# ESE305 Homework #7

**(DUE 11/30/99)**

## 11.1

(a) $z[0.8^n] = \frac{z}{z-0.8}$

(c) $z[3e^{-0.1n}] = \frac{3z}{z-e^{-0.1}}$

(e) $z[\cos 2n] = \frac{z(z-\cos 2)}{z^2 - 2z\cos 2 + 1}$

(g)

$$z[10e^{-0.2n}\cos(0.5n)] = 5\left(\sum_{n=0}^{\infty}(e^{(-0.2+j0.5)}z^{-1})^n + \sum_{n=0}^{\infty}(e^{(-0.2-j0.5)}z^{-1})^n\right)$$

$$= \frac{10z(z-e^{-0.2}\cos 0.5)}{z^2-(2e^{-0.2}\cos 0.5)z+e^{-0.4}}$$

## 11.2

**(a)** $z[e^{-0.05n}] = \frac{z}{z-e^{-0.05}}$

(c) $z[e^{-0.1(0.05n)}] = \frac{z}{z-e^{-0.005}}$

(d) $z[3e^{-j0.05n}] = \frac{3z}{z-e^{-j0.05}} = \frac{3z}{z-\cos 0.05 + j\sin 0.05}$

## 11.7

(a) $z[A\cos\Omega n] = \frac{Az(z-\cos\Omega)}{z^2-2z\cos\Omega+1} = \frac{3z(z-0.6967)}{z^2-1.3932z+1}$

$\therefore$ A = 3, $\cos\Omega = 0.6967$, $\Omega = 45.48^\circ = 0.800$ *rad*

(b) $A = 3$, $w(0.0001) = 0.8 \Rightarrow w = 8{,}000$ *rad*

## 11.12(a)

(i) $\frac{X(z)}{z} = \frac{0.4z}{(z-0.1)(z-0.6)} = \frac{1}{z-1} + \frac{-0.6}{z-0.6}$

$x[n] = 1 - 0.6^{n+1}, \quad n \ge 0$

(ii) $\frac{X(z)}{z} = \frac{0.4}{(z-0.1)(z-0.6)} = \frac{1}{z-1} + \frac{-1}{z-0.6}$

$x[n] = 1 - 0.6^n, \quad n \ge 0$

(iii) $$\frac{X(z)}{z}=\frac{0.4}{z(z-0.1)(z-0.6)}=\frac{2/3}{z}+\frac{1}{z-1}+\frac{-5/3}{z-0.6}$$

$$x[n]=\frac{2}{3}\delta[n]+1-\frac{5}{3}0.6^n, \quad n\geq 0$$

(iv) $$\frac{X(z)}{z}=\frac{1}{z^2-z+1}$$

$$z[\sin bn]=\frac{z\sin b}{z^2-2z\cos b+1}$$

$$\therefore \quad 2\cos b=1\Rightarrow b=\frac{\pi}{3}, \quad \sin b=0.866,$$

$$x[n]=\frac{1}{0.866}\sin\left(\frac{\pi n}{3}\right)$$

## 11.13(a)

$$z[x[n-1]u[n-1]]=\frac{X(z)}{z}, \quad X_1(z)=\frac{0.4z^2}{(z-1)(z-0.6)}$$

$$x_2[n]=x_1[n-1]u[n-1]$$

$$x_3[n]=x_1[n-2]u[n-2]=x_2[n-1]u[n-1]$$

## 11.16(b)

$$x[n]=u[n]\Rightarrow X(z)=\frac{z}{z-1}$$

$$Y(z)=\frac{z^2}{(z-0.5)(z-0.25)}\times\frac{z}{z-1}=z\left(\frac{8/3}{z-1}+\frac{-2}{z-0.5}+\frac{1/3}{z-0.25}\right)$$

The unit step response is:

$$y[n]=\frac{8}{3}-\frac{1}{2^{n-1}}+\frac{1}{3\cdot 4^n}, \quad n\geq 0$$

## 11.23(a)

(i) poles: $z=1,\ 0.9,$ not stable

(ii) poles: $z=0,\ 0.9,\ 1.2,$ not stable

(iii) poles: $z=0, -0.9, -1.2,$ not stable

(iv) poles: $z=0,\ 0.9, -0.9,$ stable

(v) poles: $z=1.1,\ 0.9,$ not stable