

ESE305 Homework #4
(DUE 10/14/99)

4.2(a)

(i) $w_0 = 1$

$C_0 = 3, C_1 = 2.5, C_2 = 3\angle -45^\circ, C_k = 0, k \geq 3.$

(ii) $w_0 = 0.5$

$C_0 = -10, C_2 = 1.5, C_9 = 3.5\angle -90^\circ, C_k = 0, \text{ all other } k > 0$

(iii) $w_0 = 10^6$

$C_0 = 5, C_1 = 5\angle 180^\circ, C_{10} = 2, C_{11} = 1\angle -90^\circ, C_k = 0, \text{ all other } k > 0$

4.8(a)

$$C_k = \frac{1}{T_0} \int_0^1 -3e^{-jk\omega_0 t} dt + \frac{1}{T_0} \int_{-1}^0 3e^{-jk\omega_0 t} dt, \quad \omega_0 = \frac{\pi}{2}, T_0 = 4$$

$$= \frac{j3}{k\pi} \left[1 - \cos\left(\frac{k\pi}{2}\right) \right]$$

$$C_0 = \lim_{k \rightarrow 0} C_k = \frac{j3 \left(-\frac{\pi}{2} \sin\left(\frac{k\pi}{2}\right) \right)}{\pi} \Bigg|_{k=0} = 0$$

4.8(c)

$T_0 = 2$

$$C_k = \frac{1}{2} \int_0^1 2te^{-jk\pi t} dt = \frac{-1}{k^2 \pi^2} \left[e^{-jk\pi} (-jk\pi - 1) + 1 \right]$$

$$C_0 = \lim_{k \rightarrow 0} C_k = \frac{1}{2}$$

4.8(d)

$T_0 = 2$

$$C_k = \frac{1}{2} \int_0^1 2(1-t)e^{-jk\pi t} dt = \frac{e^{-jk\pi} - 1}{-jk\pi} + \frac{1}{k^2 \pi^2} \left[e^{-jk\pi} (-jk\pi - 1) + 1 \right]$$

$$C_0 = \lim_{k \rightarrow 0} C_k = \frac{1}{2}$$

4.15

From Problem 4.8, $x_c(t) \Rightarrow C_{kc} = \frac{-1}{k^2\pi^2} [e^{-jk\pi}(-jk\pi - 1) + 1]$

$$x_d(t) \Rightarrow C_{kd} = \frac{e^{-jk\pi} - 1}{-jk\pi} + \frac{1}{k^2\pi^2} [e^{-jk\pi}(-jk\pi - 1) + 1]$$

$$C_{kc} + C_{kd} = \frac{e^{-jk\pi} - 1}{-jk\pi} = \frac{(-1)^k - 1}{-jk\pi} = \begin{cases} 0, & k \text{ even}, k \neq 0 \\ \frac{2}{k\pi} \angle -90^\circ, & k \text{ odd} \end{cases}$$

$$C_0 = 1$$

4.17(a) $x_a(0) = 0, \quad x_a(1) = -1.5, \quad x_a(3) = 1.5$

4.17(c) $x_c(1) = 1$

4.17(d) $x_d(0) = 1$

4.25

$$H(s) = \frac{10}{s+5}, \quad w_0 = \frac{2\pi}{3}, \quad T_0 = 3$$

$$H(0) = 2,$$

$$H(jw_0) = \frac{10}{5 + j\frac{2\pi}{3}} = 1.84 \angle -22.7^\circ$$

$$H(j2w_0) = \frac{10}{5 + j\frac{4\pi}{3}} = 1.533 \angle -40^\circ$$

$$H(j3w_0) = \frac{10}{5 + j2\pi} = 1.245 \angle -51.5^\circ$$

$$C_{yk} = H(jkw_0)C_{xk}$$

(a) $x(t): C_{x0} = 0, C_{xk} = \frac{40}{k\pi} \angle -90^\circ, \quad k \text{ odd}$

$$C_{y0} = 0, \quad C_{y1} = 23.4 \angle -112.7^\circ$$

$$C_{y2} = 0, \quad C_{y3} = 5.28 \angle -141.5^\circ$$

$$y(t) = 46.8 \cos\left(\frac{2\pi t}{3} - 112.7^\circ\right) + 10.56 \cos(2\pi t - 141.5^\circ) + \dots$$

(c) $x(t): C_{x0} = 10, C_{xk} = j\frac{10}{k\pi}$

$$C_{y0} = 20, \quad C_{y1} = 5.86 \angle 67.3^\circ$$

$$C_{y2} = 2.44 \angle 50^\circ, \quad C_{y3} = 1.32 \angle 38.5^\circ$$

$$y(t) = 20 + 11.72 \cos\left(\frac{2\pi t}{3} + 67.3^\circ\right) + 4.88 \cos\left(\frac{4\pi t}{3} + 50^\circ\right) + 2.64 \cos(2\pi t + 38.5^\circ) + \dots$$