Consider a split-capacitor, as in the Figure. Two planes are separated by a distance $d=1$. One has a slit $2L$, defining "source" and "drain". $L/d \equiv \lambda$. The other "whole" plate is called the gate. Source is grounded. The gate potential is V_G, the drain V_D.

Because of linearity of the Laplace equation, it is sufficient to find two solutions:

$$\phi_G(x,y) \quad \text{for} \quad V_G = 1\,,\ V_D = 0\,; \tag{1}$$

$$\phi_D(x,y) \quad \text{for} \quad V_D = 1\,,\ V_G = 0\,; \tag{2}$$

whence the general potential is given by

$$\phi(x,y) = V_G\phi_G(x,y) + V_D\phi_D(x,y)\,. \tag{3}$$

We can find the potentials ϕ_G and ϕ_D by conformal mapping of the device domain in the complex plane

$$z = x + i\,y$$

onto a strip

$$0 < \mathrm{Im}(w) < 1\,,$$

where w is the complex potential

$$w \equiv A + i\,\phi\,,$$

of which the real potential $\phi(x,y)$ of interest to us is the imaginary part. The real part $A(x,y)$ corresponds to the "vector potential" of the electric field, $\vec{E} = -\nabla\phi = \nabla\times\vec{A}$. It is useful in the capacitance calculations, because the electric field flux through any segment (a,b) of an equipotential line equals $A_b - A_a$.

For the case of Eq. (1), the transformation is

$$z = \beta\,\mathrm{cotanh}(\pi w/2) + w \tag{4}$$

whereas for Eq. (2) it is

$$z = \beta - e^{\pi w} + \frac{1}{\pi}\ln\left[1 - 2\beta\,e^{-\pi w}\right]. \tag{5}$$

In these equations, z is in units of d. The real coefficient β is given by

$$\pi\beta = (\rho^2+1)^{1/2} - 1\,, \quad \text{where} \quad \rho + \ln[\rho + (\rho^2+1)^{1/2}] = \pi\lambda\,. \tag{6}$$

These transformations are similar to those in the book by H. Kober "*Dictionary of Conformal Representations*", pp. 145-148. I have put the gate on top by taking the complex conjugate of his transformations; also I chose $\pi a = 1$ since Kober's $a \equiv d/\pi$. In Eq. (4), I replaced Kober's ζ with w, related to ζ by $\pi w = \zeta + i\pi$ (this gives the mapping of the split electrode on $\phi = 0$ and the whole electrode on $\phi = 1$). In Eq. (5) I replaced Kober's ζ with w, related to ζ by $\pi w = -\zeta$ (this transformation maps the split electrode at $x > 0$ on $\phi = 1$, while both the gate and the $x < 0$ split electrode on $\phi = 0$).

If we fix a value of ϕ and vary A, we are tracing an equipotential. As A is varied from $A_{\min} < 0$ to $A_{\max} > 0$, the gate-controlled equipotentials (Eq. 4) are traced from left to right, i.e. in order of increasing x. At the same time, the drain-controlled equipotentials are traced *right-to-left*, in order of monotonically decreasing x and y.

Numerical Procedure

The program is contained in serge/NOTES/SplitCap/FORT/compot.f:

compot.f:
MAIN:

testv:	produces splot input for $\phi(x)$ and $A(x)$ in the gap
w:	The function $w(z)$ (calls **cpot**)
prep:	Prepares guess points for **cpot**; splot input for equipotentials
beta:	Is called first by **prep**; evaluates $\beta(\lambda)$
testl:	An external function called by **beta**.
zgate:	Evaluates $z(w)$, Eq. (4)
zdrain:	Evaluates $z(w)$, Eq. (5)
cpot:	*Main engine*! For a given z calculates w_G and w_D
cgate:	An external function called by **cpot**
cdrain:	An external function called by **cpot**

cpot uses a PORT program **mullr**

```
wg=mullr(cgate,wg1,wg2,wg3,epsz,epsf,niter,iter)
wd=mullr(cdrain,wd1,wd2,wd3,epsz,epsf,niter,iter)
```

which finds the root of complex functions *cgate* and *cdrain* in the vicinity of initial guesses w_1, w_2 and w_3 (w_3 is supposed to the best guess and w_2 the second best). The functions $cgate = z - zgate$ and $cdrain = z - zdrain$. A 30×30 matrix of possible guesses is prepared by **prep**. *cpot* chooses three guesses and then calls **mullr**.

A listing of the program *compot*.f is attached. On the cray-xmp it has run in 1.776sec. The following output files are created:

open (unit=2, FILE="pot.re")	$\phi(x)$ in the gap $-\lambda < x < \lambda$
open (unit=3, FILE="pot.im")	$A(x)$ in the gap $-\lambda < x < \lambda$
open (unit=3, FILE="gate.d")	gate controlled equipotentials
open (unit=4, FILE="drain.d")	drain controlled equipotentials
open (unit=5, FILE="table.d")	30×30 matrix of guesses from **prep**