Spatial Network Traffic Intensity

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Abstract — **Calculation of the spatial intensity of network traffic in one and two dimensional networks under shortest path routing is presented. A result of this work is that traffic intensity in a network covering a circular area under assumptions of traffic uniformity and shortest path routing, is a quadratic function of spatial position.**

I. Introduction

Network traffic intensity can vary as a function of time and space. The statistical characterization of variations in time has received an increasing amount of attention in recent years [2]. In this paper a different aspect of traffic intensity variation, spatial variation, is examined. We think this is important as there are, to our knowledge, no baseline analytical studies of this topic. It has implications for our understanding of sizing and dimensioning networks. Using generic and canonical assumptions of topology, traffic uniformity and locality, and shortest path routing, we describe means to calculate network traffic intensity as a function of space. Because of the generic nature of the assumption, this work is applicable to both circuit switched and packet switched networks. A surprising result of this work is that in spite of a fairly involved derivation, traffic intensity in a network covering a circular area under assumptions of traffic uniformity and shortest path routing, is a quadratic function of spatial position. Aside from this specific result, we more generally consider calculating spatial traffic intensity for discrete and continuous network models with geographic traffic locality and preference. Canonical equations for these cases are developed. The equations and a specific case study of the effects of geographic locality on a linear, discrete network is available in a technical report [1].

II. Basic method

Assumptions for the discrete network intensity analysis are that the distances between adjacent nodes are identical and traffic follows a uniform distribution. That is, every pair of nodes has the same amount of traffic flowing between the two nodes. Then, for linear network,

$$\begin{aligned} & \text{Traffic intensity in link } x \qquad (1) \\ = \; & 2(\#\text{ nodes to the left of } x)(\#\text{ nodes to the right of } x) \end{aligned}$$

Here bidirectional traffic is assumed (accounting for the "two" in (1)). Similarly, if a linear continuous network is defined on the interval (0,L), then:

$$\text{Traffic intensity at point x} = 2x \cdot (L - x) = 2Lx - 2x^2 \qquad (2)$$

III. Two Dimensional Circular Network Case

It can be assumed that a network covers a circular region with radius R. There are two equations for calculation of network traffic density which are the line $y = ax + b$ and the circular network boundary equation $x^2 + y^2 = R^2$. Traffic is uniformly distributed between all pairs of points in the circular region. Traffic always follows a shortest path (straight line) route. To calculate the traffic "intensity" at an arbitrary point, Z, in the circular region, one can place a line through the point. Then there is an one dimensional problem involving traffic generated between pairs of points on either side of Z on the line and passing through Z. The line is rotated 180^o about Z and the intensity at Z is integrated.

In order to get the crossing points between the rotated line and the circle, substitute $y = ax + b$ into $x^2 + y^2 = R^2$. Then, the traffic intensity at $(0, b)$, I_{linear}, along the linear component of the network is

$$I_{linear} = 2(R^2 - b^2) \qquad (3)$$

This implies that the traffic intensity along the line is independent of its rotational angle. The total traffic intensity I_{total} at $Z = (0, b)$ is the integration of the above equation by α which is varying from 0 to π centered at $(0, b)$, that is,

$$\begin{aligned} I_{total} &= \int_0^{\pi} 2(R^2 - b^2) d\alpha \\ &= 2(R^2 - b^2) \cdot \pi \qquad (4) \end{aligned}$$

IV. Conclusion

The density of traffic inside the circular network is a quadratic function with a maximum of $2\pi R^2$ at b=0 (network center) and zero intensity at the boundary.

We note that the simulated result from [3] is consistent with the traffic distribution of (4).

References

[1] S.H. Kim and T.G. Robertazzi, "Spatial Traffic Intensity in One and Two Dimensional Networks," *SUNY at Stony Brook College of Engineering and Applied Science Technical Report 780*, Jan. 5, 2000

[2] W.E. Leland, M.S. Taqqu, W. Willinger and D.V. Wilson,"On the Self-Similar Nature of Ethernet Traffic," *IEEE/ACM Transactions on Networking,* vol. 2, No. 1, pp. 1-15, Feb. 1994.

[3] B. A. Abernathy and T. G. Robertazzi, "Loading and spatial location in wire and radio communications," *IEEE Milcom 91*, vol. 2, 1991, pp. 391-395.