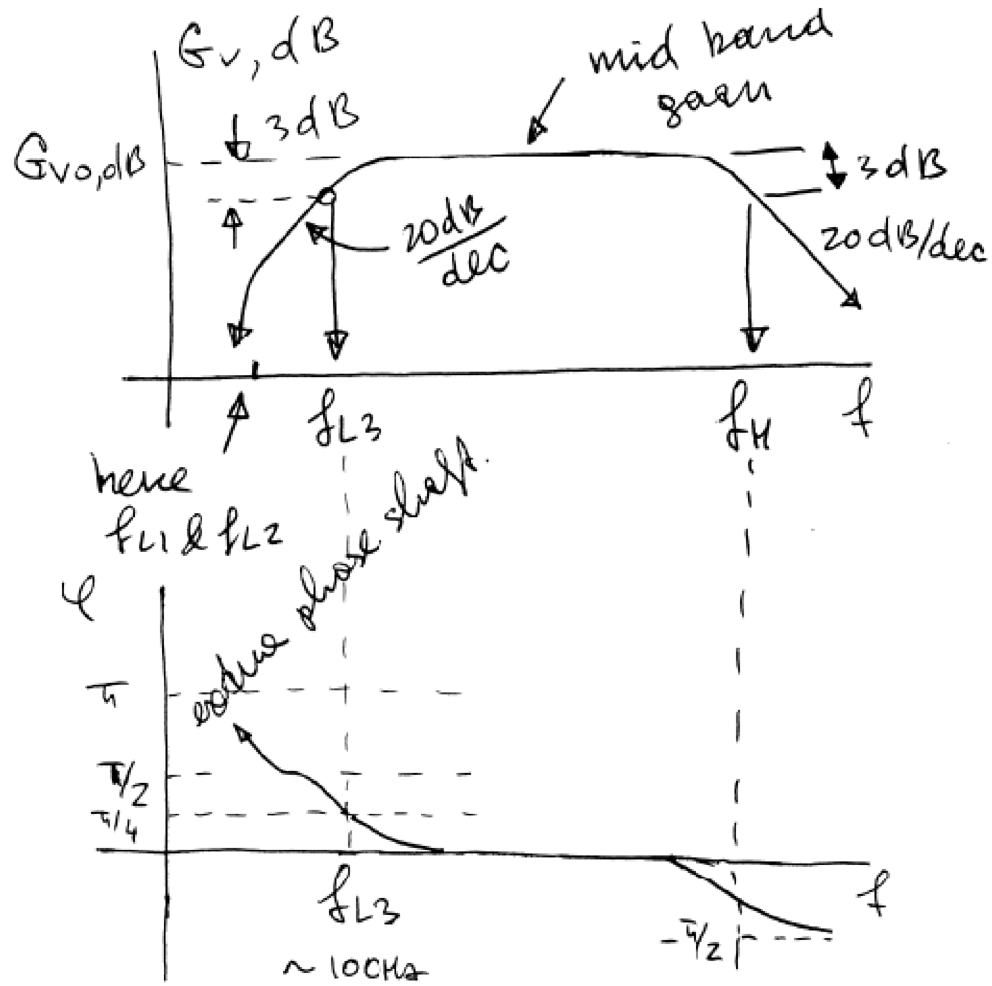
$$G_v(f) = \frac{v_{out}}{v_s} = \frac{-g_m v_{BE}(R_c \| R_L)}{\frac{R_S + R_B}{R_B} \cdot v_{eq}} \propto \frac{v_{BE}}{v_{eq}}.$$

$$\frac{v_{BE}}{v_{eq}} = \frac{\cancel{i_b} \cdot r_\pi}{(r_\pi + R_{eq})\cancel{i_b} + \cancel{i_b}(\beta+1)\frac{1}{j\omega C_E}} = \frac{r_\pi}{r_\pi + R_{eq}} \cdot \frac{j\omega (r_\pi + R_{eq})\frac{C_E}{\beta+1}}{j\omega (r_\pi + R_{eq})\frac{C_E}{\beta+1} + 1}$$

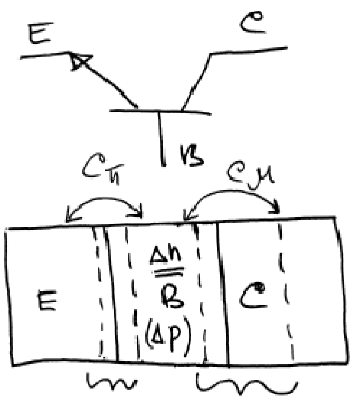
Hence: $G_v = G_{v0} \cdot \dfrac{jf/f_{L3}}{1 + jf/f_{L3}}$; $f_{L3} = \dfrac{1}{2\pi\, \underset{\sim}{(r_\pi + R_{eq})}\, \frac{C_E}{\beta+1}}$

Bandwidth of Common Emitter amplifier



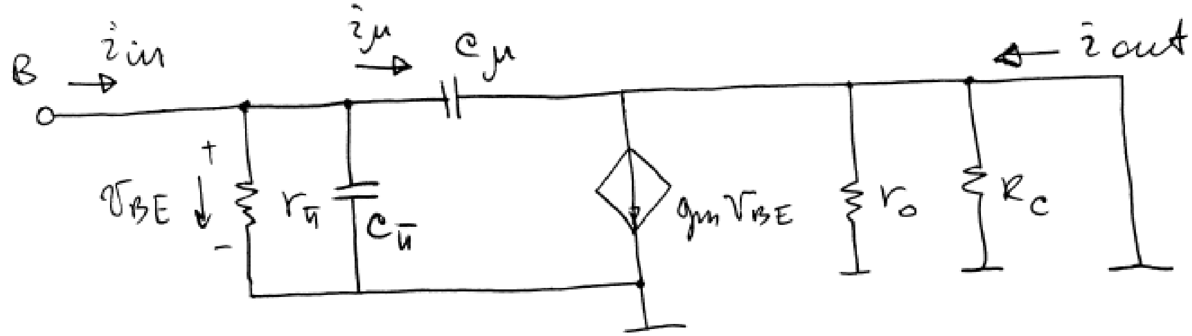


High frequency 3dB determines amplifier bandwidth.



Amplifier bandwidth is determined by BJT high frequency capabilities – determined by internal parasitic capacitances C_π and C_μ.

Frequency dependence of short circuit current gain



$$\frac{i_{out}}{i_{in}} = ?$$

① $C_\pi = C_\mu = 0$ $\quad \left(i_{out} / i_{in} \right) = \dfrac{g_m v_{BE}}{i_b} = g_m \cdot r_\pi = \beta_o$

Common emitter current gain defined earlier.

② C_π & $C_\mu \neq 0$

$$i_{out} = g_m v_{BE} - i_\mu = g_m v_{BE} - \frac{v_{BE}}{\frac{1}{j\omega C_\mu}} = (g_m - j\omega C_\mu) \cdot v_{BE}$$

$$i_{in} = \frac{v_{BE}}{r_\pi} + \frac{v_{BE}}{\frac{1}{j\omega C_\pi}} + \frac{v_{BE}}{\frac{1}{j\omega C_\mu}}$$

Negligible since << β_0 for not extreme frequencies.

$$\frac{i_{out}(\omega)}{i_{in}(\omega)} = \frac{g_m - j\omega C_\mu}{\frac{1}{r_\pi} + j\omega(C_\pi + C_\mu)} = \frac{\beta_o - j f (2\pi r_\pi \cdot C_\mu)}{1 + j f / f_\beta}$$

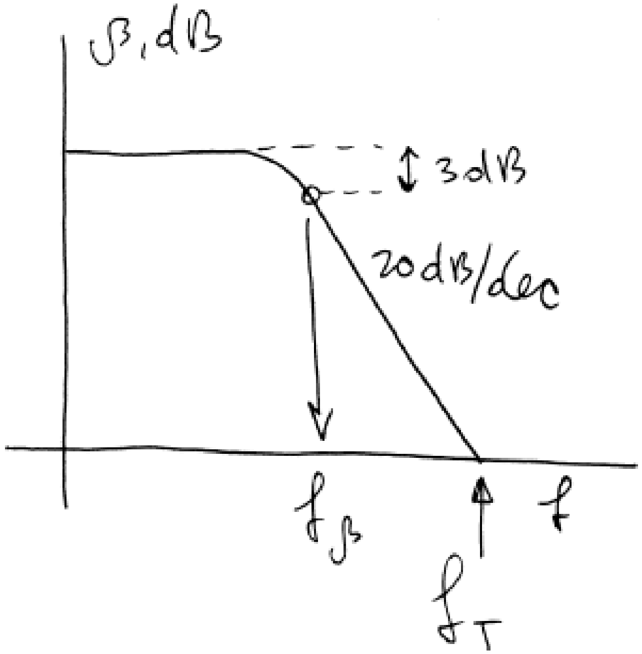
$$f_\beta = \frac{1}{2\pi r_\pi (C_\pi + C_\mu)}$$

(✱) $C_\pi \sim 1\,pF$
$C_\mu \sim 0.1\,pF$
$f_\beta \sim 10\,MHz$

Frequency dependence of short circuit current gain

$$\beta(f) \simeq \frac{\beta_0}{1 + j f / f_\beta}$$

$$f_\beta = \frac{1}{2\pi r_\pi (C_\pi + C_\mu)}$$



Unity gain bandwidth f_T.

$$\beta(f_T) = 1 = \left| \frac{\beta_0}{1 + j f_T / f_\beta} \right| \approx \frac{\beta_0 \cdot f_\beta}{f_T}$$

$$f_T = \beta_0 \cdot f_\beta = \frac{\beta_0}{r_\pi} \cdot \frac{1}{2\pi (C_\pi + C_\mu)} = \frac{g_m}{2\pi (C_\pi + C_\mu)}$$

Looks like it is supposed to improve with bias current because $g_m = \frac{I_C^Q}{V_{th}}$

Frequency dependence of short circuit current gain

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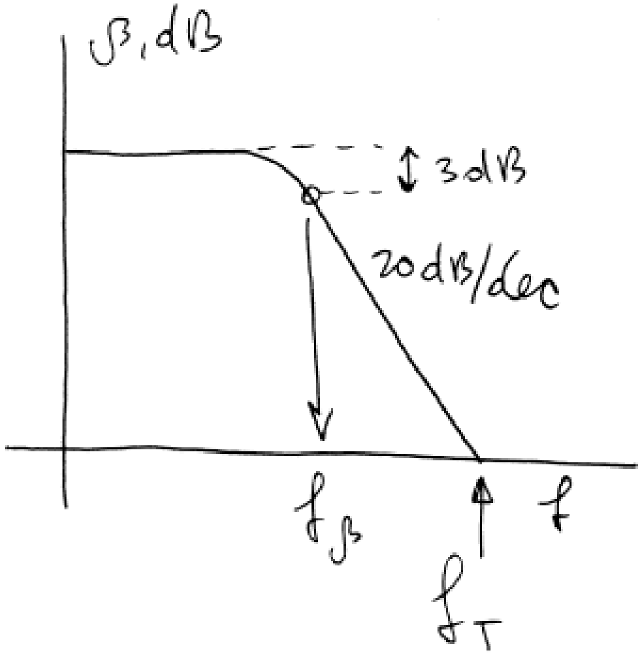

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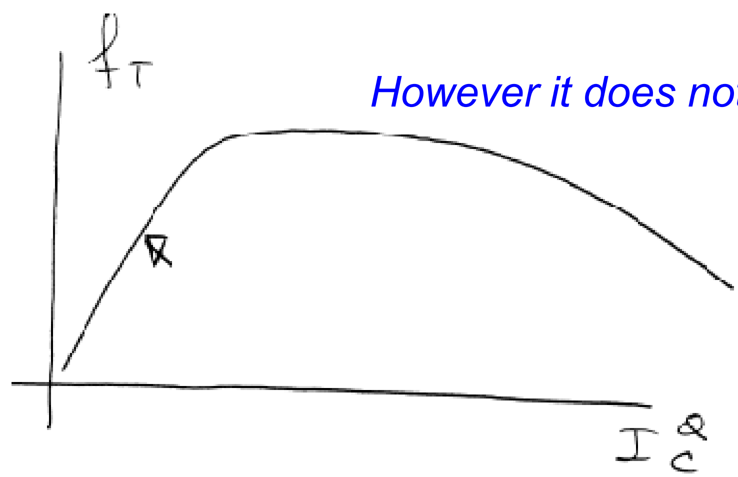
Looks like it is supposed to improve with bias current because $g_m = \frac{I_C^Q}{V_{th}}$

However it does not. Why?

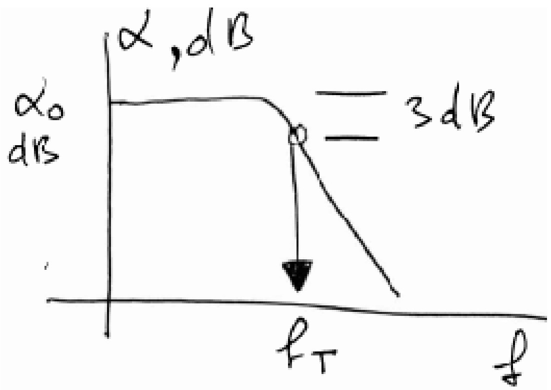


Frequency dependence of common base current gain

$$\alpha(f) = \frac{\beta(f)}{\beta(f)+1}$$

$$\alpha(f) = \frac{\beta_0}{1+\beta_0+jf/f_\beta} = \frac{\alpha_0}{1+jf/(f_\beta(1+\beta_0))} \simeq \frac{\alpha_0}{1+jf/f_T}$$

3dB frequency for α is equal to f_T.



Hence at f_T electrons from emitter can not reach collector.

Frequency dependence of common base current gain

$$\alpha(f) = \frac{\beta(f)}{\beta(f)+1}$$

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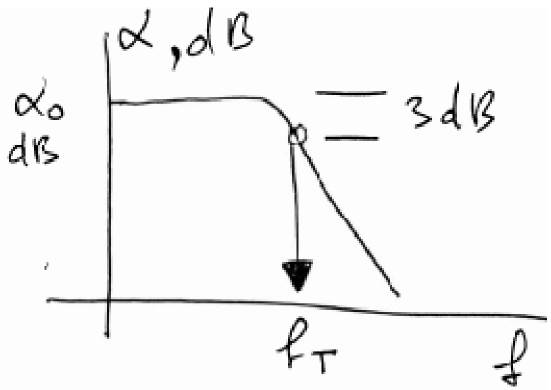
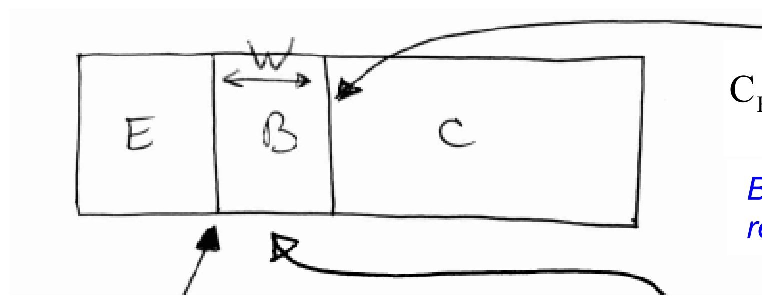




$$C_{BC}(V_{CB}) = \frac{C_{BC0}}{(1+V_{CB}/V_{bi})^{1/2}}$$

BC-junction depletion region capacitance

$$C_{BE}(V_{BE}) = \frac{C_{BE0}}{(1-V_{BE}/V_{bi})^{1/2}}$$

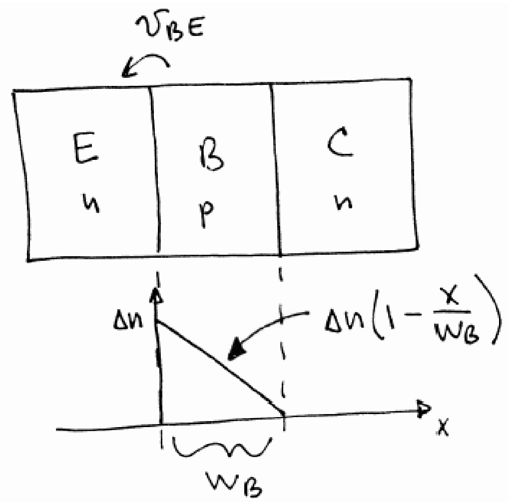
EB-junction depletion region capacitance

Base transport time – time of flight of electrons from emitter to collector.

Hence at f_T electrons from emitter can not reach collector.

**There are also several parasitic caps associated with technology limitations*

Base transport time and diffusion capacitance



$$J_{diff} = q \cdot D_n \cdot \frac{dn}{dx} = q \cdot n \cdot \underbrace{\left(\frac{D_n}{n} \cdot \frac{dn}{dx}\right)}_{v_{diff}}$$

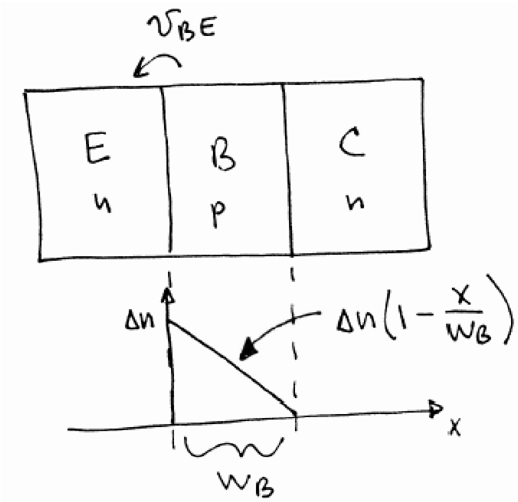
Effective velocity of diffusion electrons $\rightarrow v_{diff}$.

time of flight of electrons from emitter to collector.

$$\tau_{TF} = \frac{W_B}{v_{diff}} \simeq \frac{W_B^2}{2D_n}$$

**Need thin base for high speed operation*

Base transport time and diffusion capacitance



time of flight of electrons from emitter to collector.

$$\tau_{TF} = \frac{W_B}{v_{diff}} \simeq \frac{W_B^2}{2D_n}$$

**Need thin base for high speed operation*

Electron charge stored in base when current IC is flowing

$$Q_{TF} = I_C \cdot \tau_{TF} \approx q\,\frac{\Delta n}{2}\cdot W_B$$

$$C_{TF} = \left.\frac{dQ_{TF}}{dV_{BE}}\right|_{I_C^Q} = \tau_{TF}\cdot\left.\frac{dI_C}{dV_{BE}}\right|_{I_C^Q} = \tau_{TF}\cdot g_m$$

$$C_\pi = C_{TF} + C_{BEj}\ ; \quad C_\mu = C_{CBj}$$

Charge storage capacitance

Pn-junction depletion region capacitances and other parasitic caps

Unity gain bandwidth

$$C_\pi = \underbrace{C_{TF}}_{= \tau_{TF} \cdot g_m} + C_{BEj} \quad \& \quad C_\mu = C_{CBj}$$

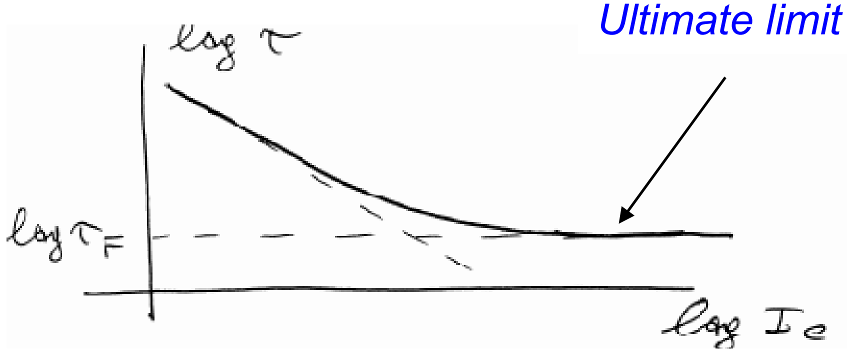
$$f_T = \frac{g_m}{2\pi (C_{BEj} + C_{CBj}) + 2\pi \cdot g_m \cdot \tau_{TF}} = \frac{1}{2\pi \tau_T}$$

Total time delay

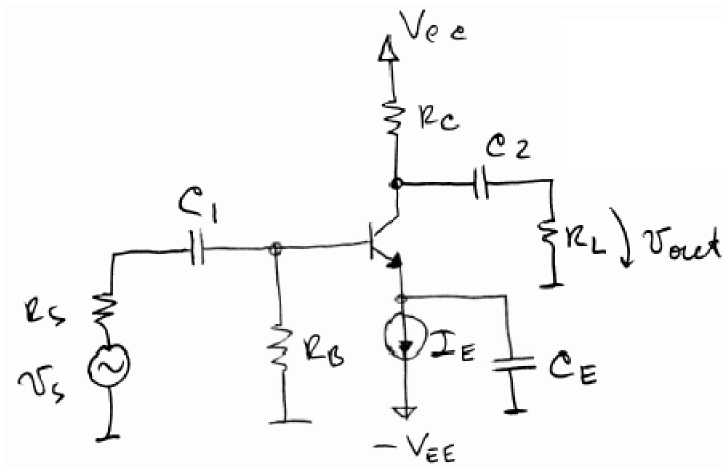
$$\tau_T = \tau_{TF} + \frac{C_{BEj} + C_{CBj}}{g_m} \quad \& \quad g_m \sim I_C^Q$$

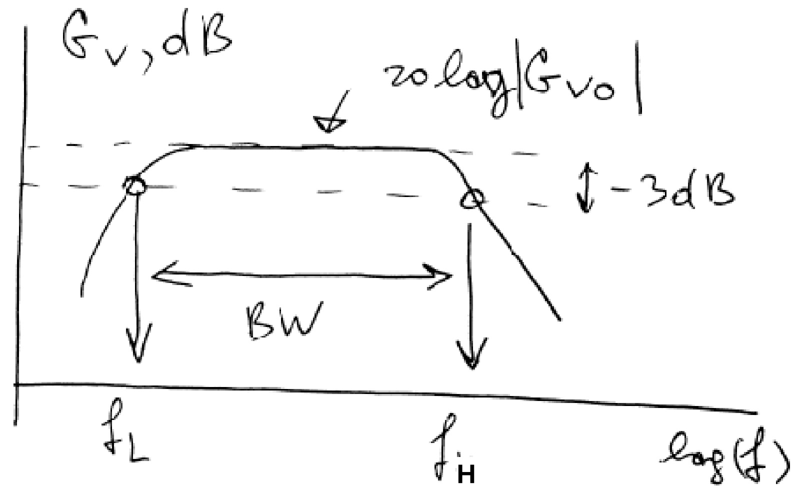
Minimum possible time delay

Ultimate limit for BJT speed

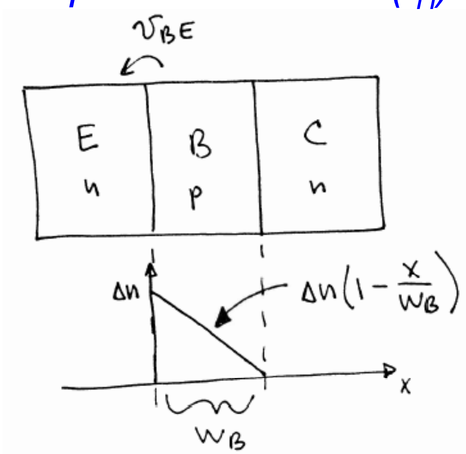


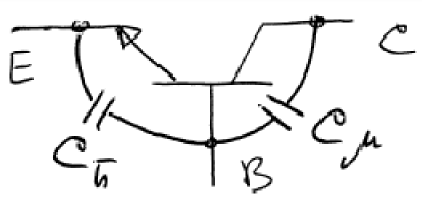
Bandwidth of Common Emitter amplifier





Amplifier bandwidth (f_H) is determined by BJT internal parasitic capacitances C_π and C_μ.





$$Q_{TF} = I_C \cdot \tau_{TF} \approx q\,\frac{\Delta n}{2}\cdot W_B \qquad \tau_{TF} = \frac{W_B}{v_{diff}} \approx \frac{W_B^2}{2D_n}$$

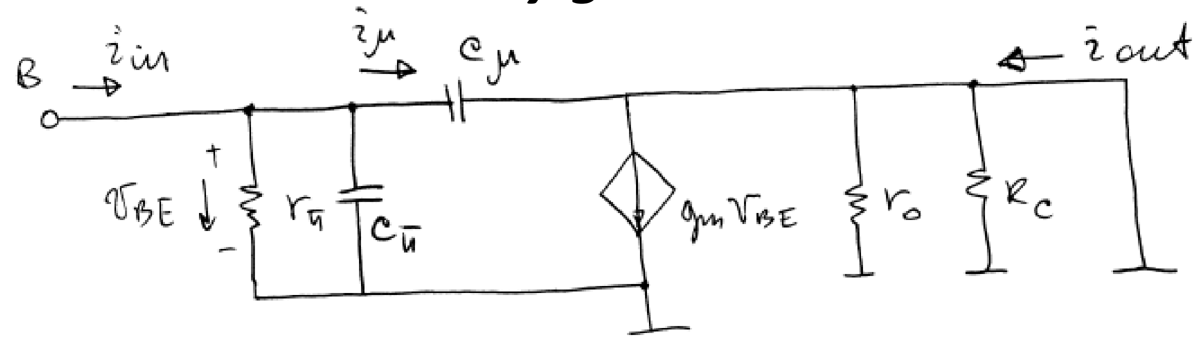
$$C_{TF} = \left.\frac{dQ_{TF}}{dV_{BE}}\right|_{I_C^Q} = \tau_{TF}\cdot\left.\frac{dI_C}{dV_{BE}}\right|_{I_C^Q} = \tau_{TF}\cdot g_m$$

$$C_\pi = C_{TF} + C_{BEj} \quad ; \quad C_\mu = C_{CBj}$$

Charge storage capacitance

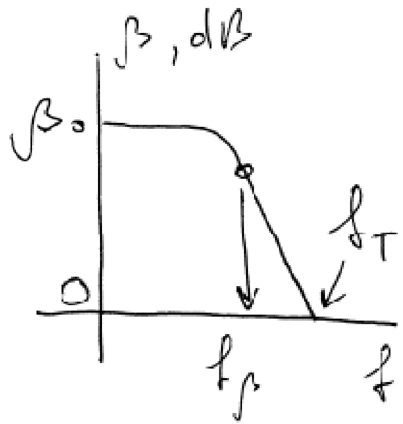
Pn-junction depletion region capacitances and other parasitic caps

Unity gain bandwidth



$$\frac{i_{out}}{i_{in}} = ?$$

$$\beta(f) = \frac{i_{out}}{i_{in}} \approx \frac{\beta_0}{1 + jf/f_\beta}$$



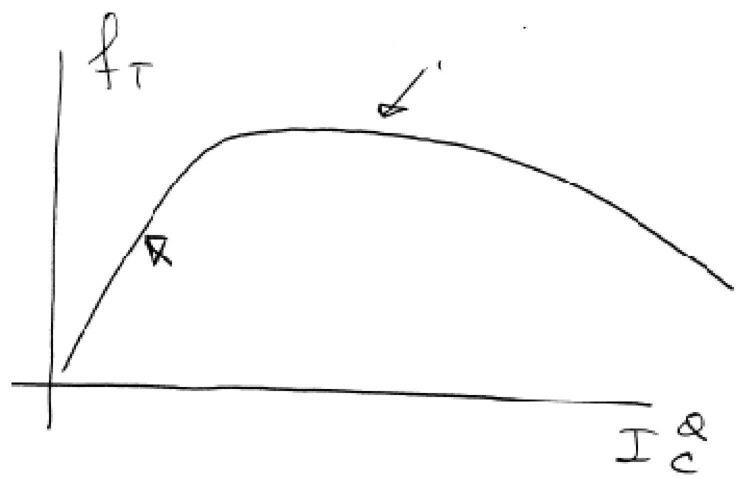
$$f_T = \beta_0 \cdot f_\beta$$

$$f_\beta = \frac{1}{2\pi r_\pi (C_\pi + C_\mu)}$$

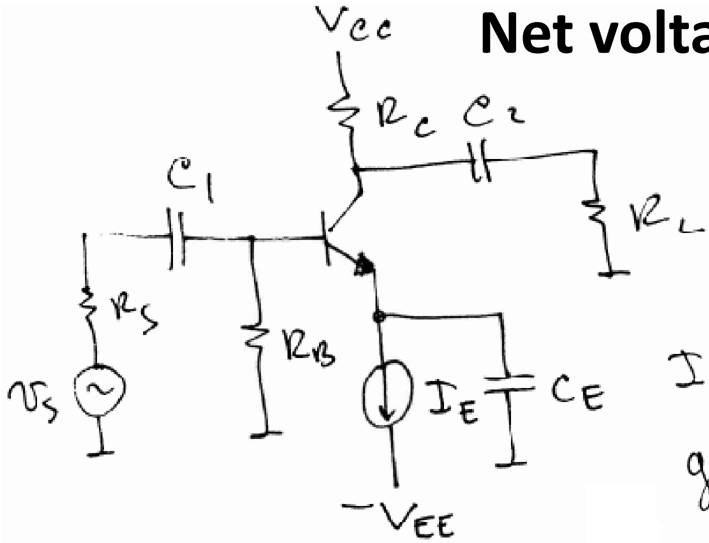
Unity gain bandwidth f_T.

$$C_\pi + C_\mu = C_{\text{junctions}} + \frac{\tau_{TF}}{V_{th}} \cdot I_C^Q$$

$$f_T = \frac{g_m}{2\pi (C_\pi + C_\mu)} \approx \frac{I_C^Q / V_{th}}{2\pi \left(C_{\text{junctions}} + \frac{\tau_{TF}}{V_{th}} \cdot I_C^Q \right)}$$



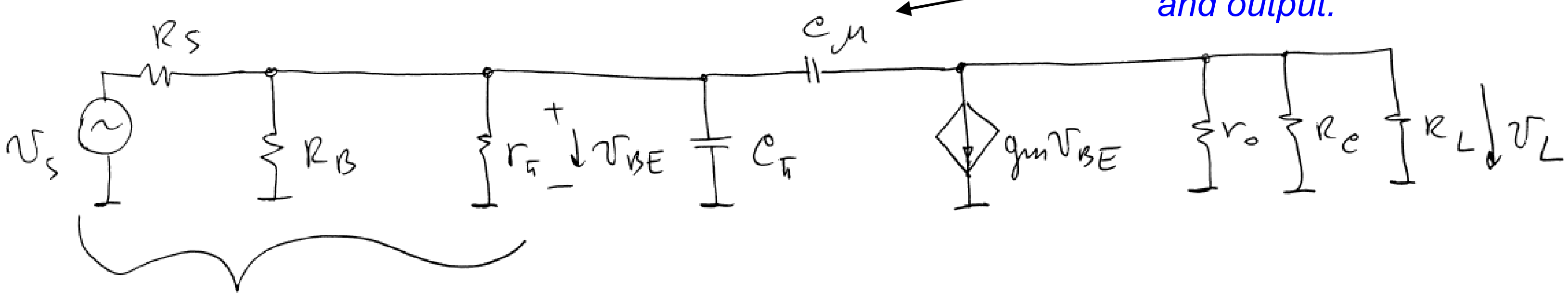
Net voltage gain bandwidth of CE BJT amplifier



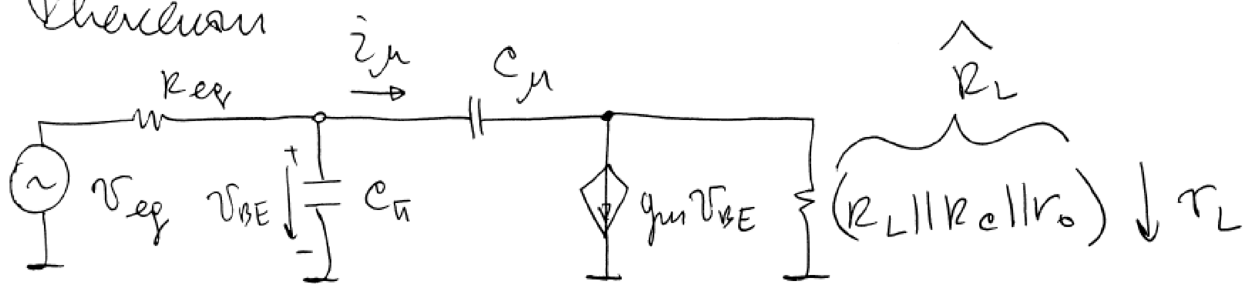
We are interested in high frequency cutoff, i.e. coupling and bypass capacitors can be replaced by short circuit for frequencies >> f_L.

I_E determines: g_m, r_π, r_o & C_π.

We have got capacitive coupling between input and output.

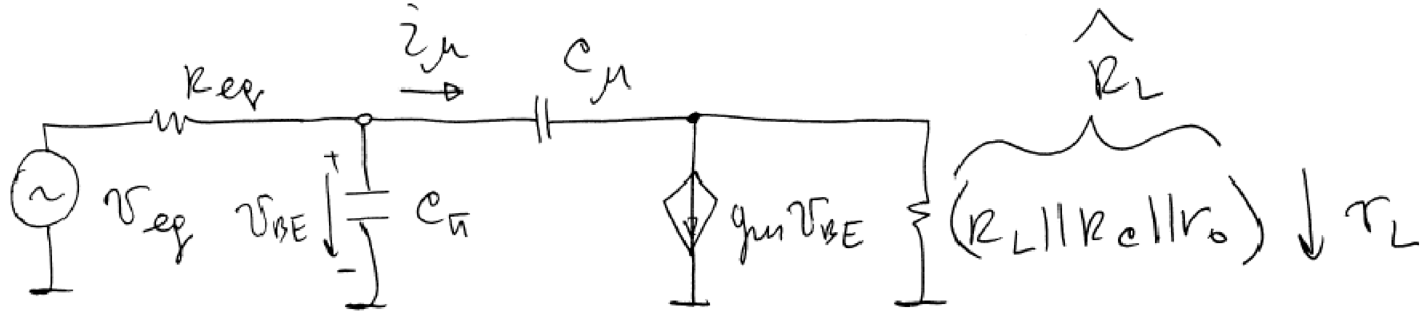


Thevenin



$$v_{eq} = v_s \frac{R_B || r_\pi}{R_S + R_B || r_\pi} \quad ; \quad R_{eq} = (r_\pi || R_B || R_S)$$

Net voltage gain bandwidth of CE BJT amplifier



$$v_L = \hat{R}_L \left(i_\mu - g_m v_{BE}\right) = v_{BE} - \frac{i_\mu}{j\omega C_\mu}, \text{ hence}$$

$$i_\mu = v_{BE} \frac{1 + g_m \hat{R}_L}{\hat{R}_L + \dfrac{1}{j\omega C_\mu}} = v_{BE} \frac{(1 + g_m \hat{R}_L) \cdot j\omega C_\mu}{1 + j\omega C_\mu \cdot \hat{R}_L}$$

Observe increase of i_μ with transconductance g_m.

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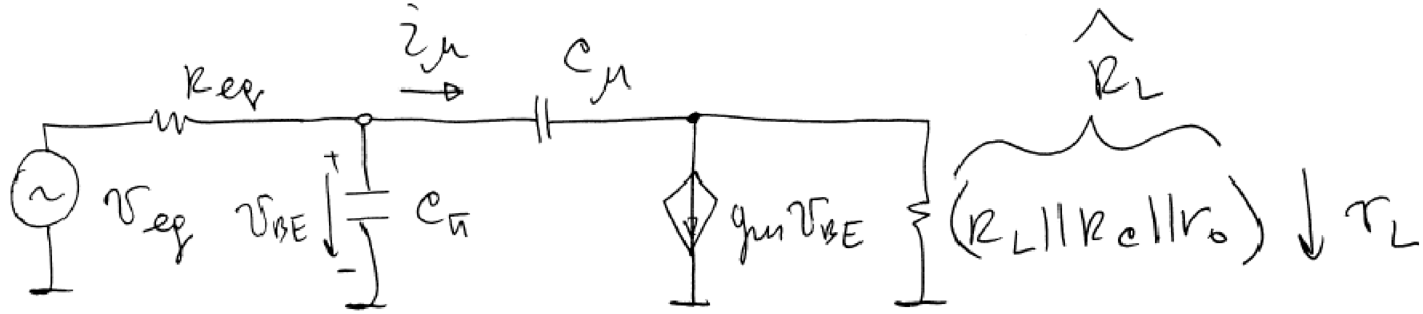
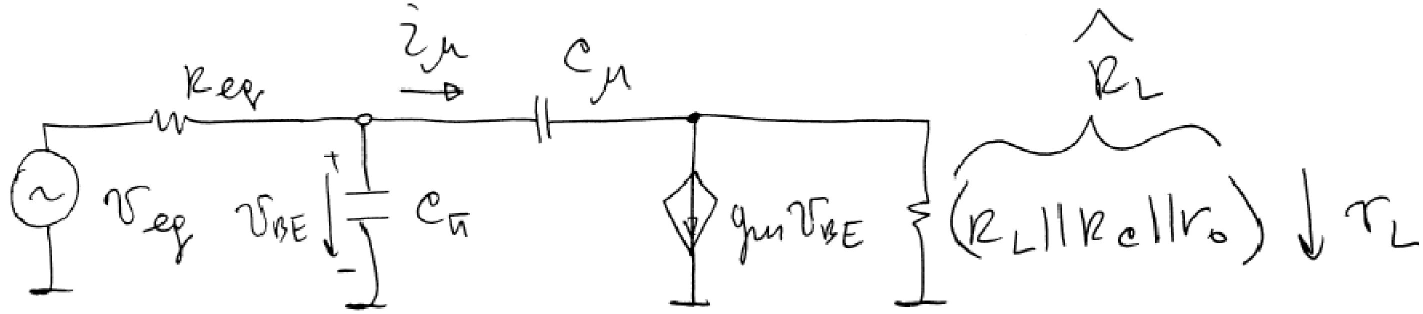


$$v_L = v_{BE} - i_\mu \frac{1}{j\omega C_\mu} = v_{BE} \frac{j\omega C_\mu \hat{R}_L - g_m \hat{R}_L}{1 + j\omega C_\mu \hat{R}_L}$$

$$v_{eq} = v_{BE} + R_{eq}\left(i_\mu + j\omega C_\pi v_{BE}\right)$$

$$\frac{v_L}{v_{eq}} = \frac{j\omega C_\mu \hat{R}_L - g_m \hat{R}_L}{(1 + j\omega C_\pi R_{eq})(1 + j\omega C_\mu \hat{R}_L) + (1 + g_m \hat{R}_L) j\omega C_\mu R_{eq}}$$

Net voltage gain bandwidth of CE BJT amplifier



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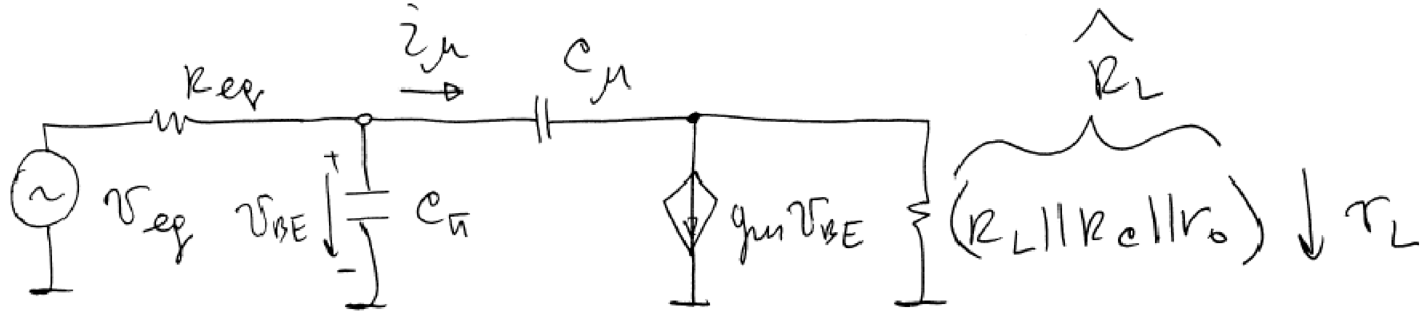

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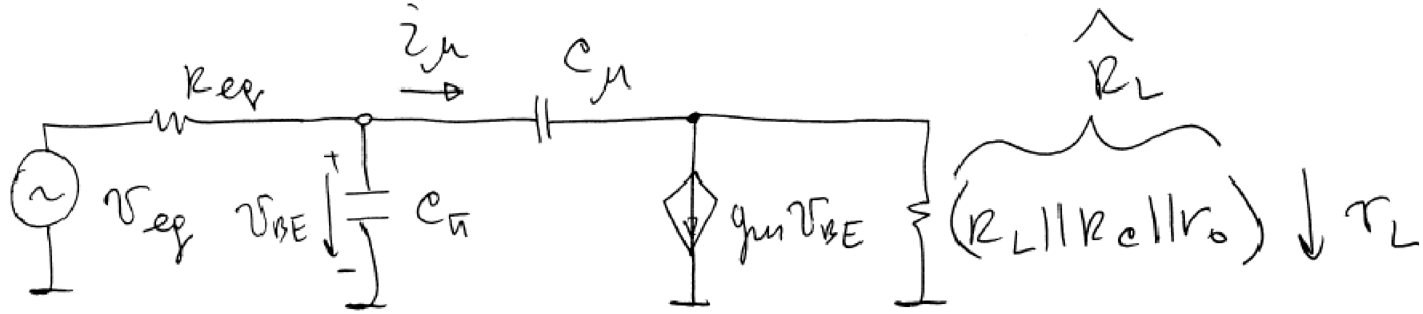
nominator

$$f_T \approx \frac{1}{2\cdot\pi\cdot\tau_{TF}} \approx 1GHz, \quad C_\mu \approx 0.1\,pF$$

$$\omega\cdot C_\mu \le 0.001\,\Omega^{-1}$$

$$g_m \approx 0.04$$

Net voltage gain bandwidth of CE BJT amplifier



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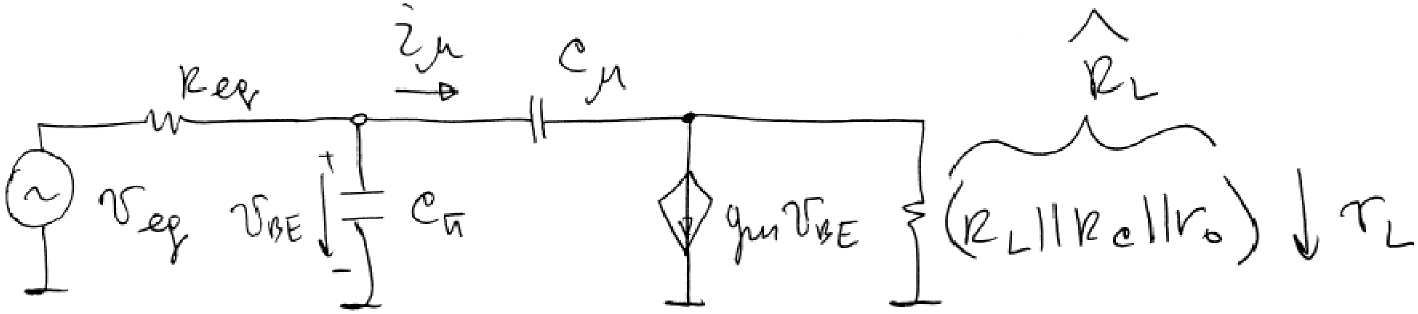

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$$f_T \approx \frac{1}{2 \cdot \pi \cdot \tau_{TF}} \approx 1GHz, \quad C_\mu \approx 0.1pF$$

$$\omega \cdot C_\mu << 0.001\,\Omega^{-1}$$

$$g_m \approx 0.04\,\Omega^{-1}$$

denominator

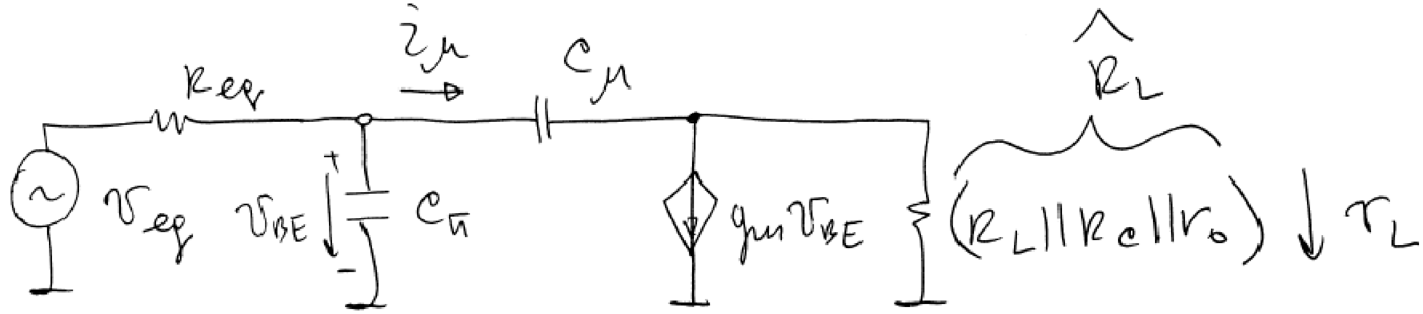
for

$$C_\mu \approx C_\pi / 10$$

and $\quad f << f_T$

$$\omega \cdot C_\mu \cdot \hat{R}_L << 1$$

Net voltage gain bandwidth of CE BJT amplifier



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nominator

$$f_T \approx \frac{1}{2 \cdot \pi \cdot \tau_{TF}} \approx 1GHz, \quad C_\mu \approx 0.1pF$$

$$\omega \cdot C_\mu << 0.001\,\Omega^{-1}$$

$$g_m \approx 0.04\,\Omega^{-1}$$

denominator

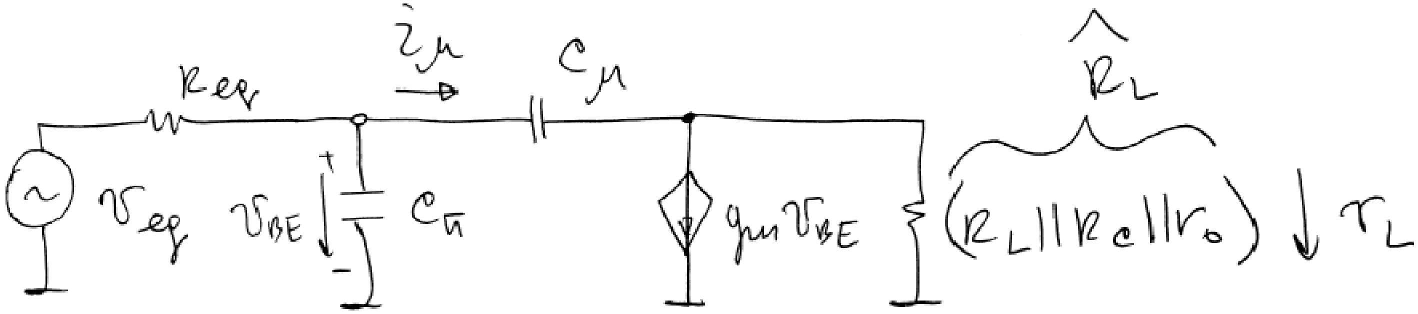
for

$$C_\mu \approx C_\pi / 10$$

and $f << f_T$

$$\omega \cdot C_\mu \cdot \hat{R}_L << 1$$

Net voltage gain bandwidth of CE BJT amplifier



$$\frac{v_L}{v_{eq}} \simeq \frac{-g_m \hat{R}_L}{1 + j\omega R_{eq}\left(C_\pi + (1+g_m \hat{R}_L)\cdot C_\mu\right)}$$

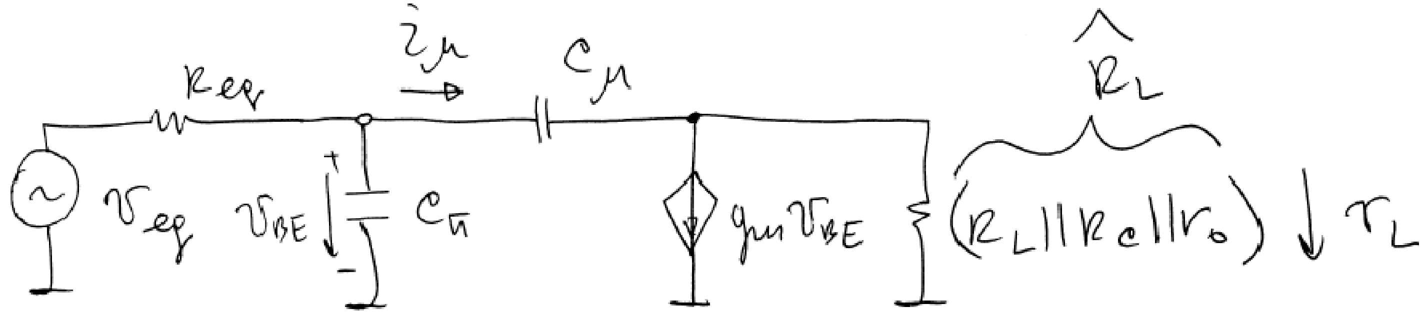
$$\frac{v_L(f)}{v_s(f)} = G_V(f) \propto \frac{-g_m (R_L \| R_C \| r_o)}{1 + j\, f/f_H}$$

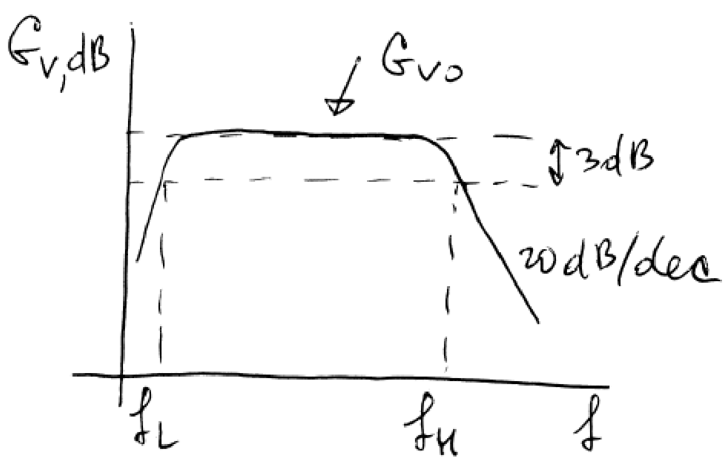
$$f_H = \frac{1}{2\pi \underset{\text{Resistor in series with input cap}}{R_{eq}} \underbrace{\left(C_\pi + (1+g_m \hat{R}_L) C_\mu\right)}_{\text{equivalent input cap}}}$$

Resistor in series with input cap

equivalent input cap

Net voltage gain bandwidth of CE BJT amplifier



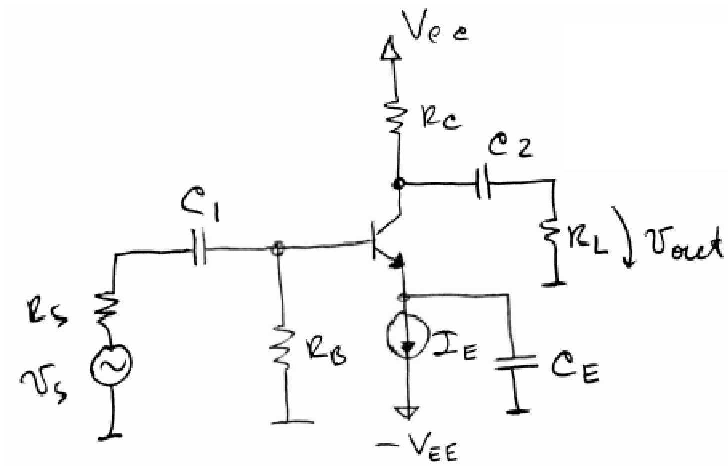


$$f_H = \frac{1}{2\pi R_{eq}\left(C_\pi + \underbrace{C_\mu\left(1 + \underbrace{g_m(R_L \| R_c \| r_o)}_{A_v}\right)}_{C_\mu(1+A_v) = C_M}\right)}$$

"Miller" capacitor

$$G_{vo} = \frac{R_{in}}{R_{in} + R_S} \cdot A_{vo} \cdot \frac{R_L}{R_{out} + R_L}$$

$$A_{vo} = -g_m(R_c \| r_o)$$



Example

$V_{CC} = V_{EE} = 10V$; $I_E = 1mA$; $R_B = 100k\Omega$

$\beta_o = 100$; $R_C = 8k\Omega$

$V_A = 100V$; $C_\mu = 1pF$; $f_T = 800MHz$; $R_S = R_L = 5k\Omega$

** confirm FA regime first …*

(1)

$$g_m = \frac{1mA}{25mV} = 40\frac{mA}{V}$$

$$r_\pi = \frac{100}{g_m} = 2.5k\Omega$$

$$r_o = \frac{100V}{1mA} = 100k\Omega$$

(2)

$$C_\pi + C_\mu = \frac{g_m}{\omega_T} = \frac{40\cdot 10^{-3}}{2\pi\cdot 800\cdot 10^6} = 8pF$$

$C_\mu = 1pF$ hence $C_\pi = 7pF$

(*) $R_L \| R_C \| r_o \simeq 3k\Omega$

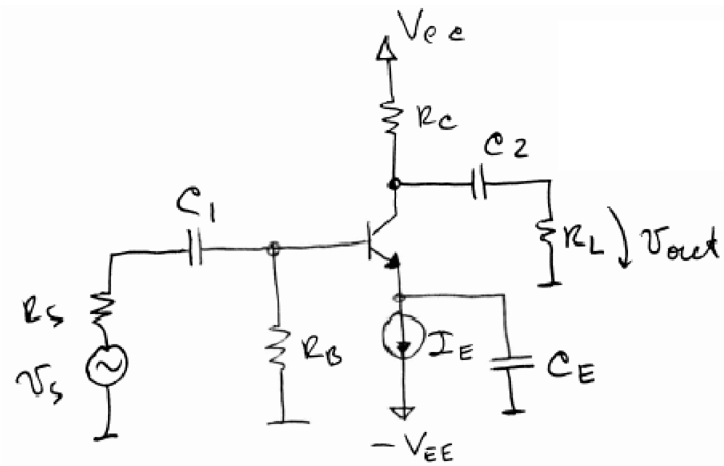
(3)

$$G_V = \frac{R_B \| r_\pi}{R_B \| r_\pi + R_S}\cdot\left(-g_m (R_L \| R_C \| r_o)\right) = -\frac{40\cdot 10^{-3}\cdot 3\cdot 10^3}{(100k \| 2.5k) + 5k}\cdot(100k \| 2.5k)$$

$$G_V \simeq -121\frac{V}{V}\cdot\frac{1}{3} \simeq -40\frac{V}{V}$$

(*) $g_m\cdot\hat{R}_L = 121\frac{V}{V}$

Example – cont.



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$V_A = 100V$; $C_\mu = 1pF$; $f_T = 800MHz$; $R_S = R_L = 5k\Omega$

$$G_V \simeq -121\frac{V}{V} \cdot \frac{1}{3} \simeq -40\frac{V}{V}$$

(4)

$$R_{eq} = (r_\pi \| R_B) \| R_S = 1{,}7k\Omega$$

$$C_{in} = 7pF + 121 \cdot 1pF = 128pF \gg C_\pi$$

$$\boxed{f_H = \frac{1}{2\pi \cdot C_{in} \cdot R_{eq}} \simeq 750kHz \ll f_T}$$

Even << f_β = 8MHz

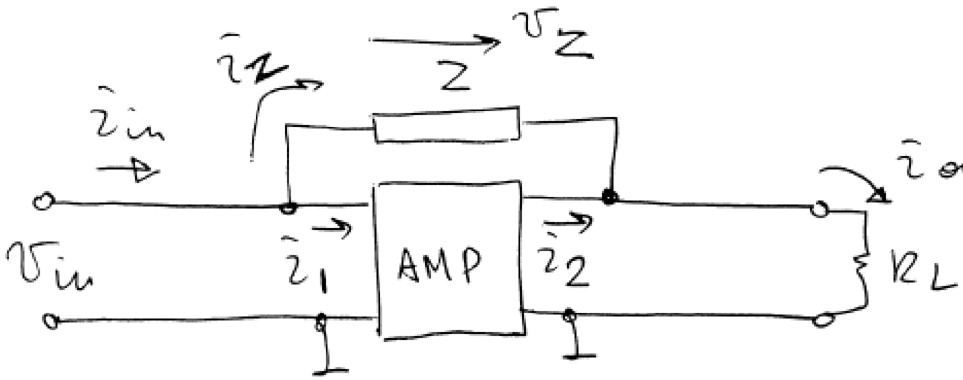
$\text{Gain} \cdot BW \simeq 3 \cdot 10^7$

Reduced gain – improved BW

$R_L = 2k\Omega$; $g_m \hat{R}_L \simeq 64\frac{V}{V}$

$f_H \simeq 1{,}4\ MHz$

Miller Effect



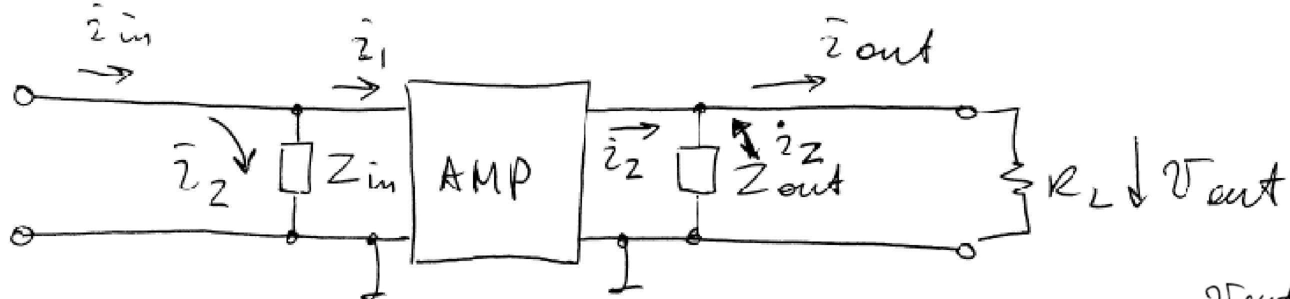
$$i_{out} = \frac{A_V \cdot v_{in}}{R_L}$$

$$v_{out} = A_V \cdot v_{in}$$

$$v_Z = v_{in} - A_V \cdot v_{in} = v_{in}(1 - A_V)$$

$$i_Z = \frac{v_Z}{Z} = \frac{v_{in}}{Z}(1 - A_V) = v_{in} \Big/ \left(\frac{Z}{1 - A_V}\right)$$

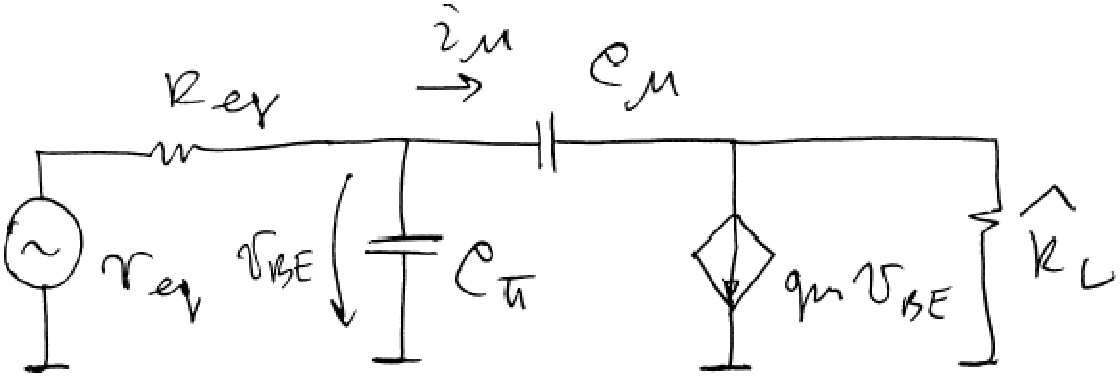
Miller transformation



$$\frac{v_{out}}{Z_{out}} = -\frac{v_{in}(1 - A_V)}{Z}$$

$$Z_{in} = \frac{Z}{1 - A_V} \qquad Z_{out} = \frac{Z \cdot A_V}{A_V - 1}$$

Miller Effect in CE amplifier



$$Z = \frac{1}{j\omega C_\mu}$$

If i_μ is not very high (small C_μ), then

$$v_{out} \simeq \overbrace{\left(-g_m \hat{R}_L\right)}^{A_v} \cdot v_{BE}$$

$$Z_{in} = \frac{1}{j\omega C_\mu (1 - A_v)}$$

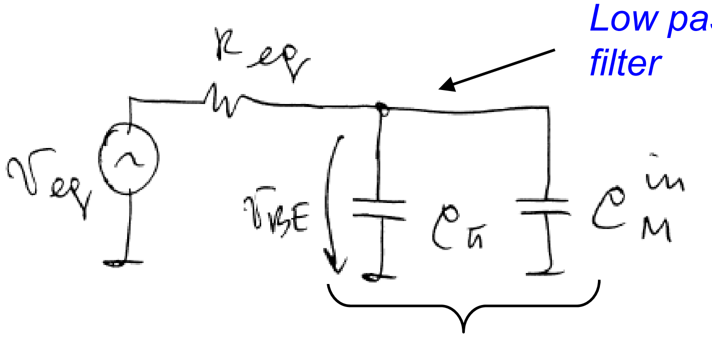
$$Z_{out} = \frac{1}{j\omega C_\mu} \cdot \frac{A_v}{A_v - 1}$$

$\longrightarrow$

$$C_M^{in} = C_\mu \left(1 + g_m \hat{R}_L\right)$$

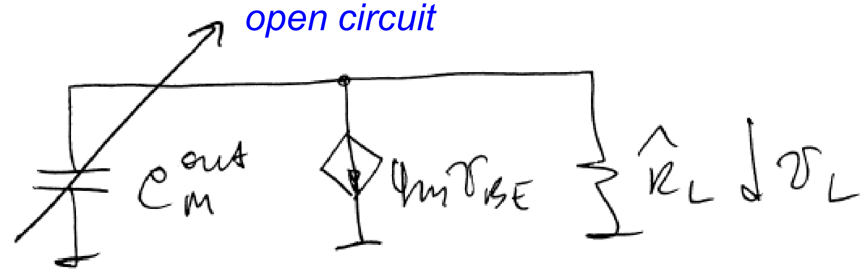
$$C_M^{out} = C_\mu$$

Low pass filter

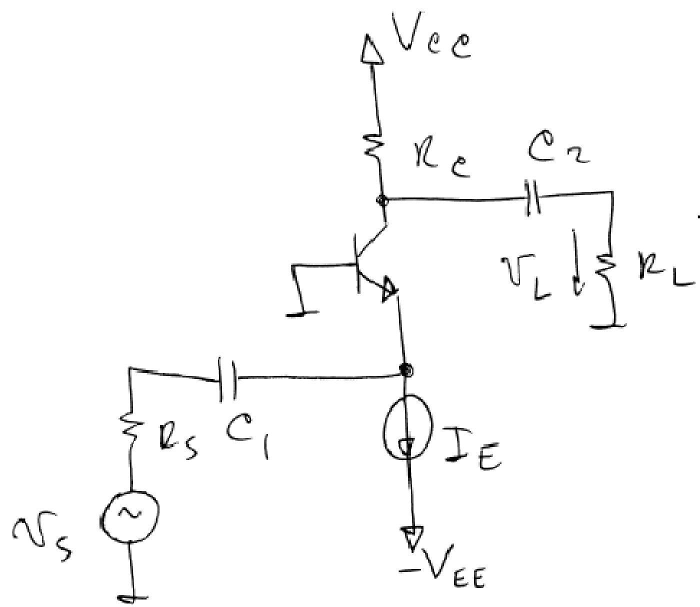


Total equivalent input cap

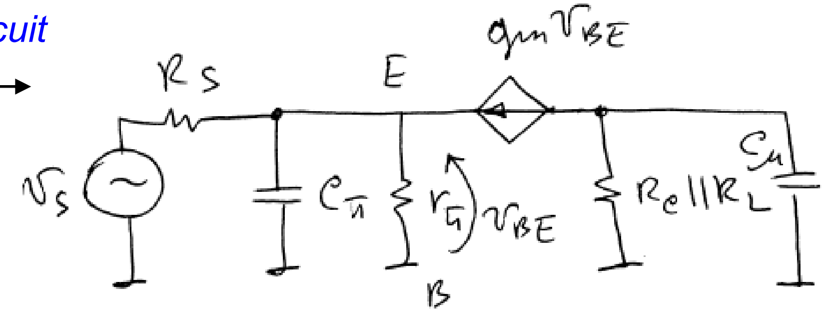
Negligible, hence open circuit



CB amplifier does not suffer from Miller effect



Neglect r_O and build equivalent circuit



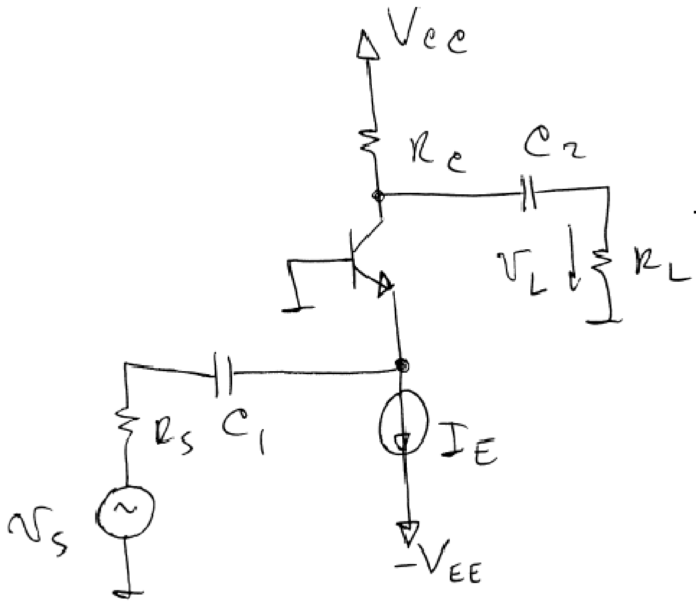
Observe no capacitor coupling output and input – No Miller effect.

$$v_L = (-g_m v_{BE})(R_C \| R_L \| C_\mu)$$

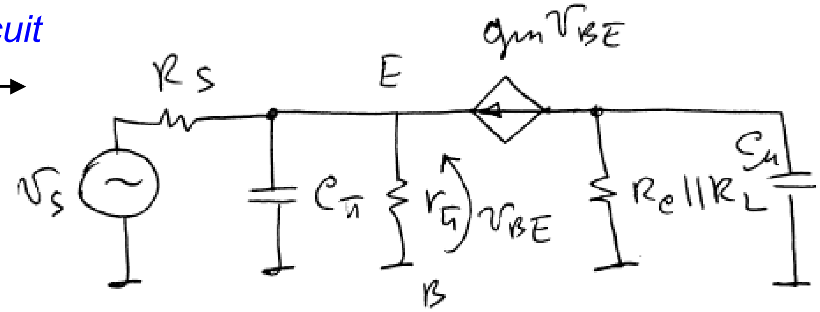
$$-v_{BE} = v_s + R_s \left(g_m v_{BE} + \frac{v_{BE}}{(r_\pi \| C_\pi)} \right)$$

$$\frac{v_L}{v_s} = \frac{g_m (R_C \| R_L \| C_\mu)}{1 + R_s \left(g_m + \dfrac{r_\pi + \frac{1}{j\omega C_\pi}}{r_\pi \cdot \frac{1}{j\omega C_\pi}} \right)}$$

CB amplifier does not suffer from Miller effect



Neglect r_O and build equivalent circuit



Observe no capacitor coupling output and input – No Miller effect.

$$\frac{v_L(f)}{v_s(f)} = \frac{g_m(R_c \| R_L)}{1 + R_s\left(g_m + \frac{1}{r_\pi}\right)} \cdot \frac{1}{1 + j\omega \underbrace{(R_c \| R_L) \cdot C_\mu}_{\tau_1}} \cdot \frac{1}{1 + j\omega \underbrace{C_\pi \frac{R_s}{1 + R_s\left(g_m + \frac{1}{r_\pi}\right)}}_{\tau_2}}$$

$$f_1 = \frac{1}{2\pi (R_c \| R_L) C_\mu} = \frac{1}{6.28 \cdot 3 \cdot 10^3 \cdot 10^{-12}} = 52 \text{ MHz}$$

$$f_2 = \frac{1}{2\pi C_\pi \left(R_s \| r_\pi \| \frac{1}{g_m}\right)} \simeq \frac{g_m}{2\pi C_\pi} = \frac{40 \cdot 10^{-3}}{6.28 \cdot 10^{-12} \cdot 7} \simeq 910 \text{ MHz}$$