ESE305 Homework #6

(DUE 11/11/99)

12.1

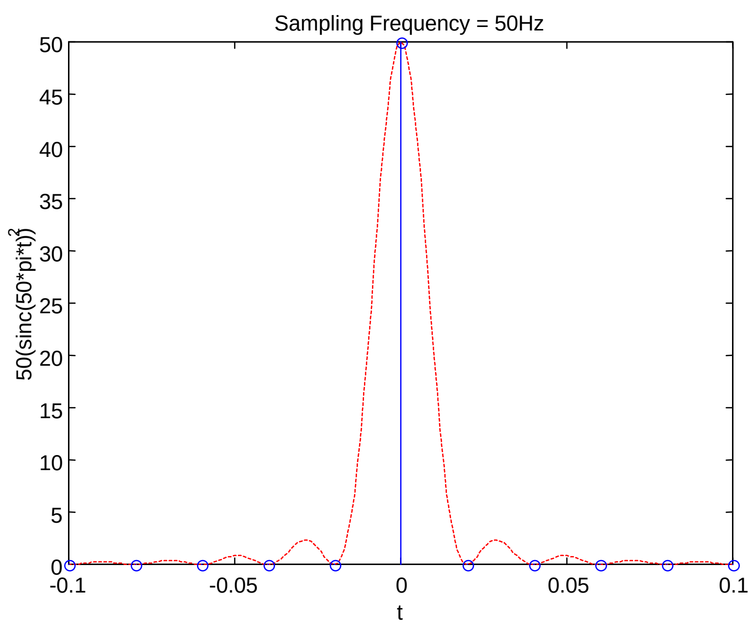
(a) $w_s \geq 400 \ \ rad/s$

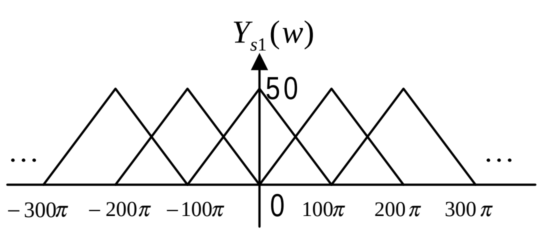
(b) $w_s \geq 200\pi \ \ rad/s$

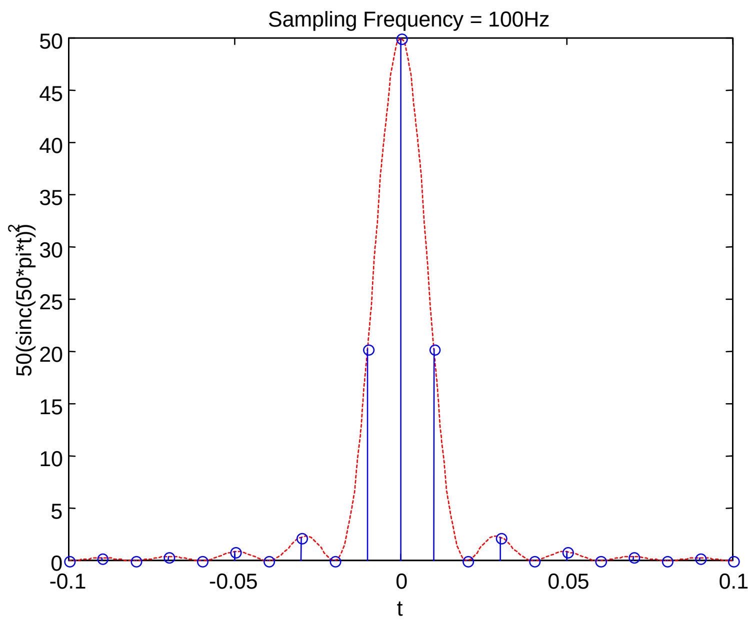
(c) $w_s \geq 400 \ \ rad/s$

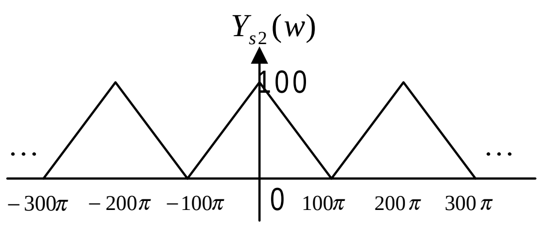
(d) $w_s \geq 200\pi \ \ rad/s$

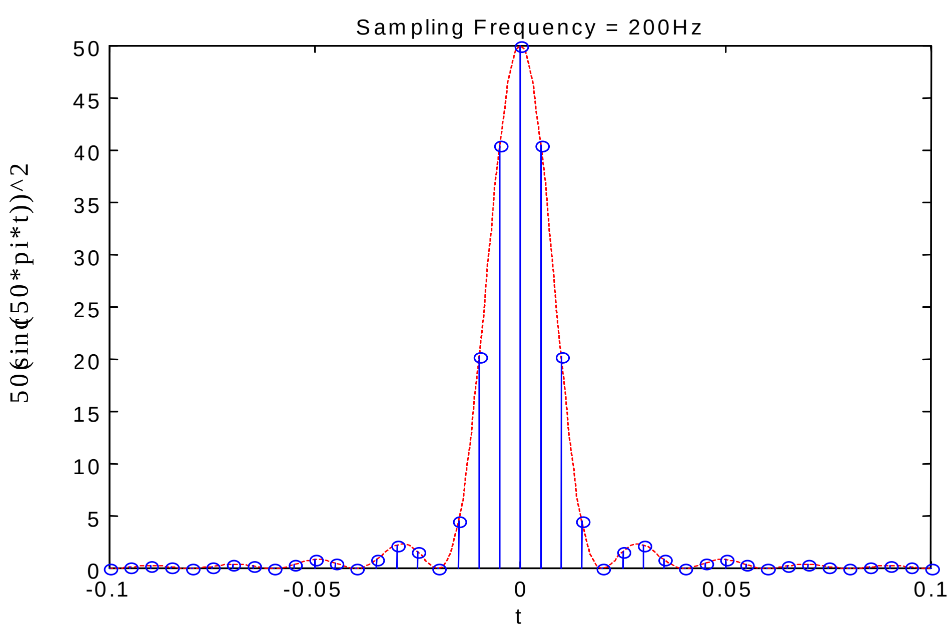
12.2 (a)

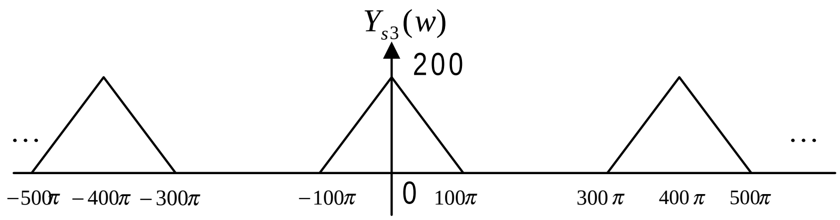





Sampling Frequency = 100Hz
50(sinc(50*pi*t))^2
t


$Y_{s2}(w)$
100
...
...
-300π
-200π
-100π
0
100π
200π
300π


Sampling Frequency = 200Hz
50(sinc(50*pi*t))^2
t

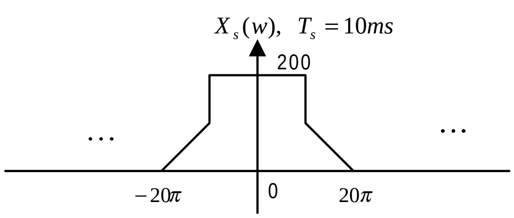


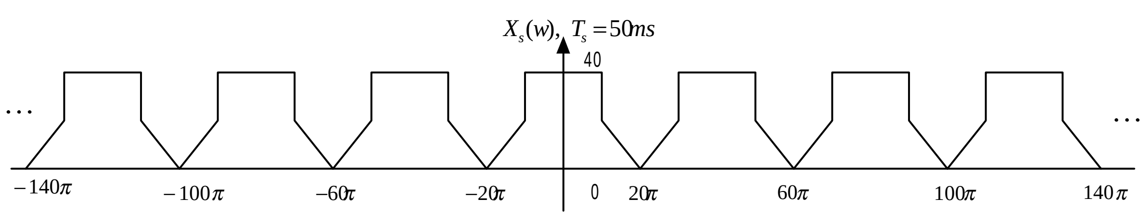
12.2(b)

$f_s = 50$ Hz is not a suitable sampling frequency for this signal. $f_s = 50$ Hz is one-half the Nyquist rate for the signal. Aliasing is seen in the frequency spectrum. $f_s = 100$ Hz is a satisfactory sampling frequency, which is the Nyquist rate for the signal.

12.3(a)

$$T_s = 10ms \Rightarrow f_s = 100Hz \Rightarrow w_s = 200\pi \quad rad/s$$
$$T_s = 50ms \Rightarrow f_s = 20Hz \Rightarrow w_s = 40\pi \quad rad/s$$





12.3(b)

Both are theoretically acceptable sampling frequencies. $T_s = 10ms$ would be preferable in most practical applications.

12.6(a)

$$x[n] = \begin{cases} 0.5^n, n \ge 0 \\ 0, \quad n < 0 \end{cases} \qquad X(\Omega) = \sum_{n=0}^{\infty} 0.5^n e^{-jn\Omega} = \frac{1}{1-0.5e^{-j\Omega}}$$

12.6(b)

$$y[n] = n(0.5)^n u[n]$$

$$X(\Omega) = \sum_{n=0}^{\infty} n0.5^n e^{-jn\Omega} = \frac{0.5e^{j\Omega}}{(e^{j\Omega} - 0.5)^2}$$

12.6(c)

$$v[n] = 2[u[n] - u[n-5]]$$

$$V(\Omega) = \sum_{n=0}^{4} 2e^{-jn\Omega} = 2e^{-j2\Omega}[e^{j2\Omega} + e^{j\Omega} + 1 + e^{-j\Omega} + e^{-j2\Omega}] = 2e^{-j2\Omega}[1 + 2\cos\Omega + 2\cos 2\Omega]$$

12.6(d)

$$w[n] = rect(n/4) + rect(n/10)$$

$$W(\Omega) = \sum_{n=-5}^{5} e^{-jn\Omega} + \sum_{n=-2}^{2} e^{-jn\Omega}$$

$$= e^{j5\Omega} + e^{j4\Omega} + e^{j3\Omega} + 2e^{j2\Omega} + 2e^{j\Omega} + 2 + 2e^{-j\Omega} + 2e^{-j2\Omega} + e^{-j3\Omega} + e^{-j4\Omega} + e^{-j5\Omega}$$

12.7

Proof:

$$x_1[n] \Rightarrow X_1(k) = \sum_{n=0}^{N-1} x_1[n] e^{-j2\pi kn/N}$$

$$x_2[n] \Rightarrow X_2(k) = \sum_{n=0}^{N-1} x_2[n] e^{-j2\pi kn/N}$$

$$x_3[n] = ax_1[n] + bx_2[n]$$

$$\Rightarrow X_3(k) = \sum_{n=0}^{N-1} x_3[n] e^{-j2\pi kn/N}$$

$$= \sum_{n=0}^{N-1} (ax_1[n] + bx_2[n]) e^{-j2\pi kn/N}$$

$$= \sum_{n=0}^{N-1} ax_1[n] e^{-j2\pi kn/N} + \sum_{n=0}^{N-1} bx_2[n] e^{-j2\pi kn/N}$$

$$= a\sum_{n=0}^{N-1} x_1[n] e^{-j2\pi kn/N} + b\sum_{n=0}^{N-1} x_2[n] e^{-j2\pi kn/N}$$

$$= aX_1(k) + bX_2(k)$$

12.8

Proof:

$$x[n] \Rightarrow X(k) = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}$$

$$x_2[n] = x[n] e^{-j2\pi k_0 n/N} \Rightarrow X_2(k) = \sum_{n=0}^{N-1} x[n] e^{-j2\pi k_0 n/N} e^{-j2\pi kn/N} = \sum_{n=0}^{N-1} x[n] e^{-j2\pi n(k+k_0)/N} = X(k+k_0)$$

12.11

$$x[n] = \begin{cases} 0.5^n, n \geq 0 \\ 0, \quad n < 0 \end{cases}$$

$$x[k] = \sum_{n=0}^{7} x[n] e^{-j2\pi kn/8} = \frac{1-(0.5e^{-jk\pi/4})^8}{1-0.5e^{-jk\pi/4}} = \frac{1-0.5^8}{1-0.5e^{-jk\pi/4}}$$

12.12

$$y[n] = n(0.5)^n u[n]$$

$$y[k] = \sum_{n=0}^{7} y[n] e^{-j2\pi kn/8}$$

$$= 0.5e^{-jk\pi/4} + 0.5e^{-jk\pi/2} + 0.375e^{-j3k\pi/4} + 0.25e^{-jk\pi} + 0.15625e^{-j5k\pi/4} + 0.09375e^{-j3k\pi/2}$$

$$+ 0.0546875e^{-j7k\pi/4}$$