ESE305 Homework #5

(DUE 10/28/99)

5.1

(a) $x(t) = 2[u(t) - u(t-4)]$

$$X(w) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt = 2\int_0^4 e^{-jwt} dt = \frac{2(1-e^{-j4w})}{jw} = \frac{4e^{-j2w}}{w}\sin(2w) = 8e^{-j2w}\sin c(2w)$$

(b) $x(t) = e^{-3t}[u(t) - u(t-4)]$

$$X(w) = \int_0^4 e^{-(3+jw)t} dt = \frac{1-e^{-12}e^{-j4w}}{3+jw} = \frac{1-6.144\times 10^{-12} e^{-j4w}}{3+jw}$$

(c) $x(t) = 2t(u(t) - u(t-4)]$

$$X(w) = 2\int_0^4 te^{-jwt} dt$$

$$\int xe^{ax} dx = \frac{e^{ax}}{a^2}(ax-1)$$

$$X(w) = \frac{2(1+j4w)e^{-j4w} - 2}{w^2} = \frac{(2-j4\sin 4w)e^{-j4w}}{w^2}$$

(d) $x(t) = \cos(4\pi t)[u(t+2) - u(t-2)]$

$$X(w) = \int_{-2}^{2} \tfrac{1}{2}(e^{j4\pi t} + e^{-j4\pi t})e^{-jwt} dt = 2[\sin c(2w-8\pi) + \sin c(2w+8\pi)] = \frac{2w\sin(2w)}{w^2 - 16\pi^2}$$

$$= \frac{(2w)^2 \sin c(2w)}{w^2 - 16\pi^2}$$

5.3

(a) $x(t) = 2[u(t) - u(t-4)] = 2rect(\tfrac{t-2}{4})$

From Table 5.2: $rect(\tfrac{t}{\tau}) \Leftrightarrow \tau \sin c(\tfrac{w\tau}{2})$

From Tale 5.1: $f(t-\tau) \Leftrightarrow F(w)e^{-jw\tau}$

$\therefore \quad X(w) = 8\sin c(2w)e^{-j2w}$

(b) $x(t) = e^{-3t}[u(t) - u(t-4)] = e^{-3tu(t)} - e^{-12}e^{-3(t-4)}u(t-4)$

From Table .2: $e^{-at}u(t) \Leftrightarrow \dfrac{1}{a+jw}$

From Tale 5.1: $f(t-\tau) \Leftrightarrow F(w)e^{-jw\tau}$

$$\therefore X(w) = \frac{1}{3+jw} - \frac{e^{-12}e^{-j4w}}{3+jw} = \frac{1-e^{-4(3+jw)}}{3+jw}$$

(c) $x(t) = 2t[tu(t) - u(t-4)] = t(2rect(\frac{t-2}{4}))$

From Table 5.2: $-jtf(t) \Leftrightarrow \frac{dF(w)}{dw} \qquad \Rightarrow tf(t) \Leftrightarrow j\frac{dF(w)}{dw}$

Let $f(t) = 2rect(\frac{t-2}{4})$, then $F(w) = 8\sin c(2w)e^{-j2w}$

$$\therefore X(w) = j\frac{dF(w)}{dw} = \frac{2(1+j4w)e^{-j4w} - 2}{w^2}$$

(d) $x(t) = \cos(4\pi t)[u(t+2) - u(t-2)] = \cos(4\pi t)rect(t/4)$

From Table 5.1: $f_1(t)f_2(t) \Leftrightarrow \frac{1}{2\pi}F_1(w) * F_2(w)$

From Table 5.2: $\cos(w_0 t) \Leftrightarrow \pi[\delta(w - w_0) + \delta(w + w_0)]$

$rect(t/\tau) \Leftrightarrow \tau \sin c(w\tau/2)$

$$\therefore X(w) = \frac{1}{2\pi}\pi[\delta(w-4\pi) + \delta(w+4\pi)] * 4\sin c(2w)$$
$$= 2[\sin c(2w - 8\pi) + \sin c(2w + 8\pi)]$$

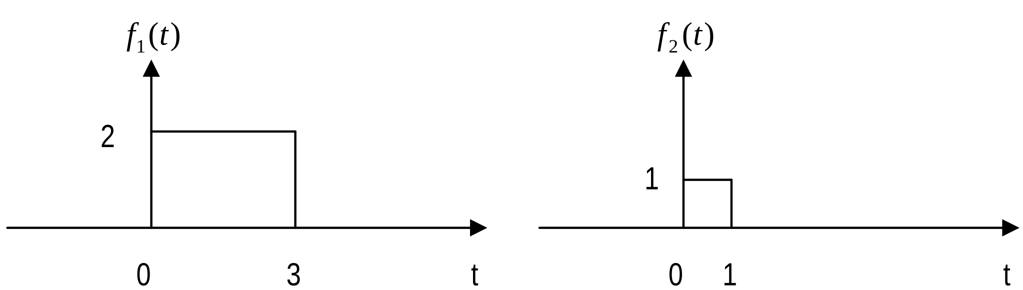
5.8

$$g(t) = 162.6\cos(377t - 0.5) = 162.6\cos[377(t - \tfrac{0.5}{377})]$$
$$G(w) = 162.6\pi[\delta(w-377) + \delta(w+377)]e^{-j(0.5/377)w}$$
$$= 162.6\pi[\delta(w-377)e^{-j0.5} + \delta(w+377)e^{j0.5}]$$

5.11

$f(t)$ can be recognized to be the result of convolving two rectangular pulses:

$f(t) = f_1(t) * f_2(t)$



$$f_1(t) = 2rect(\tfrac{t-1.5}{3}) \Leftrightarrow F_1(w) = 6\sin c(3w/2)e^{-j1.5w}$$
$$f_2(t) = rect(t - 0.5) \Leftrightarrow F_2(w) = \sin c(w/2)e^{-j0.5w}$$
$$\therefore F(w) = 6\sin c(3w/2)\sin c(w/2)e^{-j2w}$$
$$g(t) = f(10t) \Leftrightarrow G(w) = \frac{1}{10}F(w/10) = 0.6\sin c(3w/20)\sin c(w/20)e^{-j0.2w}$$

5.15

$$h(t) = \frac{\sin(2t)}{2t} = \sin c(2t) \Leftrightarrow H(w) = \tfrac{\pi}{2} rect(w/4)$$

$$x(t) = \cos(2t) + \sin(3t) \Leftrightarrow X(w) = \pi[\delta(w-2) + \delta(w+2)] + \frac{\pi}{j}[\delta(w-3) - \delta(w+3)]$$

$$y(t) = x(t) * h(t) \Leftrightarrow Y(w) = X(w)H(w)$$

$$\therefore Y(w) = \frac{\pi^2}{2} rect(w/4)[\delta(w-2) + \delta(w+2) - j\delta(w-3) + j\delta(w+3)]$$

$$= \frac{\pi^2}{2}[\delta(w-2) + \delta(w+2)]$$

$$y(t) = \frac{\pi}{2}\cos(2t)$$

5.19

(a) use the time-scaling property:

$$g(2t) \Leftrightarrow \frac{1}{2}G(w/2) = \frac{jw}{-w^2 + 10jw + 24}$$

(c) Use the time-differentiation property:

$$\frac{dg(t)}{dt} \Leftrightarrow jwG(w) = \frac{-w^2}{-w^2 + 5jw + 6}$$

(d) Use the time-scaling property:

$$g(-t) \Leftrightarrow G(-w) = \frac{jw}{w^2 + 5jw - 6}$$

5.20

(a) $f_1(t) = \sum_{n=-\infty}^{\infty} g_1(t - nT_0), \qquad T_0 = 8\times10^{-3}, \qquad w_0 = 250\pi$

$$g_1(t) = 8\cos(w_0 t) rect(t/4\times10^{-3})$$

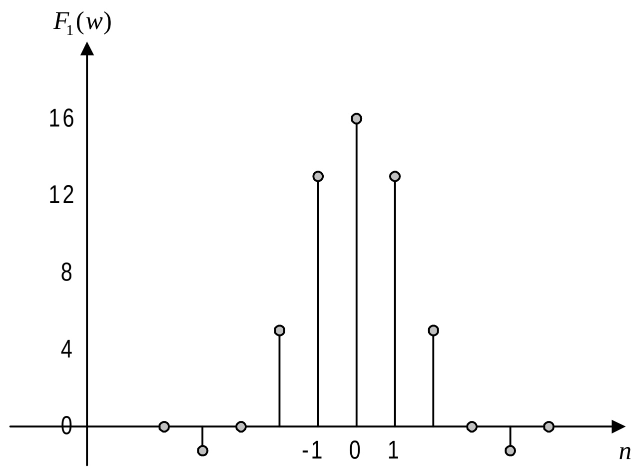
$$= 8\cos(250\pi) rect(t/4\times10^{-3})$$

$$= 4rect(t/4\times10^{-3})e^{j250\pi t} + 4rect(t/4\times10^{-3})e^{-j250\pi t}$$

$$G_1(w) = 16\times10^{-3}[\sin c(2\times10^{-3}(w-250\pi)) + \sin c(2\times10^{-3}(w+250\pi))]$$

$$F_1(w) = \sum_{n=-\infty}^{\infty} w_0 G_1(nw_0)\delta(w - nw_0)$$

$$= \sum_{n=-\infty}^{\infty} 4\pi[\sin c(\tfrac{(n-1)\pi}{2}) + \sin c(\tfrac{(n+1)\pi}{2})]\delta(w - 250\pi)$$



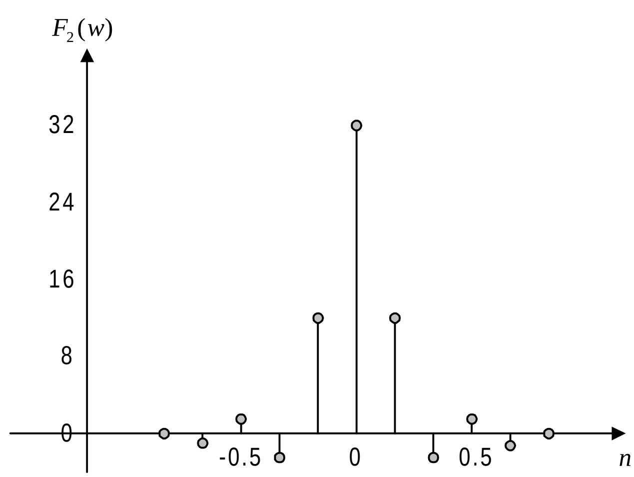
(b)

$$f_2(t) = \sum_{n=-\infty}^{\infty} g_2(t - nT_0), \qquad T_0 = 4\times 10^{-3}, \quad w_0 = 500\pi$$

$$g_2(t) = 8\cos(250\pi t)rect(t/4\times 10^{-3}) = g_1(t)$$

$$F_2(w) = \sum_{n=-\infty}^{\infty} 500\pi G_1(n500\pi)\delta(w - n500\pi)$$

$$= \sum_{n=-\infty}^{\infty} 8\pi\{\sin c[\tfrac{(2n-1)\pi}{2}] + \sin c[\tfrac{(2n+1)\pi}{2}]\}\delta(w - n500\pi)$$



(c) The zero-frequency component of $F_2(w)$, $F_2(0) = 2F_1(0)$. The impulses in $F_2(w)$ are more widely separated.