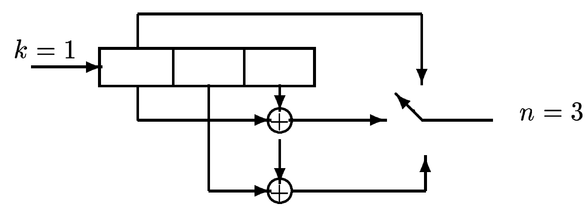
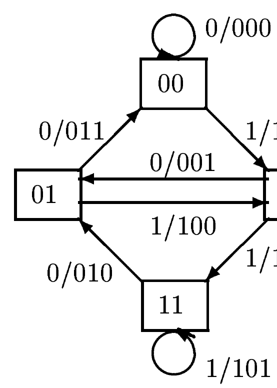
ESE 532
HW#11 Solutions

Problem 10.39

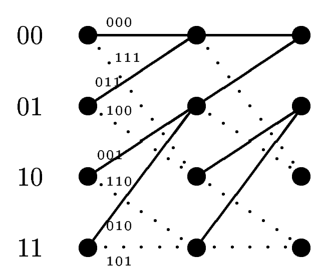
1) The encoder for the $(3, 1)$ convolutional code is depicted in the next figure.



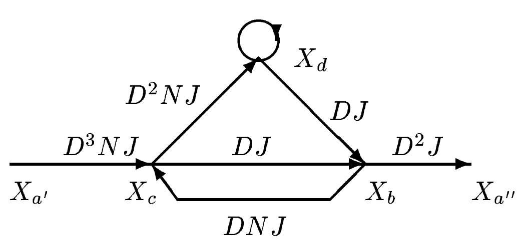
2) The state transition diagram for this code is depicted in the next figure.



3) In the next figure we draw two frames of the trellis associated with the code. Solid lines indicate an input equal to 0, whereas dotted lines correspond to an input equal to 1.



4) The diagram used to find the transfer function is shown in the next figure.



Using the flow graph results, we obtain the system

$$X_c = D^3NJX_{a'} + DNJX_b$$

$$
\begin{aligned}
X_b &= DJX_c + DJX_d \\
X_d &= D^2NJX_c + D^2NJX_d \\
X_{a''} &= D^2JX_b
\end{aligned}
$$

Eliminating X_b, X_c and X_d results in

$$T(D,N,J) = \frac{X_{a''}}{X_{a'}} = \frac{D^6NJ^3}{1 - D^2NJ - D^2NJ^2}$$

To find the free distance of the code we set $N = J = 1$ in the transfer function, so that

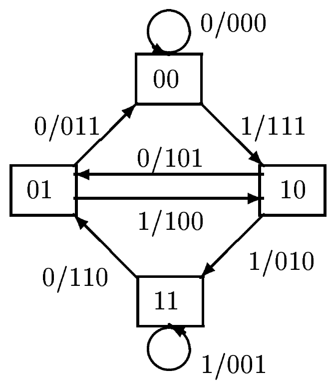
$$T_1(D) = T(D,N,J)|_{N=J=1} = \frac{D^6}{1-2D^2} = D^6 + 2D^8 + 4D^{10} + \cdots$$

Hence, $d_{\text{free}} = 6$

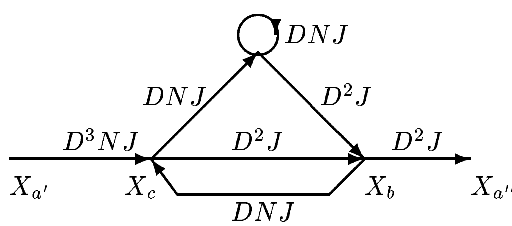
5) Since there is no self loop corresponding to an input equal to 1 such that the output is the all zero sequence, the code is not catastrophic.

Problem 10.40

1) The state diagram of the code is depicted in the next figure



2) The diagram used to find the transfer function of the code is depicted in the next figure



Using the flow graph relations we write

$$
\begin{aligned}
X_c &= D^3NJX_{a'} + DNJX_b \\
X_b &= D^2JX_c + D^2JX_d \\
X_d &= DNJX_c + DNJX_d \\
X_{a''} &= D^2JX_b
\end{aligned}
$$

Eliminating X_b, X_c and X_d, we obtain

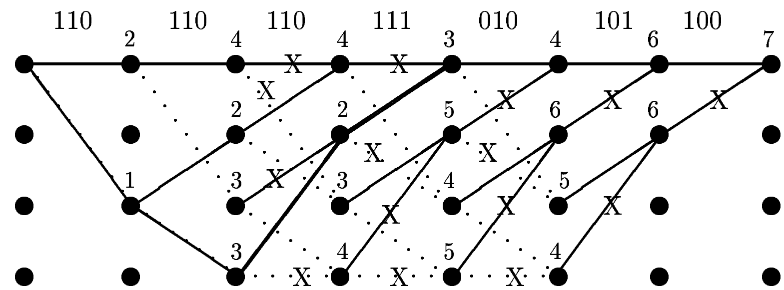
$$T(D,N,J) = \frac{X_{a''}}{X_{a'}} = \frac{D^7NJ^3}{1 - DNJ - D^3NJ^2}$$

Thus,

$$T_1(D) = T(D,N,J)|_{N=J=1} = \frac{D^7}{1-D-D^3} = D^7 + D^8 + D^9 + \cdots$$

3) The minimum free distance of the code is $d_{\text{free}} = 7$

4) The following figure shows 7 frames of the trellis diagram used by the Viterbi decoder. It is assumed that the input sequence is padded by to zeros, so that the actual length of the information sequence is 5. The numbers on the nodes indicate the Hamming distance of the survivor paths. The deleted branches have been marked with an X. In the case of a tie we deleted the lower branch. The survivor path at the end of the decoding is denoted by a thick line.



The information sequence is 11000 and the corresponding codeword 111010110011000...

5) An upper to the bit error probability of the code is given by

$$\bar{p}_b \leq \frac{1}{k} \left. \frac{\vartheta T_2(D,N)}{\vartheta N} \right|_{N=1, D=\sqrt{4p(1-p)}}$$

But

$$\frac{\vartheta T_2(D,N)}{\vartheta N} = \frac{\vartheta}{\vartheta N} \left[\frac{D^7 N}{1-(D+D^3)N} \right] = \frac{D^7}{(1-DN-D^3N)^2}$$

and since $k = 1$, $p = 10^{-5}$, we obtain

$$\bar{p}_b \leq \left. \frac{D^7}{(1-D-D^3)^2} \right|_{D=\sqrt{4p(1-p)}} \approx 4.0993 \times 10^{-16}$$