ESE 532 —*Theory of Digital Communication*

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Final Exam — N. Phamdo
Closed Books, Closed Notes, 3 hours — December 15, 1997

1. **(20 Pts Total)** Short answers. Justify your answers.

   (a) **(4 pts)** Briefly describe the Lloyd iterative algorithm. What is its shortcoming?

   (b) **(4 pts)** Draw the block diagram of a predictive scalar quantization system.

   (c) **(4 pts)** Consider an (n,k) convolutional code with N states. At each time instant in the Viterbi algorithm, how many path metrics need to be computed? How many survivors are there at each time instant?

   (d) **(4 pts)** What is a catastrophic convolutional encoder and why is it bad? Why is it incorrect to describe a *code* as catastrophic?

   (e) **(4 pts)** Under what condition is maximum a posteriori decoding equivalent to maximum likelihood decoding? For a BSC, under what condition is maximum likelihood decoding equivalent to minimum distance decoding?

2. **(20 Pts Total)** An i.i.d. ternary source, $\mathcal{X} = \{X_n\}_{n=1}^{\infty}$, has alphabet $\{a, b, c\}$. The source p.m.f. is $\Pr\{X_n = a\} = 0.9, \Pr\{X_n = b\} = 0.05, \Pr\{X_n = c\} = 0.05$. The source produce th sequence:

   aaaaaaaaaaabaaaaaaaaaaaaaaacaa

   (a) **(7 pts)** Design a first-order and a second-order Huffman code for $\mathcal{X}$. What is the average rate of each code in bits/sample?

   (b) **(6 pts)** Use the second-order Huffman code to encode the above sequence. Calculate the average number of bits used per sample.

   (c) **(7 pts)** Encode this sequence using the variable-to-variable Ziv-Lempel code. How many bits are needed to encode this sequence?

3. **(20 Pts Total)** Consider a binary Hamming code with generator matrix:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

   (a) **(3 pts)** What is the rate, R, and minimum distance, d_{min}, of this code?

   (b) **(3 pts)** A code is "cyclic" if the cyclic shift of every codeword is also a codeword. That is, if $\mathbf{c} = [c_1 c_2 c_3 c_4 c_5 c_6 c_7 c_8]$ is a codeword, then its cyclic shift $\mathbf{c}' = [c_8 c_1 c_2 c_3 c_4 c_5 c_6 c_7]$ is also a codeword. Determine whether the above code is cyclic.

(c) **(4 pts)** Suppose the received vector is $\mathbf{z} = [0001100]$. Find the most likely information vector $\mathbf{x} = [x_1 x_2 x_3 x_4]$ assuming G is the encoder.

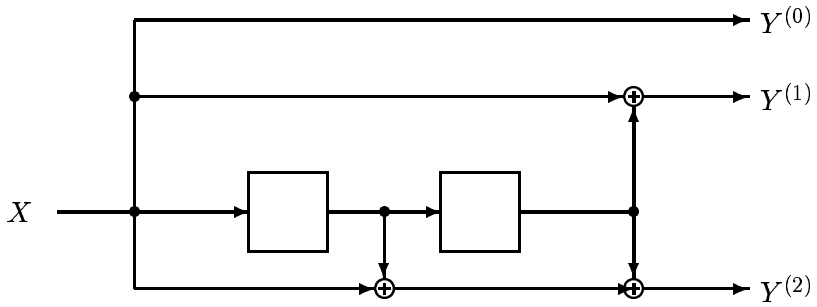
(d) **(6 pts)** The above generator matrix, G, is "equivalent" to

$$G' = \begin{bmatrix} * & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & \# & 1 \\ 1 & 1 & \star & 1 & 1 & 1 & 1 \end{bmatrix}.$$

(Two generator matrices are equivalent if they generate the same code.) Determine the values of $*, \#$ and $\star$.

(e) **(4 pts)** Repeat part (c) if instead G' is the encoder.

4. **(20 Pts Total)** Consider the (3,1) convolutional encoder shown below.



(a) **(6 pts)** Determine the generator polynomial matrix, the state transition diagram and the trellis diagram for this encoder.

(b) **(4 pts)** Which of the following terms describe this encoder: systematic, catastrophic, feedback-free, linear?

(c) **(2 pts)** What is the constraint length of this encoder? What is the rate of this encoder?

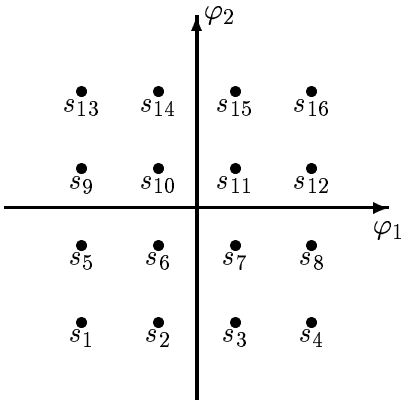
(d) **(2 pts)** What is the free distance of the code generated by this encoder?

(e) **(6 pts)** Suppose the output of this encoder is transmitted over a binary symmetric channel (crossover probability $< 1/2$) and you receive the sequence

$$\{(001), (110), (110), (100)\}.$$

Use the Viterbi algorithm to determine the most likely transmitted sequence and the most likely encoder input. Assume that the initial state of the encoder is zero.

5. **(20 Pts Total)** Consider the signal constellation



where $s_1 = (-3, -3), s_2 = (-1, -3), s_3 = (1, -3), s_4 = (3, -3)$, etc. Baseband signaling

$$\varphi_1(t) = \begin{cases} \sqrt{2/T} & 0 \le t \le T/2 \\ 0 & T/2 < t \le T \end{cases},$$

$$\varphi_2(t) = \begin{cases} 0 & 0 \le t < T/2 \\ \sqrt{2/T} & T/2 \le t \le T \end{cases}.$$

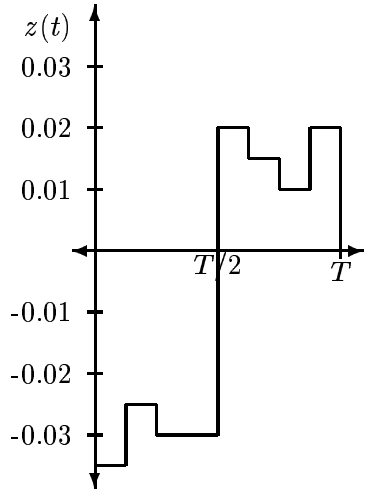
is used. The vector encoder is given by $\alpha(i) = s_i, i = 1, 2, \ldots, 16$ and $T = 1/10000$ sec.

(a) **(2 pts)** What is the data rate of this modulation scheme is bits/sec?

(b) **(2 pts)** What is the minimum channel bandwidth required for this system? Provide the correct unit.

(c) **(4 pts)** Draw the optimal decoding regions assuming equally likely messages and AWGN channel.

(d) **(6 pts)** Suppose we receive the signal $z(t)$ is as follows:



Find the most likely transmitted message. [Hint: First project $z(t)$ into the signal space and then do minimum distance decoding.]

(e) **(6 pts)** Assume an AWGN channel with *non-uniform* messages:

$$\Pr\{X = i\} = \begin{cases} 1/8 & i = 7 \\ 7/120 & i \neq 7 \end{cases} .$$

That is, all messages are equally likely except the seventh one which is more probable. Assume we know that $N_0/2 = 1$. Make a rough sketch of the optimal (MAP) decoding regions.

Enjoy your holiday !!!
