

## Fall 1997 ESE 305 Final Exam Solution

- ①
- (a) linear, time-invariant, causal, unstable, ~~invertible~~ <sup>not memoryless</sup>
  - (b) non-linear, time-varying, non-causal, stable, not memoryless
  - (c) non-linear, time-invariant, causal, stable, not memoryless
  - (d) linear, time-invariant, causal, stable, not memoryless

- ②
- (a) It is causal because  $h[n] = 0$  for  $n < 0$
  - (b) It is stable because  $\sum_{n=-\infty}^{\infty} |h[n]| = \frac{1}{1-0.9} = 10 < \infty$   
or because ROC of  $H(z)$  contains unit circle

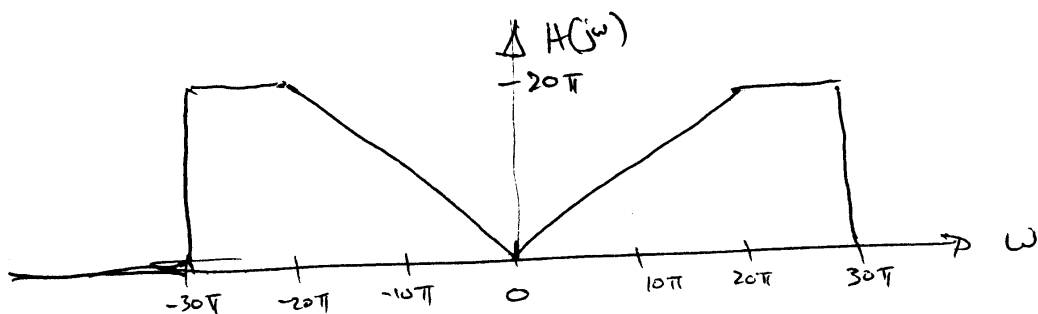
(c)  $H(z) = \frac{1}{1-0.9z^{-1}}$   $X(z) = \frac{1}{1-0.8z^{-1}}$

$$Y(z) = H(z) \cdot X(z) = \frac{1}{(1-0.9z^{-1})(1-0.8z^{-1})} = \frac{9}{1-0.9z^{-1}} + \frac{-8}{1-0.8z^{-1}}$$

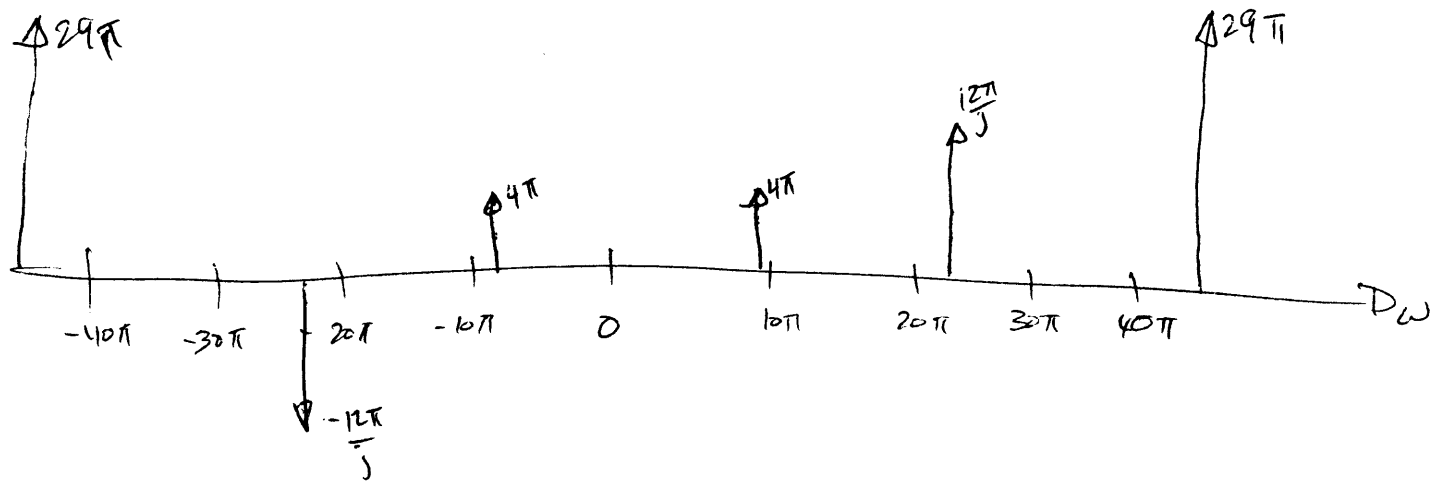
ROC =  $|z| > 0.9$  (1)

$\Rightarrow y[n] = 9(0.9)^n u[n] - 8(0.8)^n u[n]$  (1)

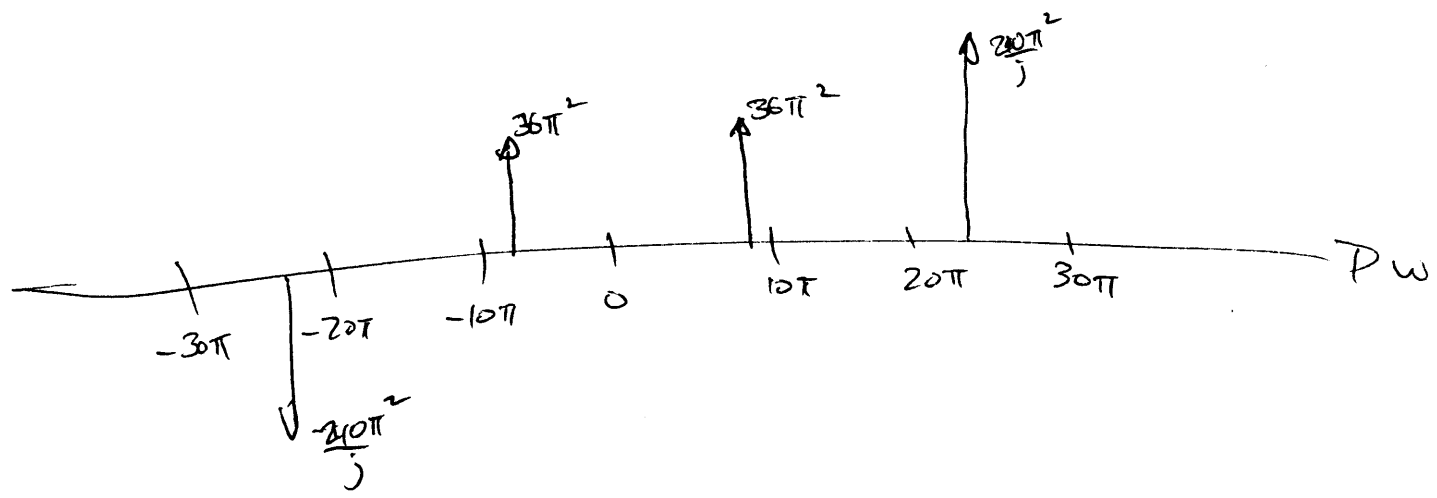
③ (a)



$$(b) \quad X(j\omega) = 4\pi [\delta(\omega - 9\pi) + \delta(\omega + 9\pi)] + \frac{12\pi}{j} [\delta(\omega - 23\pi) - \delta(\omega + 23\pi)] + 29\pi [\delta(\omega - 44\pi) + \delta(\omega + 44\pi)]$$



$$(c) \quad Y(j\omega) = 36\pi^2 [\delta(\omega - 9\pi) + \delta(\omega + 9\pi)] + \frac{240\pi^2}{j} [\delta(\omega - 23\pi) - \delta(\omega + 23\pi)]$$



$$(d) \quad y(t) = 36\pi \cos(9\pi t) + 240\pi \sin(23\pi t)$$

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III

(a) <sup>(3pts)</sup>  $X(z) = \frac{1}{1-0.5z^{-1}} + \frac{-1}{1+3z^{-1}}$       ROC:  $0.5 < |z| < 3$

(b) <sup>(3pts)</sup>  $H(z) = \frac{1}{1-0.7z^{-1}}$       ROC:  $|z| > 0.7$

(c) <sup>(4pts)</sup>  $Y(z) = H(z)X(z) = \frac{1}{(1-0.5z^{-1})(1-0.7z^{-1})} + \frac{-1}{(1+3z^{-1})(1-0.7z^{-1})}$

ROC:  $0.7 < |z| < 3$

(d) <sup>(5pts)</sup>  $Y(z) = \frac{A}{1-0.5z^{-1}} + \frac{B}{1-0.7z^{-1}} + \frac{C}{1+3z^{-1}} + \frac{D}{1-0.7z^{-1}}$

$$= \frac{-5/2}{1-0.5z^{-1}} + \frac{7/2}{1-0.7z^{-1}} + \frac{-30/37}{1+3z^{-1}} + \frac{-7/37}{1-0.7z^{-1}}$$

$$= \frac{-5/2}{1-0.5z^{-1}} + \frac{-30/37}{1+3z^{-1}} + \frac{245/74}{1-0.7z^{-1}}$$

$$y[n] = \underbrace{\left(-\frac{5}{2}\right)}_{-2.5} (0.5)^n u[n] + \underbrace{\left(\frac{245}{74}\right)}_{3.31} (0.7)^n u[n] + \underbrace{\left(\frac{30}{37}\right)}_{0.81} (-3)^n u[n-1]$$

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IV

(a) False. Because  $x(t) \rightarrow 0$  as  $t \rightarrow \infty$

(b) True. Because  $x[-n] = \frac{\sin(-n)}{-n} = -\frac{\sin(n)}{-n} = x[n]$

(c) True. Shown in class

(d) True

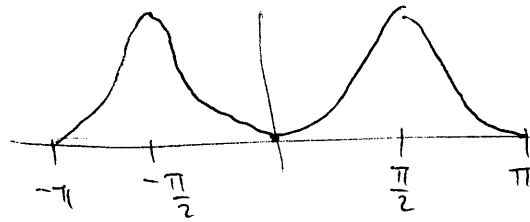
(e) True

(f) False. Nyquist rate = 2000 samples/sec

(g) False. It is used in A-D-C

(h) False. ROC must contain  $j\omega$ -axis

(i) True



(j) True.  $H(z) = \frac{\gamma(z)}{1+z^{-1}}$