ESE 532 – Quiz # 5 Name _____

Consider a random variable X with p.d.f.

$$f_X(x) = \begin{cases} |x|/3 & \text{if} |x| < 3 \\ 0 & \text{if} |x| \geq 3 \end{cases} .$$

We wish to design a two-level scalar quantizer for X to minimize the mean squared error. Suppose we have initialized the levels at $(y_1, y_2) = (-1, +1)$. We have three thresholds: $T_0 = -3 < T_1 < T_2 = 3$. Only T_1 is unknown.

1. **(1 pt)** Write an expression for the mean squared error as a function of the quantization threshold T_1.
2. **(3 pt)** Using basic differential calculus, show that the optimum value of T_1 given that $(y_1, y_2) = (-1, +1)$ is $T_1 = 0$.
3. **(4 pt)** Given that $T_1 = 0$, determine the optimum values of (y_1, y_2) by calculating the conditional expected values of X: $y_1 = E[X|X < 0], y_2 = E[X|X \geq 0]$.
4. **(2 pt)** Calculate the mean squared error for the two-level quantizer found in part 3.