A. From the textbook: Problems 8.1, 8.2, 8.3, 8.4, 8.5, 8.6, 8.7, 8.8, 8.9, 8.10

B. Consider a discrete memoryless channel with input X and output Y. Assume that the input alphabet is $\mathcal{X} = \{1, 2\}$, the output alphabet is $\mathcal{Y} = \{0, 1, 2, 3\}$, and the transition probabilities $p(y|x) \triangleq Pr(Y = y|X = x)$ are given by

$$p(y|x) = \begin{cases} 1 - 2\epsilon, & \text{if } x = y; \\ \epsilon, & \text{if } |x - y| = 1; \\ 0, & \text{otherwise,} \end{cases}$$

where $0 < \epsilon < 1$.

(a) Determine the channel probability transition matrix $Q \triangleq [p(y|x)]$, and draw the channel transition diagram.

(b) Compute the capacity of this channel. What is the maximizing input distribution that achieves capacity ?

(c) Assuming the input distribution obtained in (b), what is the joint entropy of X and Y – i.e., what is $H(X, Y)$? Under the same assumption, what is $H(X|Y)$?

C. Let X be a binary random variable with alphabet $\mathcal{X} = \{0, 1\}$. Let Z denote another random variable that is independent of X and taking values in $\mathcal{Z} = \{0, 1, 2, 3\}$ such that $\Pr\{Z = 0\} = \Pr\{Z = 1\} = \Pr\{Z = 2\} = \epsilon$, where $0 < \epsilon < 1$. Consider a discrete memoryless channel with input X, noise Z, and output Y described by the equation:

$$Y = 3X + (-1)^X Z,$$

where X and Z are as defined above.

(a) Determine the channel transition probability matrix $Q \triangleq [p(y|x)]$, and draw the channel transition diagram.

(b) Compute the capacity C of this channel in terms of ϵ. What is the maximizing input distribution that achieves capacity ?

(c) If $\epsilon = 1/4$, find the value of C. Comment on the result.