ESE 535 —*Information Theory and Reliable Communications*

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Final Exam	N. Phamdo
Closed Books, Closed Notes, 3 Hours	May 12, 1997

1. **(25 Pts Total)** Decide whether each of the following statements is true or false. Prove the validity of those that are true and give counterexamples or arguments based on known facts to disprove those that are false.

   (a) **(5 pts)** Given a binary lossless source code, α, with codeword lengths $\{l_i\} = \{3,3,3,4,4,4,4,5,5\}$. It is possible, using the Kraft inequality, to determine whether or not α is uniquely decodable.

   (b) **(5 pts)** The capacity of a 32-input, 32-output noisy typewriter DMC is 4 bits/channel use.

   (c) **(5 pts)** Given an i.i.d. Gaussian source which produces 10 samples/second and a binary symmetric channel (BSC) which is used 60 times/second with crossover probability 0.11, there exists a coding scheme with expected squared-error distortion (in SQNR) of 16 dB or better. [Hint: $h_b(0.11) = 0.5$.]

   (d) **(5 pts)** If X and Y are real-valued, independent random variables and $Z = X + Y$, then
   $$h(Z) = h(X) + h(Y).$$

   (e) **(5 pts)** There exists a continuous random variable with differential entropy $-\sqrt{2}\pi$.

2. **(20 Pts Total)** Consider two binary p.m.f.'s $p = (0.5, 0.5)$ and $q = (0.9, 0.1)$. Let $A_\epsilon^{(n)}(p)$ and $A_\epsilon^{(n)}(q)$ be the typical sets defined w.r.t. p and q, respectively. Let $\mathbf{X} = (X_1, X_2, \ldots, X_n)$ be a binary i.i.d. random vector with marginal p.m.f. p.

   (a) **(5 pts)** Determine $\left|A_\epsilon^{(n)}(p)\right|$ for each n and each $\epsilon > 0$.

   (b) **(5 pts)** Determine $\left|A_\epsilon^{(n)}(q)\right|$ for $n = 100$ and $\epsilon = 0.005$. [An analytical expression will do.]

   (c) **(5 pts)** Determine $\Pr\{\mathbf{X} \in A_\epsilon^{(n)}(p)\}$ for each n and each $\epsilon > 0$.

   (d) **(5 pts)** Determine $\Pr\{\mathbf{X} \in A_\epsilon^{(n)}(q)\}$ for $n = 100$ and $\epsilon = 0.005$. [An analytical expression will do.]

   [Hint: $h_b(0.9) = 0.469$]

3. **(20 Pts Total)** Consider a zero-mean stationary Gaussian source, $\{X_n\}$, with p.s.d.
   $$S_X(f) = \begin{cases} 5 & \text{if } |f| \le 1/4, \\ 2 & \text{if } 1/4 < |f| \le 1/2. \end{cases}$$
   Determine and sketch the rate-distortion function of $\{X_n\}$ w.r.t the squared-error distortion measure.

4. **(20 Pts Total)** Consider a stationary binary Markov source $\mathcal{V} \triangleq \{V_n\}_{n=1}^{\infty}$ with

$$\Pr\{V_{n+1} = j | V_n = i\} = \begin{cases} \alpha & \text{if } i = j = 0 \\ \beta & \text{if } i = j = 1 \end{cases},$$

$n \geq 1$, where $0 < \alpha, \beta < 1$.

(a) **(5 pts)** Calculate $\Pr\{V_n = 0\}$.

(b) **(5 pts)** Find the entropy rate in bits/sample of $\mathcal{V}$.

(c) **(10 pts)** Assume now that $\alpha = 1/4$, $\beta = 1/8$, and that the source $\mathcal{V}$ is generated at a rate of 8000 samples per second. Suppose further that you wish to encode the output of this source and transmit the result over a binary erasure channel (BEC) with input alphabet $\mathcal{X} = \{0, 1\}$, output alphabet $\mathcal{Y} = \{0, E, 1\}$, and transition probability matrix given by

$$[\Pr(Y = y | X = x)] = \begin{pmatrix} 1-\epsilon & \epsilon & 0 \\ 0 & \epsilon & 1-\epsilon \end{pmatrix}.$$

Assume that you are allowed to use the channel 10^5 times per second. What value of ϵ is required to ensure the existence of an encoder/decoder providing arbitrarily good performance (i.e., arbitrarily low probability of decoding error) for such a communication system ?

5. **(15 Pts Total)** Answer only one of the following five:

(a) [**Network IT**] Define the multiple access channel. Define the capacity region of a multiple access channel. Explain why the capacity region of a multiple access channel is convex.

(b) [**Gambling & Stock Market**] What is the meaning of a portfolio $\mathbf{b} = (b_1, b_2, \ldots, b_m)$? What is a log-optimal portfolio? In a horse race, what is a fair odd? a superfair odd? a subfair odd?

(c) [**Kolmogorov Complexity**] Define the Kolmogorov complexity of a binary string. Explain how the Kolmogorov complexity is computer independent. How is Kolmogorov complexity related to the Shannon entropy?

(d) [**Maximum Entropy & Spectral Estimation**] Explain the meaning of a maximum entropy distribution. What is Burg's maximum entropy theorem?

(e) [**IT & Statistics**] What is a type class? What is Sanov's theorem? How is it related to the conditional limit theorem?

Have an enjoyable summer !!!
