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# Cayley Pseudo-Random (CPR) Protocol: A Novel MAC Protocol for Dense Wireless Sensor Networks

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*Abstract* - With the rapid growth of wireless sensor technology, there is a need for MAC protocols that support efficient simultaneous peer-to-peer communications in large and dense wireless sensor networks. To meet such a need, we propose a MAC protocol that uses a novel channel assignment scheme based on the pseudo-random connection of a dense Cayley graph as an underlying graph. By utilizing all or most of the available frequency channels, the proposed protocol can support many, simultaneous peer-to-peer communications. Other features of the protocol include minimal collisions due to fixed channel assignments and a decentralized routing algorithm that avoids global time synchronization. The effectiveness of using Cayley graphs as the underlying topology for such frequency assignment is evaluated and compared with that of the Manhattan Street Network via a simulator with power model parameterized to CrossBow MICA 2 sensors.

## I. INTRODUCTION

The recent development of small and affordable microsensors that can communicate with each other via radio transceivers have resulted in the rapid growth of wireless sensor networks [1], [2], [3]. When deployed in large numbers, they provide unprecedented opportunities for monitoring applications such as real-time traffic monitoring, nuclear factory surveillance, military sensing and reconnaissance, disaster relief networks, wildfire detection, wildlife tracking, . . . [4].

In response to the popularity of wireless sensor networks, quite a number of Medium Access Control (MAC) protocols have been proposed [5]. However, with the exception of SMACS [6], most of the protocols described in [5] consider a single channel or a small number of multiple channels. We are interested in exploring the potential benefits of utilizing the entire communication bandwidth in a dense sensor network. Inspired by SMACS, our intriguing thought is that, to ensure robustness and manageability, what if each node only transmits to a small number of nodes via a dedicated frequency channel and yet the large bandwidth is exploited through simultaneous peer-to-peer communications. The underlying assumption here is that the nodes in the sensor network all have computational power and are performing some sort of local computation and therefore communications tend to be peer-to-peer. Furthermore, the density and mobility of the nodes in a defined space implies that all nodes are within transmission range of each other, albeit there is a different transmission energy requirement between each pair of nodes.

We propose to use a "dense" graph as the underlying pattern for channel assignment in wireless sensor networks. In our vision, each node of the sensor network is capable of transmitting to a small number of nodes with a dedicated and assigned frequency channel. Such an assignment is based on the connection pattern of an underlying graph. Before deployment, each sensor node is programmed to transmit and receive with a set of small and dedicated frequency channels. Once deployed, this frequency allocation is fixed. Each sensor node will only transmit, listen and receive messages with the allocated frequency channels. Since each link is only dedicated to a specific pair of sensor nodes, there are almost no collisions, and hence minimal collision induced energy. Furthermore, channels are divided according to frequencies and a decentralized routing algorithm is used. Global time synchronization and its associated communication overheads are therefore avoided.

To assure that communication is efficient, the key of this approach is to assign the frequency channels such that any peer-to-peer communications can be achieved via a small number of hops through intermediate nodes. In graph theory terminology, that means the graph must be "dense" – each node connects to a small number of neighbors and yet the diameter (maximum of the shortest distance between nodes) is small [7]. Currently, Cayley graphs over the Borel group, indeed, are the densest known degree-4 graphs over a range of diameters [8,9]. The density of Cayley graph is a result of its almost random connection. Our proposed protocol is based on such pseudo-random connections, hence the name CPR (Cayley Pseudo-Random) MAC protocol.

However, we also emphasize that the idea of using a graph as the underlying pattern for channel assignment has been applied in wavelength division multiplexed lightwave networks such as ShuffleNet, BanyanNet, Manhattan Street Network and CayleyNet [10,11]. To demonstrate how much performance advantage is provided by the dense property of Cayley graph, in this paper, we compare the performance of our proposed MAC protocol based on both Cayely graph topology and the well-known Manhattan Street Network.

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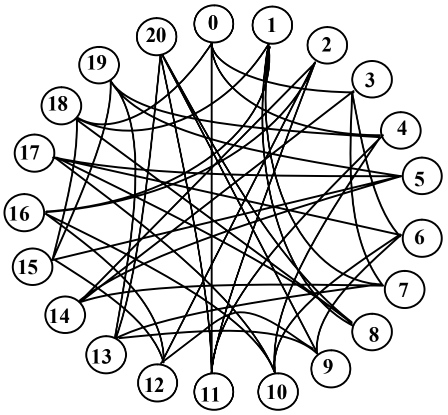

**Figure 1: A Borel Cayley Graph**

The paper is organized as follow: Section II is an overview of Cayley graphs. Our novel MAC protocol is described in Section III. Section IV present simulation results and Section V the conclusions.

## II. OVERVIEW OF CAYLEY GRAPHS

The crux of our approach is to use Cayley graphs for channel assignment in a large and dense wireless sensor network.

Cayley graphs were first constructed by A. Cayley[12]. The construction of these graphs is described by finite algebraic group theory. A group $(G,*)$ consists of a set $G$ of elements and an associative binary operation $*$. The group $G$ contains a unique element, called the identity element, and denoted $e$, such that for every $g \in G$, $g*e = e*g = g$. For each element $g \in G$, there exists a unique element in $G$, called the inverse of $g$, and denoted $g^{-1}$, such that $g*g^{-1} = e$. The set $G$ is closed under the inversion and composition or multiplication of its elements by $*$. A group is finite if $|G|$, i.e., the number of elements in $G$ is finite. We will deal with finite groups in this paper. The definition of Cayley graph is:

*Definition: Let $\Delta$ be a subset of a group $G$ of elements excluding the identity and such that $g \in \Delta$ if and only if $g^{-1} \in \Delta$. A graph $\Gamma = (G, E)$ is a Cayley graph with a vertex set $G$ that is identified with the elements in $G$ and generated by $\Delta$, if two vertices $v_1$, $v_2$ in $G$ are adjacent, that is $(v_1,v_2)$ is in $E$ iff $v_1 = v_2*g$ for some $g \in \Delta$.*

The set $\Delta$ is called the *generator set* of the corresponding Cayley graph. The exclusion of the identity element from the generator set prevents the corresponding Cayley graph from having self-loops. Also, including the inverse of each element in the generator set insures that the resulting Cayley graph is undirected and regular with a vertex degree = $|\Delta|$.

Why Cayley graphs? This is mainly because (1) Cayley graphs are generally *dense*; and (2) all Cayley graphs are *vertex transitive*. The *dense* property implies that they can connect a large number of nodes via a small number of hops through intermediate nodes. Each node in the sensor network corresponds to a vertex of a Cayley graph. Each edge of the Cayley graph corresponds to a dedicated frequency channel between two nodes of the sensor network.

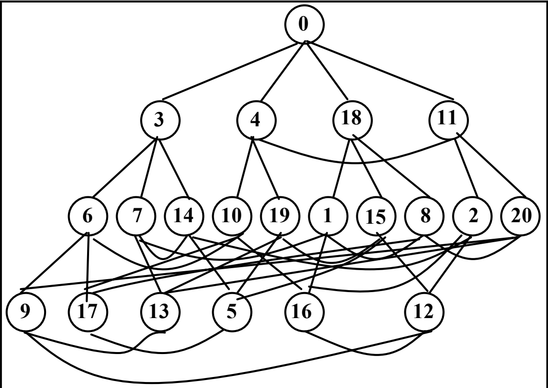

**Figure 2: Tree with root node 0.**

The vertex-transitive property means that a Cayley graph "looks the same from any node" [7,12]. Mathematically this implies that for any two vertices $a$ and $b$, there exists an automorphism of the graph that maps $a$ to $b$ [7]. Based on this symmetric property, we developed a distributed routing algorithm for Cayely graphs in our earlier work [13-15]. The main advantage of the algorithm is that identical routing table can be used at every node, making decentralized routing feasible. Due to space limitation, the details of the routing algorithm are not included. Readers interested are referred to [13-15].

In our previous work [13-15], we provided algorithms to transform a Cayley graph from the group theoretic domain to the integer domain which simplifies routing. Figure 1 is an example of a 21-node, degree-4 Cayley graph in the integer domain. At first sight, the connection pattern looks random. But in fact, the graph is vertex-transitive. Figure 2 shows the Cayley graph in a tree-like form with root node 0. The Cayley graph will have the same tree-like form with any other root node.

In other words, two trees with different root nodes will have the same structure. The only difference among them is in the vertex labels. Once a tree with a certain root node is constructed, we have established a formula to identify the node labels of the tree with a different root node [13-15]. This vertex-transitive property is very useful for routing. It provides the basis for a distributed routing algorithm [13-15] that can be efficiently implemented in sensor networks.

## III. CAYLEY PSEUDO-RANDOM (CPR) PROTOCOL

In the previous sections, we indicated that the *dense* and *symmetric* properties of Cayley graph make them attractive as a virtual topology for frequency assignment. In this section, we describe the proposed CPR protocol. Before describing the protocol in details, we summarize our assumptions:

### A. Assumptions

**Assumption 1**: The sensor network consists of large and dense number of sensors deployed in an ad hoc fashion. Say for example, in a room of 20 m x 20m = 400 $m^2$, there are several hundreds to thousands of sensor nodes. This density is not an exaggeration because the research community is predicting the density to be as high as 20 nodes/$m^3$ [6]. Because of this density and the current technology allows each sensor node having a transmission range of up to 1,000 feet, the sensor nodes in the network are therefore all within communication range of each other. However, with that many nodes in the network, a simple contention-based protocol such as the IEEE 802.11 will not work well. Furthermore, these sensor nodes are expected to have mobility and may be drifted to a location out of transmission range of some nodes.

**Assumption 2**: There are large numbers (hundred to thousands) of transmission frequency channels. Since the required bandwidth for sensor data is expected to be low, on the order of 1-100 kb/s [16], assuming the radios operate in the 902-928 MHz ISM (Industrial, Scientific, and Medical) band, and that the transmission data rate from each node is 10 kb/s, as many as 2,600 frequency channels can be available [8]. It is also assumed that none of these channels overlap so there is no interference among channels.

**Assumption 3**: Each node is equipped with a single half-duplex transceiver that can be tuned to transmit or receive at different frequency channels. However, having only a half-duplex transceiver, a sensor node can either transmit or listen at a time but cannot do both simultaneously. Some other researchers assume that multiple transceivers are available at a sensor node or that the transceiver is capable of carrier sensing multiple channels simultaneously [17]. If that's the case, the proposed CPR MAC protocol will be even simpler to implement and more efficient.

**Assumption 4**: The transceiver is capable of switching its frequency band dynamically. The switching time is less than 1 μs [36,37], a negligible overhead.

### B. The Protocol

For a sensor network with $n = p \times k$ number of nodes, the protocol involves two steps before deployment:

***Step 1:*** Frequency Assignments of Sensor Nodes. Before deployment, each node is programmed to receive and transmit via a small set of dedicated frequency channels, according to the connection pattern of an underlying Borel Cayley graph. This process should be done on a central computer with reasonable computational power and is not expected to be real-time.

***Step 2:*** Establishment of a Universal Routing Table. By constructing a breadth-first tree, a routing table that lists all optimal outgoing links from node 0 to all other nodes in the network is generated. The size of the table is $(n-1) \times \delta$, where $n$ is the number of nodes and $\delta$ is the number of dedicated frequency channels at a node. Again, the generation of this routing table is expected to be completed before deployment and at a central computer with reasonable computational power and is not a real-time process. Once such a table is generated, the same identical table is loaded to each of the sensor nodes.

After the above two steps of preparation, the sensor nodes in the network are ready for deployment and the proposed protocol works as follows: because there is only one single half-duplex transceiver, the multiple channels have to be used in a "distributed time-division manner". The proposed protocol borrows the well known RTS (Request to Send), CTS (Consent to Send), and ACK (Acknowledgement) concept of the IEEE 802.11 DCF protocol. Each sensor node has a Transmit, Receive, Listen and Sleep cycle.

The sensor node "jumps" to the Transmit cycle whenever there is data to send, provided that it is not busy receiving data. If it is busy receiving data from a neighbor node, it will wait until the data has been completely received and an ACK has been sent out before switching to the Transmit Cycle. The Transmit cycle is completed once the sensor receives an ACK from the destination sensor.

During the Receive cycle, a sensor node receives data from its designated frequency neighbor. Once data message is received, an ACK message is sent to the source sensor.

During the Listen cycle, a sensor tunes its antenna to each adjacent sensor for a fix time period $\tau$. Sensor cycles through each adjacent sensor (D sensors) until it receives an RTS control message or it has a message ready for transmission. Upon receiving an RTS, a sensor sends out an CTS and switches over to Receive Cycle. One listen cycle lasts $D\tau$ time units if there is no activity.

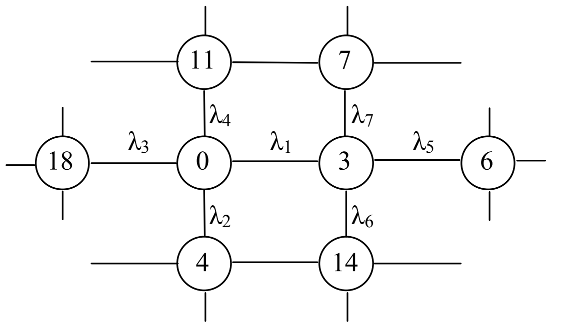

**Figure 3a:** Frequency assignments at nodes 0 & 3.

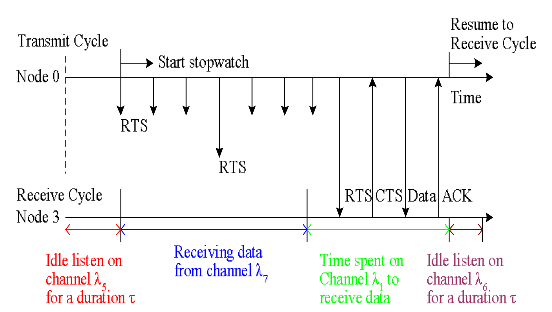

**Figure 3b:** Time sequence of events.

During the optional Sleep cycle, sensor places itself in power saving mode for a fixed time duration (either a listen cycle duration or a transmit cycle estimated duration).

An Example: Assume that the underlying Borel Cayley graph dictates that channel $\lambda_1$ connects node 0 to node 3 (Figure 3a). Suppose node 3 is in the Receive Cycle (Figure 3b) but is monitoring other channels or receiving data from other channels, no CTS will be received by node 0. Node 0 will repeat sending RTS at a period smaller than the "listening time" to ensure that a RTS arrives at node 3 when it begins monitor channel $\lambda_l$ for the duration $\tau$. If a CTS is received before the stopwatch exceeds a preset time, the source node 0 will begin transmission of the data and expects an ACK from the receiving node, node 3 (Figure 3b). If however, no CTS is received after the stopwatch has exceeded a preset value, the link/node is considered dead or too busy.

In summary, the key features of the CPR protocol are that (1) because Cayley graphs are dense, the number of intermediate nodes is small, and (2) because of the vertex-transitive property, the same distributed routing algorithm with the same database can be used at every node; and (3) there is no synchronization, scheduling or updating of routing tables.

## IV SIMULATION RESULTS

We have developed a simulator to implement the CPR protocol. The simulator was written in the C programming language. It is discrete event driven and includes a heap scheduler.

We investigate the effectiveness of the unidirectional Cayley graphs as a virtual topology for frequency assignment when compared with the Manhattan Street Network (MSN), a well-known unidirectional topology for communications. Within the simulation model, sensors are randomly placed in a circular region (coordinates are selected from a uniform distribution) with sensor transmission range (500 feet) as the diameter. All sensors are logically connected with one another according to either a Cayley or MSN. Because sensors are pre-programmed with the topology, there is no direct relationship between logically adjacent and geographically nearby sensors.

In the following sections, we first describe the sensor model, power model, traffic patterns, and performance metrics. Preliminary results that show Cayley graphs having a superior performance than MSN is then presented.

### *A. The Sensor Model*

Our model for sensors is derived from the CrossBow Mica2 sensor. Each sensor is modeled as an input queue for transit messages, an input queue for new data, a server (triangular service time distribution), and output queues (one per adjacent sensor).

Communication to and from adjacent nodes traverses the radio module. New data queued in the local queue is moved to the output queue destined to the appropriate sensor just before its transmission. All queues are FIFO served. Processing delays, message lengths, and battery consumption are derived from Mica2 data sheets and user manual [53,54].

### *B. The Power Model*

Currently, our simulator only accounts for transmission power consumption. We used Crossbow Mica2 user manual [54] to identify the transmission energy consumed by the sensor as a function of distance. In the future, we plan to account for energy consumed by all sensor operations.

### *C. Performance Metric*

Based on Crossbow Mica2 datasheet and user manual, we use 40 bytes and 38,400 bps as the message size and transmission data rate. Based on our own experience, we use 16B as the size for all control messages (RTS, CTS, and ACK); the data messages processing delays are assumed to be between 5 and 10 ms (ranges of triangular distribution of mean 7.5ms). The size of the local buffer Lx is 10.

To measure the sensor network performance, we consider the following: *sensor utilization* is the fraction of time a sensor is processing messages in a transmit or a receive cycle; when sensor utilization is high, more energy is consumed in the sensor and its lifetime is shorter; *mean transmission delay* and *packet loss probability*.

### *D. Results*

In our preliminary studies, we compare a 52-node Cayley network with a *13x4* Manhattan Street Network (MSN). The traffic pattern considered is *single-node accumulation*.

For ***single node accumulation***, all sensors in the network send exponentially distributed data messages to an accumulation sensor at the same transmission rate. Two specific rates are considered: *0.1* message per second (msg/s) and *0.7* msg/s. We found that beyond 0.7 msg/s, end-to-end delay and message loss grow significantly indicating that the tested sensor networks throughput limit was reached. Overall, Cayley network exceeds MSN (Manhattan Street Network) in all performance metrics.

For **mean transmission delay and packet loss probability**, We found that the mean dealy of Cayley is about 67% and 68% of MSN for transmission rate of 0.1 msg/s and 0.7 msg/s respectively.

Figure 4 shows the histogram of the **end-to-end delay distribution** of Cayley and MSN networks. From this figure, we observe that Cayley's distribution is more centered at its *mode* than that of MSN. It indicates that, overall, message delay is shorter for Cayley.

Figure 5 provides the sensor utilization for MSN (top figure) and Cayley (bottom figure) for single node accumulation at 0.1 msg/s. We observe that for Cayley network, sensor utilization is less than that of the Manhattan Street network (MSN). Recall that sensor utilization is defined as the fraction of time a sensor is processing messages in a transmit or a receive cycle. In other words, a higher utilization implies more energy consumed and a smaller lifetime for the sensor. This figure indicates that the CPR protocol, because of Cayley's pseudo random connections, is potentially more energy efficient.

Furthermore, we also note that, even for a centralized traffic pattern such as single node accumulation, sensor utilization is much more evenly distributed within the Cayley network than that of MSN. This is an important advantage as it implies that the lifetime of sensors are more evenly distributed. A similar observation can also be made for single node accumulation at 0.7 msg/s.

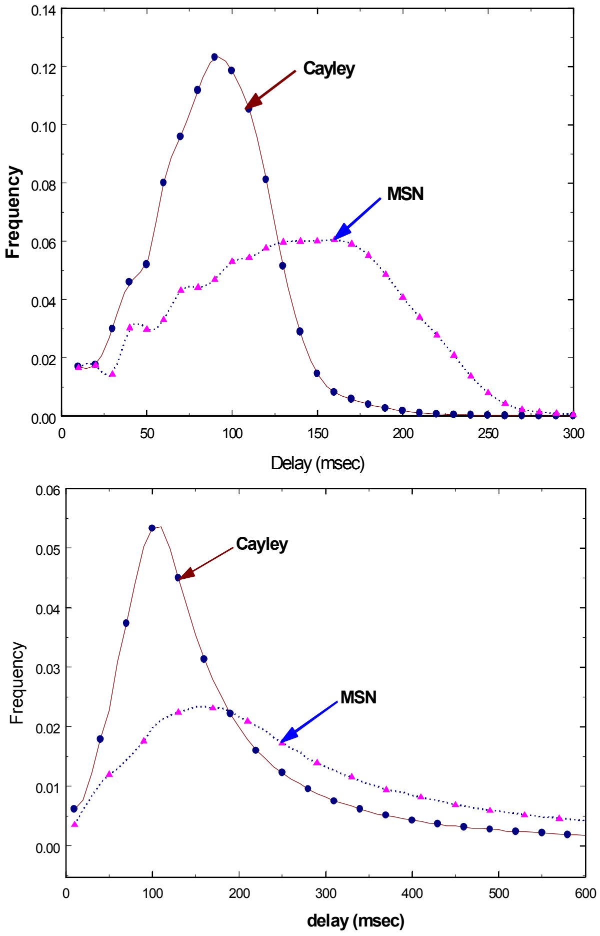

**Figure** 4: End-to-End Delay Histogram (10ms bins). Transmission rate=0.1 msg/s (Top); transmission rate = 0.7 msg/s (Bottom).

## V. CONCLUSIONS

In this paper, we propose the novel idea of using a "dense" graph as the underlying pattern for channel assignment in wireless sensor networks. Each node of the sensor network is capable of transmitting to a small number of nodes with a dedicated and assigned frequency channel. Such an assignment is based on the connection pattern of an underlying graph. Before deployment, each sensor node is programmed to transmit and receive with a set of small and dedicated frequency channels. Once deployed, this frequency allocation is fixed. Each sensor node will only transmit, listen and receive messages with the allocated frequency channels. Since each link is only dedicated to a specific pair of sensor nodes, there are almost no collisions, and hence minimal collision induced energy wastes. Furthermore, channels are divided according to frequencies and a decentralized routing algorithm is used. Global time synchronization and its associated communication overheads are therefore avoided.

To investigate the effectiveness of using Cayley graphs as the underlying topology for such frequency assignment, we compare the performance of the protocol based on both Cayley graph and Manhattan Street Networks. To make our simulation model more realistic, we parameterized the power model of the simulator according the datasheet of CrossBow Mica2 sensor. We found that our proposed protocol based on Cayley graphs performed better than that of the Manhattan Street Network. Ongoing efforts are being made to expand the comparisons to larger network size and to other networks.

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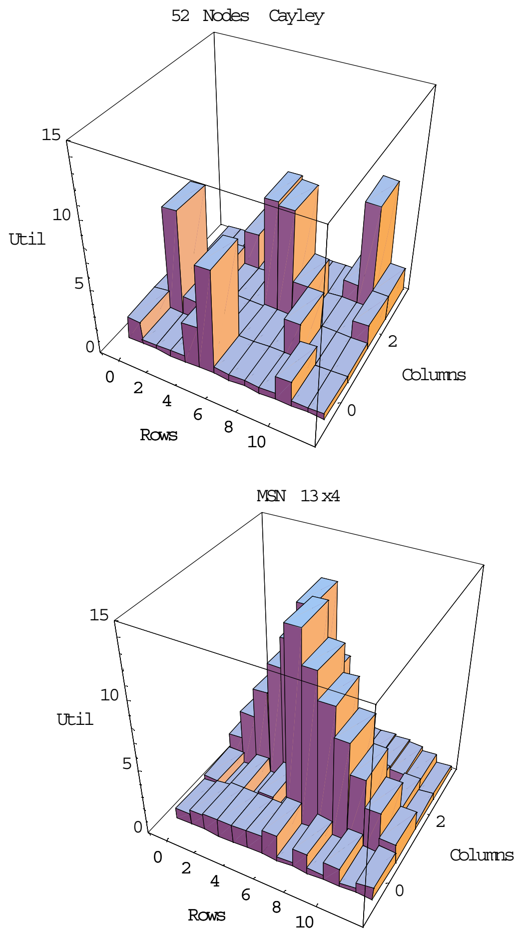

**Figure 5:** Sensor Utilization for MSN (Top) and Cayley (Bottom) for single node accumulation at 0.1 msg/s.

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