CayleyNet: A Multihop WDM-Based Lightwave Network

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Abstract

A multihop, wavelength division multiplex (WDM)-based network, CayleyNet, is proposed for the realization of terabit lightwave networks. CayleyNets are attractive virtual topologies because they support the largest number of nodes for four-neighbor connection in a range of diameters. In general, CayleyNets are bi-directional and there are $N = p \times k$ nodes, where p is a prime and k is a factor of $p-1$. Our analysis shows that CayleyNet has similar performance in channel efficiency, total and user throughput as the ShuffleNet with $N = 4^k \times k$. However, CayleyNet has the advantages of a more flexible network size and symmetric transmission distance. Compared with the $N = 2^k \times k$ ShuffleNet and toroidal mesh, the CayleyNet has a superior performance.

1 Introduction

With the recent advances in fiber optics, *lightwave networks* composed of optical fibers have embarked on an important role in telecommunications. In the realization of *terabit lightwave networks*, wavelength division multiplexing (WDM) is used for large-scale concurrency on a single fiber [1]. There are two classes of WDM-based systems, *single-hop* and *multi-hop* [2, 3]. While the single hop approach requires wavelength-agile transmitters and receivers, the multihop approach, on the other hand, assigns fixed transmission frequency to each communication link, and therefore, eliminates the need for pre-transmission communications and rapidly tunable devices [3]. Each node has a small number of transmitters, transmitting and receiving signals in an assigned and fixed wavelength. This arrangement allows simultaneous transmission among multiple users and thus attaining the terabit capacity of the network. As the word *multihop* suggested, a message may be required to route through intermediate nodes, each retransmits the message on a different wavelength until it reaches the destination.

The establishment of an efficient multihop lightwave network relies heavily on the proper assignment of wavelengths to communication links of each node. The goals are to ensure that there is at least one path between any pair of nodes and that the average and maximum number of hops for a message to reach its destination should be small. Such assignments are based on an interconnection topology. Since this topology is not directly related to the physical connection of nodes, it is referred to as a *virtual topology*. A number of virtual topologies have been proposed [3, 4]. Of the many options, ShuffleNet and the toroidal mesh are among the most popular topology [3]. Performance comparison of these two networks are found in [4, 10, 11].

For the ShuffleNet, there are $N = p^k \times k$ nodes arranged in p^k rows and k columns [6, 5]. Interconnection between adjacent columns is a perfect shuffle [12] and transmission is uni-directional. Each node has p transmitters and p receivers, each transmitting and receiving in a fixed and assigned frequency. Figure 1 shows an $N = 2^2 \times 2 = 8$ ShuffleNet. The symbol λ_{ij}, $i, j = 0, \ldots, 7$ corresponds to different transmitting frequencies.

The toroidal mesh (or torus) is basically a 2-dimensional grid. In general, there are $N = p \times k$ nodes, where p and k are any integers. Applications of toroidal meshes are found in both communication networks and parallel computers. A uni-directional version of the torus is implemented as the Manhattan Street Network [9] whereas a bi-directional case is studied in [13]. In applying the torus as a virtual topology for lightwave networks, there are four or two transmitters and receivers, depending if the links are bi-directional or uni-directional.

Whether it is a ShuffleNet or a torus, a virtual topology is for wavelength assignments. Physically, the network topology can be arbitrary, provided that direct transmission exists between adjacent node in the virtual network. Popular topologies in local and

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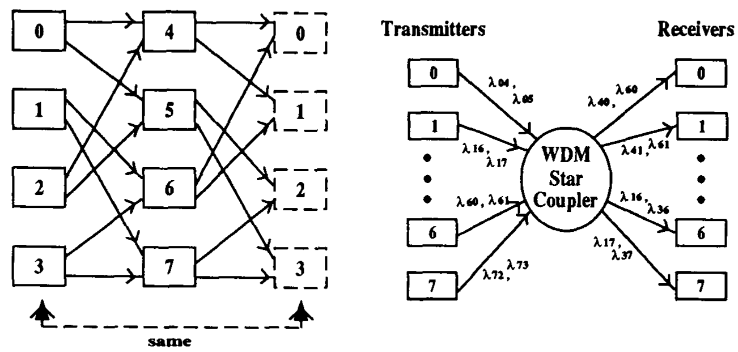

(a) ShuffleNet (b) Star Implementation

Figure 1: *An 8-Node ShuffleNet and Its Implementation.*

metropolitan area networks such as the bus, star or tree networks are sufficient. As indicated in [4], Figure 1 (b) shows a star implementation of the 8-node ShuffleNet.

One advantage of the ShuffleNet and the torus is their simple routing algorithm. Since messages usually require multiple hops to get to destinations, the goal of routing is to determine an appropriate outgoing links for each incoming message. A simple, *distributed, self-routing* algorithm that can identify shortest paths based only on address of the destination exists for the ShuffleNet and torus [6, 14]. The maximum of the minimal distance (in hops) between two nodes is called the *diameter*[15]. Obviously, a small diameter implies potentially a small communication delay. The diameter of an $N = p^k \times k$ ShuffleNet is $2k-1$ whereas that of an $N = p \times k$ torus is $\lfloor \frac{p}{2} \rfloor + \lfloor \frac{k}{2} \rfloor$.

While the main disadvantage of a torus is its relatively large diameter, the disadvantages of a ShuffleNet include non-symmetric node distance and limited number of nodes. Due to ShuffleNet's unidirectional property, distance from node i to j does not equal to that from j to i. In most cases, if node i sends a message to its immediate neighbor, the reply/acknowledge message needs to traverse more than half the diameter. Furthermore, the number of nodes for a ShuffleNet is restricted to $N = p^k \times k$. When p is large, the possible number of nodes becomes limited. For example, when $p = 4$, there are only three possible network configurations in the range of $1,000$ to $100,000$ users. The number of feasible configurations reduces to two when $p = 8$ for this range of users. With the growing number of computer users and network size, the design of an efficient virtual topology that can accommodate thousands of nodes with minimum delay is desirable.

In this paper, we propose a new interconnection graph as the virtual topology for wavelength assignment in large-scale lightwave networks. This new topology is based on a special class of symmetric graphs, the Borel Cayley graphs [16, 17]. For simplicity, we call the resultant network, the *CayleyNet.* These Cayley graphs are attractive candidates for virtual topologies because they are currently, the *densest* known degree-4 graphs for a range of diameters, $D = 7, \ldots, 13$. (The *degree* of a network is the number of neighbors at each node.) This property means that for a degree-4 connection and the given range of diameter, CayleyNet connects the largest number of nodes known. This dense property also implies that the number of intermediate hops between any two nodes is small for CayleyNet. Furthermore, unlike the ShuffleNet, CayleyNet is a bi-directional network. This property implies symmetric node distance, i.e., the distance (in hops) from node i to node j is the same as from nodes j to i.

2 CayleyNet

The original definitions of Cayley graphs and Borel Cayley graphs are based on finite algebraic group theory [18]. In this section, we first review these original definitions and then present a more practical definition.

Definition 1 A graph $\mathbf{C} = (\mathbf{V}, \mathbf{G})$ is a Cayley graph with vertex set $\mathbf{V}$ if two vertices $v_1, v_2 \in \mathbf{V}$ are adjacent $\Leftrightarrow v_1 = v_2 * g$ for some $g \in \mathbf{G}$ where $(\mathbf{V}, *)$ is a finite group and $\mathbf{G} \subset \mathbf{V} \setminus \{I\}$. $\mathbf{G}$ is called the generator set of the graph and I is the identity element of the finite group $(\mathbf{V}, *)$.

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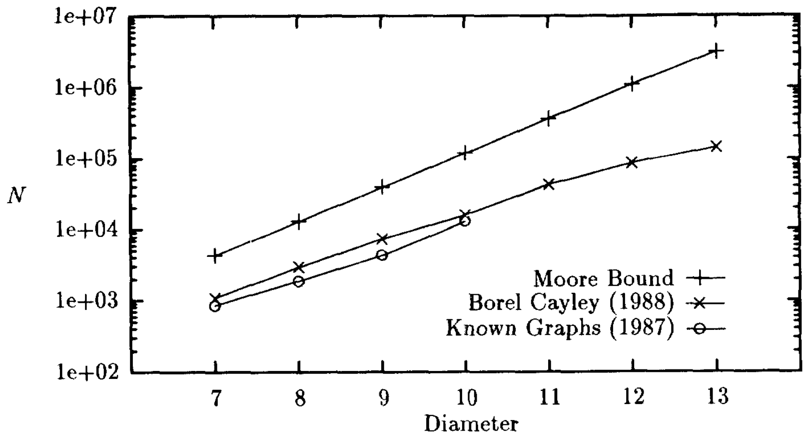

Figure 2: *Size of Degree-4 Graphs.*

This original definition of a Cayley graph requires nodes to be elements in a group but does not specify a particular group. A *Borel Cayley graph* is a Cayley graph defined over the *Borel* group of matrices:

Definition 2 Let $\mathbf{V}_{(p,a)}$ be a set of Borel matrices, then

$$\mathbf{V}_{(p,a)} = \left\{ \begin{pmatrix} x & y \\ 0 & 1 \end{pmatrix} : x = a^t \pmod p, \right. \\ \left. y \in \{0, \ldots, p-1\},\ t \in \{0, \ldots, k-1\} \right\}$$

where p and a are fixed parameters, p is a prime, $a \in \{2, \ldots, p-1\}$, and k is the *order* of a. That is, $a^k = 1 \pmod p$.

In other words, the nodes of Borel Cayley graphs are 2×2 Borel matrices, and matrix multiplication (mod p) is the group operation $*$. Connections of a vertex are generated by post-multiplying the vertex by elements in the generator set, $\mathbf{G}$. The number of elements in $\mathbf{G}$ determines the number of neighbors at a node and hence the degree of the graph. For bi-directional networks, the generator set $\mathbf{G}$ is closed under inversion. In general, we can construct a degree-δ CayleyNet by choosing the appropriate generator set ($\delta = \mid \mathbf{G} \mid$). In this paper, we concentrate our efforts on degree-4 networks.

For degree-4, bi-directional networks, we have 4 elements (generators) in the generator set $\mathbf{G}$, $\mathbf{A} = \begin{pmatrix} a^{t_1} & y_1 \\ 0 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} a^{t_2} & y_2 \\ 0 & 1 \end{pmatrix}$, and their inverses. These Cayley graphs are attractive interconnection models because they are ***node-symmetric*** and are currently the densest known, degree-4 graphs. The node-symmetric property is useful for routing and is reviewed in Section 3. Figure 2 shows the Moore bound, Borel Cayley graphs discovered in 1988 and known graphs in 1987. The Moore bound is an upper bound for the number of nodes for a given degree and diameter. However, this upper bound has been proved to be unattainable except for a few trivial cases where the diameter, $D = 2$ [19]. Given this general impossibility of constructing graphs with the Moore bound, there has been a longstanding search to find the densest regular graphs of a given degree and diameter. In a technical report published in 1988, Chudnovsky *et al.* announced that the Borel Cayley graphs are the densest known degree-4 graphs for a range of diameters, $D = 7, \ldots, 13$. Since then, these Cayley graphs remained the densest for that range of diameter [20].

However, with vertices of the graph defined as 2×2 matrices and connection specified through modulo p matrix multiplications, the original definition of Borel Cayley graphs is abstract and complex. In studying the degree-4 case, we have used their inherent properties to develop a more practical definition in the integer domain. For simplicity, we call the resultant degree-4, bi-directional network, the *CayleyNet.*

Definition 3 A degree-4, bi-directional, $N = p \times k$ CayleyNet where p is prime and k is a factor of $p-1$ is specified by choosing two generators $t_1, t_2 \in \{0, \ldots, k-1\}$, $t_1 \neq t_2$, and an element $a \in \{2, \ldots, p-1\}$ such that $a^k = 1 \pmod p$. Connections of nodes are defined as: for any node $j \in \mathbf{V} = \{0, \ldots, N-1\}$, if $j = i \pmod k$,

$$j \sim j + \alpha_i,\ j + \alpha_i^{-1},\ j + \beta_i,\ j + \beta_i^{-1},\ \pmod N$$

where

$$\begin{array}{rcll} \alpha_i & = & < i + t_1 >_k + < a^i >_p & k - i; \\ \alpha_i^{-1} & = & < i - t_1 >_k + < -a^{i-t_1} >_p & k - i; \\ \beta_i & = & < i + t_2 >_k + < a^i >_p & k - i; \\ \beta_i^{-1} & = & < i - t_2 >_k + < -a^{i-t_2} >_p & k - i; \end{array} \quad (1)$$

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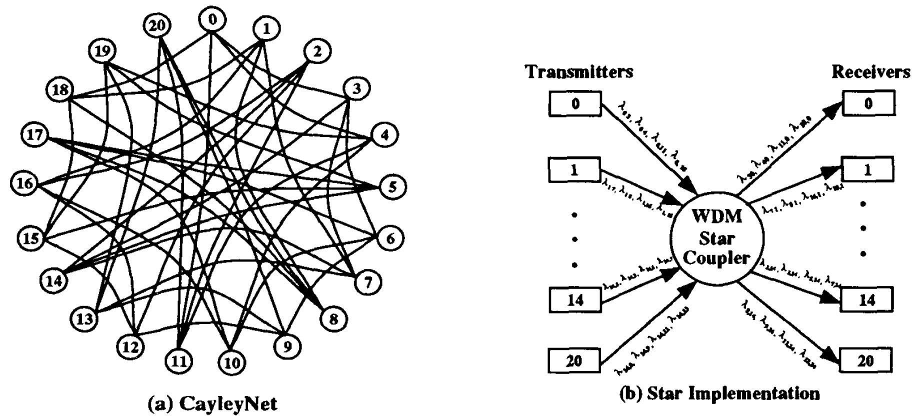

Figure 3: *A 21-Node CayleyNet and Its Implementation.*

the symbol $\sim$ represents "adjacent to" and $< i+t >_k$ denotes $(i+t) \pmod k$.

This new definition of Borel Cayley graphs is actually a Generalized Chordal Ring (GCR) representation of the graph. Generalized Chordal Rings are integer-domain graphs in which nodes are divided into k classes according to modulo k arithmetics. Nodes belonging to the same congruence class $i \pmod k$ have the same connection constants, $\alpha_i, \beta_i, \alpha_i^{-1}, \beta_i^{-1}$. In earlier reports, we have proved that any Cayley graphs can be transformed into an a GCR with k classes. The connection constants, $\alpha_i, \beta_i, \alpha_i^{-1}, \beta_i^{-1}$ corresponds to the generators, **A, B** $\mathbf{A}^{-1}$, $\mathbf{B}^{-1}$ of the original definition. The resultant transformation is isomorphic to the original graph and hence retains all properties including the node-symmetric and dense properties. Readers interested in the proof and transformation algorithm are referred to [21, 22].

As an example, consider a CayleyNet with $p = 7$, $k = 3$ and $N = 21$ nodes. We choose $a = 2$ ($a^k = 1 \bmod p$), $t_1 = 0$ and $t_2 = 1$. With these choices, the diameter $D = 3$.

Connections can be defined as: For any $j \in \mathbf{V}$, if $j \bmod 3 = i$: j is connected to $j + \alpha_i$, $j + \alpha_i^{-1}$, $j + \beta_i$, and $j + \beta_i^{-1} \pmod N$, where according to Equation 1

$$\alpha_i = \begin{cases} 3 & \text{for } i = 0; \\ 6 & \text{for } i = 1; \\ -9 & \text{for } i = 2; \end{cases} \quad \alpha_i^{-1} = \begin{cases} -3 & \text{for } i = 0; \\ -6 & \text{for } i = 1; \\ 9 & \text{for } i = 2; \end{cases}$$

$$\beta_i = \begin{cases} 4 & \text{for } i = 0; \\ 7 & \text{for } i = 1; \\ 10 & \text{for } i = 2; \end{cases} \quad \beta_i^{-1} = \begin{cases} -10 & \text{for } i = 0; \\ -4 & \text{for } i = 1; \\ -7 & \text{for } i = 2. \end{cases}$$

Figure 3(a) shows an $N = 7\times3 = 21$ CayleyNet represented in the integer domain as GCR graphs. Similar to the ShuffleNet, the CayleyNet is only a virtual topology. Physical implementation is arbitrary. Figure 3(b) shows a star-implementation. But unlike the ShuffleNet, CayleyNet is bi-directional and there are four transmitters and four receivers at each node.

3 Routing for CayleyNet

Since CayleyNet is a multi-hop network, the purpose of routing is to identify intermediate nodes between and source and destination pair. Ideally, an *optimal* routing algorithm identifies the shortest path between any two nodes thorough a computational scheme with a space (storage) requirement independent of the graph size, i.e., space complexity is of O(1). However, to our knowledge, such an algorithm does not exist for CayleyNet. In this section, we review an routing algorithm, the *Vertex-Transitive Routing*, that identifies shortest path with a table of $O(N)$ at every node.

Vertex-Transitive Routing exploits the node-symmetric (or vertex-transitive) property of CayleyNet. By definition, a *node-symmetric* graph implies that there is an automorphism (mapping) that maps vertex i to vertex j for any i, j in the vertex set [18]. A useful interpretation of this property for CayleyNet can be summarized in the following proposition [22].

Proposition 1 Let $i = m_1k + c_1$, $j = m_2k + c_2$, and $i' = m'k + c'$. If i connects to j through a sequence of generators $\alpha, \beta, \ldots$, then i' connects to j' through the same sequence of generators, where

$$j' = < m' + a^{<c'-c_1>_k}(m_2 - m_1) >_p k + < c' - c_1 + c_2 >_k . \tag{2}$$

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Given source $i = m_1 k + c_1$ and destination $j = m_2 k + c_2$.

While $(i \neq j)$ do Steps 1-4

Step 1: Identify new destination,
$j' =< a^{k-c_1}(m_2 - m_1) >_p k + < c_2 - c_1 >_k$

Step 2: From row j' of database, determine which link to take.

Step 3: Identify new source, $i' = mk + c$ and
$m = 1, \quad c = t_1, \quad$ if link α was chosen
$m = 1, \quad c = t_2, \quad$ if link β was chosen
$m = p- < a^{k-t_1} >_p, \quad c =< -t_1 >_k, \quad$ if α^{-1} was chosen
$m = p- < a^{k-t_2} y_2 >_p, \quad c =< -t_2 >_k, \quad$ if β^{-1} was chosen

Step 4: $i = i'$ and $j = j'$

Table 1: *Vertex-Transitive Routing for CayleyNet.*

The proof of the above proposition is included in [22] and is not repeated here. This interpretation of vertex-transitivity is useful for routing. It allows us to transform the original problem of routing between nodes i and j to a new problem of routing between node $i' = 0$ and j', where using Equation 2

$$j' =< a^{<k-c_1>_k}(m_2 - m_1) >_p \; k + < c' - c_1 + c_2 >_k . \quad (3)$$

This suggests that a table-based routing algorithm can be used to determine all shortest paths between any two nodes. At each node, we store a table with $N-1$ rows and 4 columns. Row i of the table indicates all optimal links from node 0 to node i. We emphasize that the *same* routing table is used at every node. When a message is generated at source i to destination j, this is equivalent to routing between 0 and j' where j' is identified by Equation 3. Then using row j' of the routing table, we can determine the link that corresponds to a shortest path. Once a link is identified, we can find the neighboring node by using the appropriate connection constants, $\alpha, \alpha^{-1}, \beta, \beta^{-1}$ for the corresponding class. We then have a new problem of routing between this neighboring node and j'. This procedure is repeated until the source and destination are the same. Table 1 consists of a pseudo-code for the algorithm. We observe that this routing algorithm finds all possible shortest path(s) and in addition, is capable of determining the entire path from the source using the same routing table.

As an example, we consider the CayleyNet with $p = 7$ and $k = 3$, given in section 2. There are $N = p \times k = 21$ nodes. At each node, we store a size 20×4 routing table shown in Figure 4. Again, row i of the table indicates all optimal links from node 0 to node i. To illustrate that vertex-transitive routing is capable of identifying all optimal shortest paths, we use the algorithm to route a message from node 0 to 16. According to Figure 3(a) there are three shortest paths between the two nodes, namely:

$$\begin{aligned}
\text{path } 1: &\quad 0 \xrightarrow{\beta} 4 \xrightarrow{\alpha} 10 \xrightarrow{\alpha} 16 \\
\text{path } 2: &\quad 0 \xrightarrow{\alpha^{-1}} 18 \xrightarrow{\beta} 1 \xrightarrow{\alpha^{-1}} 16 \\
\text{path } 3: &\quad 0 \xrightarrow{\beta^{-1}} 11 \xrightarrow{\alpha} 2 \xrightarrow{\beta^{-1}} 16
\end{aligned}$$

At iteration 0, $i = 0$ and $j = 16$. According to Step 1 of Table 3, we identify j' as 16. From row 16 of the routing table, there are three choices corresponding to the three shortest paths. Arbitrarily we choose link β. According to step 3, the new source $i' = 4$. Now at iteration 1, we have $i = 4$ and $j = 16$. Step 1 identifies j' to be 6. From row 6 of the routing table, we pick link α, which determines the new source $i' = 3$. Then at iteration 2, j is 3 and i is 6. For this source and destination pair we have $j' = 3$, which means link α should be taken according to row 3 of the routing table. The equations in step 3 then determine i' to be 3. Finally at iteration 3, both source and destination are 3 and the algorithm terminates. We have thus successfully found path 1 ($\beta \; \alpha \; \alpha$) between 0 and 16. The iterations of this example are summarized in Table 4. Recall that at iteration 0, there are three optimal links, $\beta, \alpha^{-1}, \beta^{-1}$. We have arbitrarily chosen β which leads to shortest path 1. Had we chosen link α^{-1}, we would have found path 2. And similarly, the choice for link β^{-1} will lead to path 3.

4 Performance

In this section, we compare the performance of CayleyNet with other popular networks, such as the ShuffleNet and Torus. Analogous to the work in [5] and [6], the performance attributes considered are channel efficiency, η; network throughput, C; and user throughput, c. Assuming the traffic load is uniformly dis-

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	α	β	α^{-1}	β^{-1}
1			1	
2				1
3	1			
4		1		
5	1	1	1	
6	1			
7	1			
8			1	
9	1			1
10		1		

	α	β	α^{-1}	β^{-1}
11				1
12			1	1
13	1	1		1
14	1			
15			1	
16		1	1	1
17	1	1	1	
18			1	
19		1		
20				1

Figure 4: *A Routing Table for a CayleyNet*

tributed, these attributes are defined as:

$$\begin{aligned} \eta &= \frac{1}{\text{E[number of hops]}} \\ C &= \eta\, W \\ c &= \eta\, \omega \end{aligned}$$

where W is the total number of channels in the network and ω is the number of channels per user. For an $N = p^k \times k$ ShuffleNet, according to [6], the channel efficiency, η_S is

$$\eta_S = \frac{2(p-1)\,(p^k k - 1)}{k\, p^k\, (p-1)\,(3k-1) \,-\, 2k\,(p^k - 1)}$$

The total number of channels, the number of channels per user, the total and the user throughput are respectively [6],

$$\begin{aligned} W_S &= k\, p^{k+1} \\ \omega_S &= p \\ C_S &= k\, p^{k+1}\, \eta_S \\ c_S &= p\, \eta_S \end{aligned} \tag{4}$$

For an $N = p \times k$ bi-directional toroidal mesh, the channel efficiency, η_T is

$$\eta_T = \frac{4}{p+k}$$

The total number of channels, the number of channels per user, the total and the user throughput are respectively,

$$\begin{aligned} W_T &= 4\, pk \\ \omega_T &= 4 \\ C_T &= W_T\, \eta_T \\ c_T &= 4\, \eta_T \end{aligned} \tag{5}$$

For an $N = p \times k$ bi-directional CayleyNet, we do not have a closed form solution for the expected number of hops. Instead, we use the average path length obtained by our computer implementation of the Vertex-Transitive routing algorithm to determine channel efficiency, η_C. For the total number of channels, the number of channels per user, the total and the user throughput, we have

iteration	i	j	j'	link	i'
0	0	16	16	β	4
1	4	16	6	α	3
2	3	6	3	α	3
3	3	3	—	—	—

Table 2: *A Summary of Iterations*

$$\begin{aligned} W_C &= 4\, pk \\ \omega_C &= 4 \\ C_C &= W_C\, \eta_C \\ c_C &= 4\, \eta_C \end{aligned} \tag{6}$$

Assuming a 1-Gb/s user transmission rate, the channel efficiency and network throughput for different sizes of the ShuffleNet (S-N, $p = 2, 4$), degree-4 toroidal mesh (T-M) and degree-4 CayleyNet (C-N) are investigated and plotted in Figures 5 and 6. In the case of the torus, we have assumed the best configuration in which $p = k = \sqrt{N}$.

Of the degree-4 networks (i.e, C-N, T-M, and S-N with $p = 4$), we observe that the CayleyNet and the ShuffleNet with $p = 4$ have the best and comparable performance. However, we recall that the ShuffleNet has non-symmetric node distance and limited number of nodes. In the range of $1,000 - 100,000$ nodes, there are only three possible configurations for $p = 4$ ShuffleNet. In other words, the bi-directional Cayleynet provides similar performance to the $p = 4$ ShuffleNet without the problems associated with ShuffleNet.

The $p = 2$ ShuffleNet provides more possible network configurations. However, because of its smaller degree, its performance is significantly less than that of the CayleyNet. The toroidal mesh has the most flexible network size, $N = p \times k$ where p, k can be any integers. Nevertheless, because of its relatively large path length, the toroidal mesh has the worst performance compared with the ShuffleNet and CayleyNet.

5 Conclusions

Wavelength division multiplexed (WDM) systems are useful in the realization of terabit lightwave networks. For WDM systems, there are the *single-hop*

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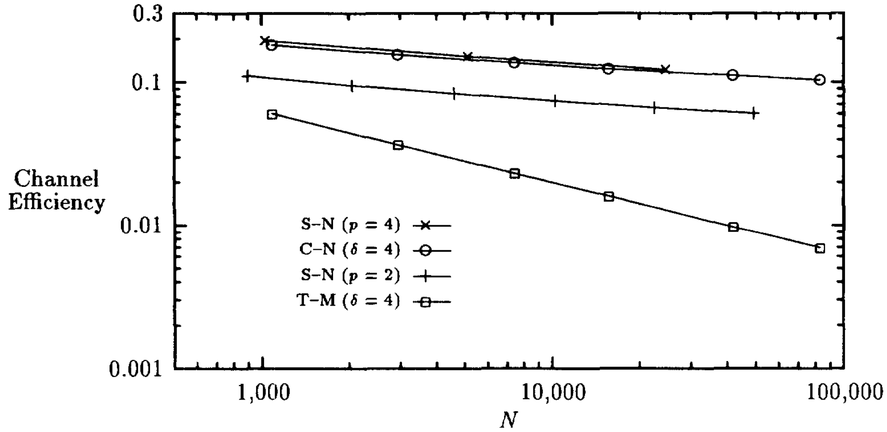

Figure 5: *Channel Efficiency vs. Size of Networks*

and *multi-hop* approaches. By eliminating the need for pre-transmission communication and wavelength-agile transmitters, the multihop approach is readily implementable [5]. In a multihop network, a user has a small number of transmitters and receivers, each transmitting and receiving signals in a fixed and assigned wavelength. The wavelength assignment is based on a virtual topology. Existing examples include the ShuffleNet, MSN, and hypercube.

An efficient virtual topology implies that a large number of users are connected through a small number of hops. Of the many topologies, the ShuffleNet and toroidal mesh are among the most popular options [3]. However, ShuffleNet's limitations and disadvantages include a restricted number of nodes, $p^k \times k$, and an asymmetric transmission distance between two nodes. In most cases, if node i connects to node j in one hop, a message from node j to node i takes more than half the diameter. The toroidal mesh, on the other hand, has symmetric transmission distance and a very flexible network size. However, its relatively long path length implies messages need to route through large number of intermediate nodes and therefore not suitable for large-scale networks.

To alleviate these problems, we propose the use of Cayley graphs from the Borel group [16]. These networks are attractive because they have been known as the densest degree-4 graphs for an interesting range of diameter, $D = 7, \ldots, 13$ [16, 17]. This property implies that the number of hops between two nodes is small, compared with the size of the network. Furthermore, it is a bi-directional network with four transmitters and receivers at each node. Analogous to the uni-directional ShuffleNet, we called the resultant network, *CayleyNet*. In general, a CayleyNet has $p \times k$ nodes, where p is a prime and k is a factor of $p-1$. This more flexible network size is another advantage of the CayleyNet.

We further evaluate performance of the CayleyNet, ShuffleNet ($p = 2, 4$) and toroidal mesh in terms of *channel efficiency*, network and user *throughput*. We found that the CayleyNet provides similar performance as the $p = 4$ ShuffleNet with the added advantages of (i) a symmetric transmission distance and (ii) more available network size. When compared with the $p = 2$ ShuffleNet and toroidal mesh, CayleyNet has superior performance. However, we caution readers that this is achieved at a higher cost of bi-directional links and more complex routing algorithm. Finally, as it has been noted in [4], we emphasize that there is no ideal universal topology. An efficient topology is application dependent and is subject to various physical and economical constraints. One of the purposes of this paper is to suggest CayleyNet as a design option in pursuit of terabit lightwave networks.

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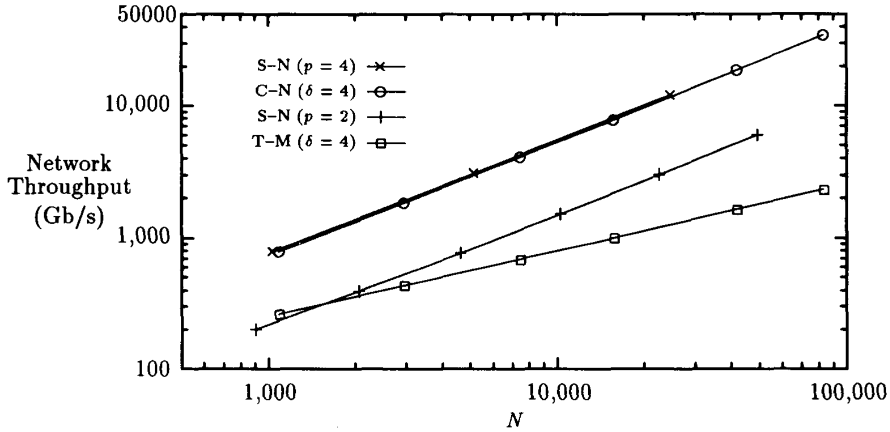

Figure 6: *Network Throughput vs. Size of Networks*

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