Nonblocking WDM Switching Networks With Full and Limited Wavelength Conversion

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***Abstract*—In recent years, with the rapid exhaustion of the capacity in wide area networks led by Internet and multimedia applications, demand for high bandwidth has been growing at a very fast pace. Wavelength-division multiplexing (WDM) is a promising technique for utilizing the huge available bandwidth in optical fibers. In this paper, we consider efficient designs of nonblocking WDM permutation switching networks. Such designs require nontrivial extensions from the existing designs of electronic switching networks. We first propose several permutation models in WDM switching networks ranging from no wavelength conversion, to limited wavelength conversion, to full wavelength conversion, and analyze the network performance in terms of the permutation capacity and network cost, such as the number of optical cross-connect elements and the number of wavelength converters required for each model. We then give two methods for constructing nonblocking multistage WDM switching networks to reduce the network cost.**

***Index Terms*—Multistage networks, nonblocking, optical networks, permutation, switching networks, wavelength conversion, wavelength-division multiplexing (WDM).**

I. Introduction

In recent years, with the rapid exhaustion of the capacity in wide area networks (WANs) led by Internet and multimedia applications, demand for high bandwidth has been growing at a very fast pace. Optical networks which employ optical fiber for transmission are very attractive because optical fiber provides a huge bandwidth (nearly 50 THz), low signal attenuation (as low as 0.2 dB/km), and very low bit error rate (BER) (less than 10^{-11}) [1]. Wavelength-division multiplexing (WDM) is an important approach to utilizing the huge available bandwidth in optical fibers. WDM is basically frequency-division multiplexing in the optical frequency domain, where on a single optical fiber there are multiple communication channels operated by different wavelengths concurrently. In a traditional (electronic) switching network which consists of one or more stages of switches, each source node can only be connected to exactly one of the destination nodes at a time. Adopting WDM provides a way to enable each source node to send the same or different messages to different destination nodes concurrently.

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In an all-optical WDM network, the signals remain in the optical domain throughout their paths except at the ends. Such paths are termed *lightpaths* [2]. To keep the signal in the optical domain, in the absence of any wavelength converters (WCs) [3]–[5], a lightpath is required to be on the same wavelength channel throughout its path in the network; this requirement is known as the *wavelength continuity constraint* [2], [6]. This requirement may not be necessary if there are WCs in the network. A WC converts a signal on one wavelength to another wavelength. WCs can be distinguished into two types: 1) a full-range wavelength converter (FWC) [4] that can convert an incoming wavelength to any outgoing wavelengths of the WDM network; and 2) a limited-range wavelength converter (LWC) [4], [8], [12] that can convert an incoming wavelength to a subset of the full wavelength set. A single lightpath in a wavelength-convertible network can use a different wavelength along each of the links in its path. Thus, wavelength conversion may improve the efficiency in the network by resolving the wavelength conflicts of the lightpaths, but the disadvantage of allowing wavelength conversion is the increased cost and complexity. This implies potential tradeoffs between the performance of a WDM network and the number of WCs needed, along with other design parameters.

The technology of optical wavelength conversion [3], [7]–[10], has received great attention in the optical network community [2], [4], [5], [11]–[13], [15], [19]–[21]. Xiao and Leung [4], Lee and Li [5], and Subramaniam *et al.* [11] investigated the optimal WC placement in all-optical WDM networks. Ramaswami and Sasaki [12] considered using limited WCs to support lightpaths efficiently. Yates *et al.* [19], Tripathi and Sivarajan [2], and Barry and Humblet [13] analyzed the blocking probability in all-optical networks with limited wavelength conversion. Recently, Sharma and Varvarigos [14] analyzed limited-range wavelength conversion in wavelength routed mesh and hypercube WDM networks, and demonstrated that limited wavelength conversion of fairly small degree is sufficient to obtain benefits comparable to those obtained by full-range wavelength conversion. Also, Yang *et al.* [15] studied WDM switching networks under another important type of traffic, multicast, with or without wavelength conversion.

In this paper, we consider efficient designs of nonblocking WDM switching networks under unicast traffic, as one-to-one or unicast connections remain as a dominant type of traffic pattern in communications. Such designs of WDM switching networks require nontrivial extensions from the existing designs of electronic switching networks. This is because, in addition to the great difference between a WDM switching network and an electronic switching network in terms of how connections be-

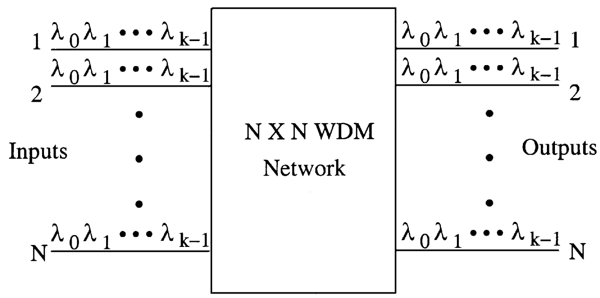

Fig. 1. $N \times N$ WDM switching network with k wavelengths.

tween source and destination nodes are supported as mentioned above, a major challenge in designing a WDM switching network is how to keep the signal in the optical domain, eliminating the need for conversions between optical and electronic signals, hence, avoiding the so-called electronic bottleneck. Note that, in order to keep the signal in the optical domain throughout its path, it is also desirable for a WDM switching network to be nonblocking, since blocked signal will be dropped, or lost, due to the lack of optical random-access memory (RAM), or buffer.

The paper is organized as follows. In the next section, we first describe several permutation models in WDM networks which specify wavelengths that can be used by source and destination nodes of a connection. Then, under these models, we analyze the nonblocking permutation capacity (to be defined later), and calculate the network cost in terms of the number of optical cross-connect elements and the number of WCs required in a nonblocking crossbar-based design. We then propose two methods for constructing nonblocking multistage WDM switching networks to reduce the network cost in Section III. Finally, in Section IV we summarize our results and conclude the paper.

II. Nonblocking Permutation in WDM Switching Networks

An $N \times N$ WDM switching network is a photonic switch with N input ports and N output ports, and each input port can be connected to any output port without optical-electrical-optical (OEO) conversion, although the switch may still be controlled by electronic signals. Considering a WDM switching network (or simply a network) as shown in Fig. 1 with k wavelengths (denoted as $\{\lambda_0, \lambda_1, \ldots, \lambda_{k-1}\}$) and N input/output ports, each input port in such a WDM network is usually equipped with k fixed-tuned optical transmitters, and each output port is equipped with k fixed-tuned optical receivers, or optical filters. In this section, we will assume that such an $N \times N$ k-wavelength WDM network is a crossbar-like switching fabric, which may be implemented using light branches (also called splitters) and light combiners, as well as optical cross-connect elements. A light branch splits a signal carried by a specific wavelength into a set of signals on the same wavelength, while a light combiner operates with the opposite purpose, combining multiple input signals on the same or different wavelengths into one output signal. Optical cross-connect elements may be implemented using the gate switch technology, in which semiconductor optical amplifier (SOA) gates are employed to select and filter input signals to specific output ports by means of being turned on or off.

To establish a connection in such a network, a channel (or wavelength) at an input port can be paired to a channel at one of the output ports either on the same wavelength or on a different wavelength, depending on whether the network is subjected to the wavelength continuity constraints or not. Generally speaking, a node at the input side can be involved in up to k connections simultaneously; also, it can be connected to up to k output nodes. A one-to-one connection is a pairing between an input channel and an output channel in the network (hereafter, simply referred to as a connection). In a WDM permutation switching network, the admissible connection patterns are in the form of permutations. A set of Nk connections forms an input/output connection pattern or connection state, which is referred to as a *permutation assignment* in this paper. A nonblocking permutation switching network can realize any permutation assignment without conflicts.

In general, switching networks with a larger number of realizable permutation assignments offer more degrees of freedom to the network, which can improve network performance. We define permutation capacity of a WDM network as the number of permutation assignments realizable in the network, and denote it as C_p in this paper. Clearly, the greater the permutation capacity (or the more functionality) a network has, the more flexible is the network in reacting to fluctuating user demand, changing loads, and equipment problems, and the better it will perform under all types of conditions. Thus, permutation capacity is a deterministic measure to quantify the network performance in terms of throughput, connectivity, flexibility, and survivability. It should also be pointed out that under the same traffic load, a network with higher permutation capacity will have a lower blocking probability as more connection patterns can be realized without blocking in such a network.

A. Permutation Models in WDM Switching Networks

We further categorize WDM networks by different permutation models according to their utilization of WCs. A WC converts an incoming wavelength to a different wavelength without loss of any information modulated on the incoming wavelength. As we mentioned earlier, there are two types of WCs, namely, FWCs and LWCs. With no wavelength conversion, a connection could only use the same wavelength along its lightpath and is referred to as the permutation with same wavelength (PSW) model. With limited-range wavelength conversion, a connection can be set up by assigning some limited wavelengths to its destination node in addition to the wavelength of the source node, and is referred to as the permutation with limited wavelengths (PLW) model. Finally, with full-range wavelength conversion, a connection can use any of the wavelengths and is referred to as the permutation with any wavelength (PAW) model. Based on the above definitions, the PAW is the strongest model among the three permutation models, and the PSW is the weakest permutation model. This is because a connection under the PSW model can always be realized under the PLW model and a connection under the PLW model can always be realized under the PAW model, but not vice versa. Also note that a traditional electronic switching network is a special case under the PSW model, since

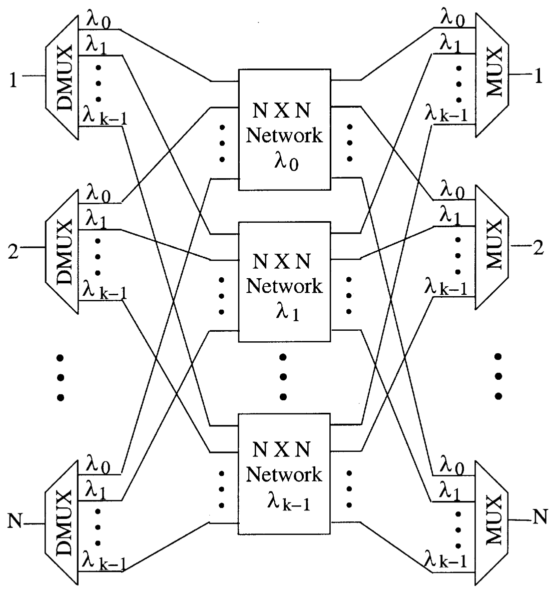

Fig. 2. $N \times N$ k-wavelength WDM network consisting of k parallel $N \times N$ one-wavelength networks.

it can be viewed as a one-wavelength WDM network. In the following, we analyze these three models in more detail in terms of their permutation capacity, as well as their network cost.

B. Permutation Capacity Under PSW and PAW Models

In an $N \times N$ k-wavelength WDM switching network, the permutation capacity is determined by the number of permutation assignments that can be realized. Clearly, the larger the permutation capacity, the better the network performance, or, in other words, the stronger the permutation model. We start with the simplest model, the PSW model. Then, we turn to the PAW model, which is easier to analyze than the PLW model. We use a separate subsection to exploit the PLW model due to the complexity involved in the analysis.

First, we have the following lemma concerning the permutation capacity under the PSW model:

Lemma 1: For an $N \times N$ k-wavelength WDM switching network under the PSW model, the permutation capacity is $C_p = (N!)^k$.

Proof: Note that, under the PSW model, the same wavelength has to be used by a connection at both input and output sides. In fact, an $N \times N$ k-wavelength WDM switching network under this model is equivalent to k parallel $N \times N$ one-wavelength networks as shown in Fig. 2, where all the same wavelength from different links are directed to an $N \times N$ one-wavelength network [21]. Since we have a total of k wavelengths on each fiber link, there are k $N \times N$ one-wavelength networks. For each network, the number of permutation assignments is the same as that of a traditional electronic crossbar network, i.e., $N!$. Given that each wavelength can be involved in different permutation assignments independently from each other, there are

$$\underbrace{N!N!\cdots N!}_{k} = (N!)^k$$

different permutation assignments. Therefore, $C_p = (N!)^k$. $\square$

We observe that an $N \times N$ k-wavelength WDM network under the PSW model is not the same as an $Nk \times Nk$ electronic network when $k > 1$, whose permutation capacity is $(Nk)!$. As will be shown, the strongest permutation model for a WDM network, i.e., the PAW model, will be reduced to an $Nk \times Nk$ electronic network, but not the PLW model.

Lemma 2: For an $N \times N$ k-wavelength WDM switching network under the PAW model, the permutation capacity is $C_p = (Nk)!$.

Proof: Under the PAW model, there are no restrictions on how wavelengths can be assigned to a connection. Any channel of an input port can be paired to any channel of an output port. In fact, this model can be viewed as an $Nk \times Nk$ traditional electronic network [21]. Therefore, there are $(Nk)!$ possible permutation assignments under the PAW model, i.e., $C_p = (Nk)!$. $\square$

C. Permutation With Limited Wavelength Conversion

From *Lemmas 1* and *2*, we know that the permutation capacity under the PSW model is $C_p = (N!)^k$, and the permutation capacity under the PAW model is $C_p = (Nk)!$. Clearly, the permutation capacity under the PAW model is much greater than that under the PSW model, which implies that network performance of a PAW model is much better than that of a PSW model. Note that FWCs are needed to implement a PAW model, while no WCs are needed to implement a PSW model. However, implementing all-optical full-range wavelength conversion is quite difficult and expensive due to technological limitations [2], [12]. A realistic all-optical WC may only be able to convert to a limited number of output wavelengths for any given input wavelength. Thus, it is interesting to investigate network performance for limited-range wavelength conversion, i.e., the PLW model.

We characterize the limited-range wavelength conversion capability by wavelength degree, which is defined next. An LWC has wavelength degree w (for some integer w, $1 \leq w \leq k$) if an input wavelength can be converted to $w-1$ output wavelengths in addition to the input wavelength itself. Due to the complexity of the analysis, in this paper, we consider WDM networks with wavelength degree two ($w = 2$) under the PLW model. In this case, incoming wavelength λ_i can be converted to outgoing wavelength $\lambda_{(i+1)\mathrm{mod}k}$, where $0 \leq i \leq k-1$. For notational convenience, we will use λ_{i+1} instead of $\lambda_{(i+1)\mathrm{mod}k}$ in the rest of the paper, with the understanding that $(i+1) \equiv (i+1)\mathrm{mod}k$.

In the following, we will start deriving the permutation capacity for an example network with $k = 3$, we then consider the general case for any k.

For an $N \times N$ three-wavelength WDM network, each fiber link has three channels, i.e., $\{\lambda_0, \lambda_1, \lambda_2\}$. With the limited wavelength conversion of wavelength degree $w = 2$, λ_0 can be converted to λ_1, λ_1 can be converted to λ_2, and λ_2 can be converted to λ_0 (circular conversion). We define the following matrix to represent the relationship among these conversions, where c_{ij} $(0 \leq i, j \leq 2)$ is the number of λ_i's being converted to λ_j's.

$$\begin{pmatrix} c_{00} & c_{01} & 0 \\ 0 & c_{11} & c_{12} \\ c_{20} & 0 & c_{22} \end{pmatrix}_{3\times 3}.$$

Note that the sum of row i in the matrix is the total number of wavelengths coming from λ_i, and the sum of column j is the total number of wavelengths going to λ_j. Since this is an $N \times N$ three-wavelength WDM network, the total number of wavelengths in the network should be exactly N. Therefore, the constraints of the matrix are

$$\begin{cases} c_{00} + c_{01} = N \\ c_{11} + c_{12} = N \\ c_{20} + c_{22} = N \end{cases} \qquad \begin{cases} c_{00} + c_{20} = N \\ c_{01} + c_{11} = N \\ c_{12} + c_{22} = N. \end{cases} \tag{1}$$

From (1), we have

$$\begin{cases} c_{00} = c_{11} = c_{22} \\ c_{01} = c_{12} = c_{20}. \end{cases} \tag{2}$$

Let

$$\begin{cases} s = c_{00} = c_{11} = c_{22} \\ N - s = c_{01} = c_{12} = c_{20} \end{cases} \quad \text{for } 0 \le s \le N. \tag{3}$$

In fact, (3) represents the dependency between the number of input λ_i's ($0 \le i \le 2$) converted into λ_{i+1}'s and the number of input λ_i's not converted. From (3), we also have that for all input wavelengths, the number being converted (and not being converted) is the same.

Now, the permutation capacity can be calculated as

$$C_p = \sum_{\substack{0 \le s \le N \\ c_{00}=c_{11}=c_{22}=s \\ c_{20}=c_{01}=c_{12}=N-s}} \binom{N}{c_{00}\ c_{20}} N! \binom{N}{c_{11}\ c_{01}} N! \binom{N}{c_{22}\ c_{12}} N!. \tag{4}$$

This is because for any N λ_i's, say, N λ_0's, at the output side of the network, connections could only be set up with those input wavelengths on λ_0 and λ_2 under the PLW model we consider. Within these N connections, c_{00} represents the connections from input wavelength λ_0's, and c_{20} represents the connections from input wavelength λ_2's. By utilizing the multinomial coefficients [16], we have $\binom{N}{c_{00} c_{20}}$ ways to assign c_{00} out of N input wavelength λ_0's, and c_{20} out of N input wavelength λ_2's to form N connections to output λ_0's. Furthermore, there are $N!$ different connections in every such assignment. Therefore, we have $\binom{N}{c_{00} c_{20}} N!$ possible ways to form connections to output wavelength λ_0's. Similarly, we can obtain that there are $\binom{N}{c_{11} c_{01}} N!$ and $\binom{N}{c_{22} c_{12}} N!$ ways to form connections to output wavelength λ_1's and λ_2's, respectively. Noting that the dependency between the number of input λ_i's converted to λ_{i+1}'s $(c_{i\,i+1})$ and the number of input λ_i's not converted (c_{ii}) is given by (3), (4) indeed calculates the permutation capacity for an $N \times N$ three-wavelength WDM network. Finally, (4) can be simplified to

$$\begin{aligned} C_p &= \sum_{\substack{0 \le s \le N \\ c_{00}=c_{11}=c_{22}=s \\ c_{20}=c_{01}=c_{12}=N-s}} (N!)^3 \binom{N}{c_{00}} \binom{N - c_{00}}{c_{20}} \binom{N}{c_{11}} \\ &\quad \cdot \binom{N - c_{11}}{c_{01}} \binom{N}{c_{22}} \binom{N - c_{22}}{c_{12}} \\ &= \sum_{\substack{0 \le s \le N \\ c_{00}=c_{11}=c_{22}=s}} (N!)^3 \binom{N}{c_{00}} \binom{N}{c_{11}} \binom{N}{c_{22}} \\ &= \sum_{s=0}^{N} \left[\binom{N}{s} (N!) \right]^3. \end{aligned} \tag{5}$$

Using a similar method, we can form a general matrix and calculate the permutation capacity for an $N \times N$ k-wavelength WDM network as follows:

$$\begin{pmatrix} c_{00} & c_{01} & 0 & \cdots & \cdots & 0 \\ 0 & c_{11} & c_{12} & 0 & \cdots & 0 \\ \vdots & & \ddots & \ddots & & \vdots \\ \vdots & & & \ddots & \ddots & \vdots \\ 0 & & & & c_{k-2\ k-2} & c_{k-2\ k-1} \\ c_{k-1\ 0} & 0 & \cdots & \cdots & 0 & c_{k-1\ k-1} \end{pmatrix}_{k \times k}$$

where the dependency between the number of input λ_i's converted to λ_{i+1}'s and the number of input λ_i's not converted is given by

$$\begin{cases} c_{00} + c_{01} = N \\ c_{11} + c_{12} = N \\ \qquad \vdots \\ c_{k-1\ 0} + c_{k-1\ k-1} = N \end{cases} \qquad \begin{cases} c_{00} + c_{k-1\ 0} = N \\ c_{01} + c_{11} = N \\ \qquad \vdots \\ c_{k-2\ k-1} + c_{k-1\ k-1} = N. \end{cases} \tag{6}$$

From (6), we obtain the following relationship:

$$\begin{cases} c_{00} = c_{11} = \cdots = c_{k-1\ k-1} = s \\ c_{01} = c_{12} = \cdots = c_{k-1\ 0} = N - s \end{cases} \tag{7}$$

where $0 \le s \le N$. Therefore, the permutation capacity can be derived as

$$\begin{aligned} C_p &= \sum_{\substack{0 \le s \le N \\ c_{00}=c_{11}=\cdots=c_{k-1\ k-1}=s \\ c_{01}=c_{12}=\cdots=c_{k-1\ 0}=N-s}} \binom{N}{c_{00}\ c_{k-1\ 0}} N! \binom{N}{c_{11}\ c_{01}} N! \\ &\quad \cdots \binom{N}{c_{k-1\ k-1}\ c_{k-2\ k-1}} N! \\ &= \sum_{\substack{0 \le s \le N \\ c_{00}=c_{11}=\cdots=c_{k-1\ k-1}=s}} \binom{N}{c_{00}} (N!) \binom{N}{c_{11}} (N!) \\ &\quad \cdots \binom{N}{c_{k-1\ k-1}} (N!) \\ &= \sum_{s=0}^{N} \left[\binom{N}{s} (N!) \right]^k. \end{aligned} \tag{8}$$

We summarize the above result into the following theorem:

Theorem 1: The permutation capacity for an $N \times N$ k-wavelength WDM network with limited wavelength conversion under λ_i being converted to λ_{i+1} is

$$\sum_{s=0}^{N} \left[\binom{N}{s} (N!) \right]^k.$$

D. Network Cost Under Different Models

In this section, we analyze the network cost of a WDM switching network under different models. As we discussed earlier, light branches, light combiners, and optical cross-connect elements such as SOA gates are needed to construct a crossbar-like WDM switching network. Also, the LWCs or

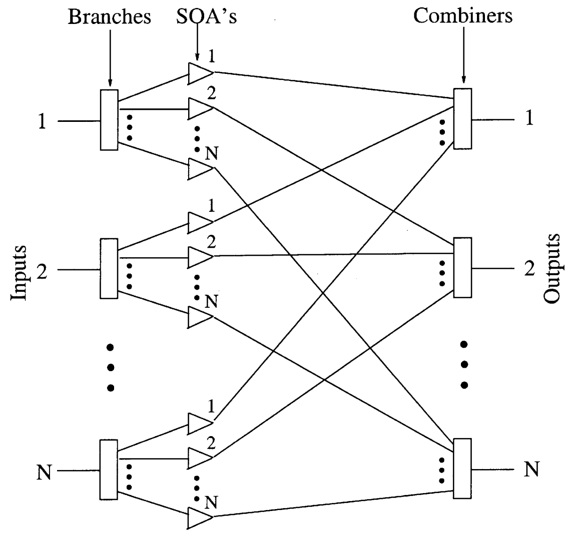

Fig. 3. $N \times N$ one-wavelength switching network.

FWCs are required to implement the PLW model or PAW model. While light branches and light combiners are passive optical components, SOA gates and WCs are active devices. Such passive components are made of glass, hence, inexpensive, but the active devices are not. Thus, we characterize the cost of a WDM network by the number of optical cross-connect elements in addition to the number of WCs. For simplicity, we will refer to optical cross-connect elements as *crosspoints*, which is a well-known term representing the hardware cost for traditional switching circuits. In the following, we analyze the network cost of a nonblocking WDM under these three models, i.e., PSW, PLW, and PAW models.

1) Number of Crosspoints: For an $N \times N$ k-wavelength WDM switching network under the PSW model, the number of crosspoints is kN^2. In fact, an $N \times N$ k-wavelength WDM network under this model is equivalent to k parallel $N \times N$ one-wavelength networks (see Fig. 2) as we pointed out earlier. Each of these $N \times N$ one-wavelength networks may be implemented as shown in Fig. 3. In such a network, each input signal first passes through a $1 \times N$ light branch. The signals then pass through an array of N^2 SOA gate elements, and are then recombined in $N \times 1$ light combiners and sent to the N outputs. Note that at the input side of a combiner, only one of the channels actually carries signal, because only a selected signal can be directed to one of the combiners by means of turning on or off the SOA gates, which is controlled by circuits.

For an $N \times N$ k-wavelength WDM network under the PLW model with wavelength degree two, the number of crosspoints is $2kN^2$, since any of the Nk wavelengths at the input side may be connected to two wavelength groups (each wavelength group contains N wavelengths) at the output side. An example when $N = 2$, $k = 3$, and with wavelength degree two is shown in Fig. 4(a). Finally, the number of crosspoints is k^2N^2 for an $N \times N$ k-wavelength network under the PAW model, since any of the Nk wavelengths at the input side may be connected to any of the Nk wavelengths at the output side. An example when $N = 2$, $k = 3$, and with full wavelength conversion is shown in Fig. 4(b).

2) Number of WCs: Clearly, for an $N \times N$ k-wavelength WDM network under the PSW model, no converter is needed. However, for a WDM network of the same size under the PLW model, Nk LWCs are required as shown in Fig. 4(a). Similarly, under the PAW model, Nk FWCs are required as shown in Fig. 4(b). These Nk WCs can be placed immediately after the light combiners to convert a source wavelength to a possibly different wavelength, which is then multiplexed into an output fiber link.

E. Comparison of Different Models

We summarize the network performance in terms of the permutation capacity and network cost measured by the number of crosspoints as well as the number of WCs required for nonblocking WDM networks under different models in Table I. As we expected, a nonblocking WDM network under the weakest model (PSW model) has lower cost than that of a stronger model (PLW model), which in turn has lower cost than that of the strongest model (PAW model). Although under either the PLW model or the PAW model, each model needs the same number of LWCs or FWCs, note that FWCs are much more expensive than LWCs. Obviously, there exist cost–performance tradeoffs between these models. As we will see in the next section, such cost analysis leads us to construct more efficient nonblocking multistage networks where the number of crosspoints can be greatly reduced.

III. Multistage WDM Switching Networks

In this section, we investigate how to use a multistage network to reduce the network cost in terms of crosspoints. We consider the well-known $N \times N$ three-stage network called $v(m, n, r)$ network [17] as shown in Fig. 5, which has r $(n \times m)$ switches in the input stage, m $(r \times r)$ switches in the middle stage, and r $(m \times n)$ switches in the output stage with $N = nr$ and $m \geq n$. In general, a multistage network can have any odd number of stages with the middle stage switches being built in a recursive fashion of the three-stage networks. A critical issue in designing such a network is how to ensure that the network is nonblocking, and in the meantime, minimize the number of middle-stage switches (m), hence, reduce the number of crosspoints. In the traditional electronic domain, for a $v(m, n, r)$ three-stage network, Clos [17] showed that if $m \geq 2n - 1$, the network is nonblocking, and if $m \geq n$, the network is rearrangeable. In the case of nonblocking, for any legitimate connection request from an idle input port to an idle output port, it is always possible to provide a connection path through the network to satisfy the request without disturbing any existing connections. In the case of rearrangeable, it is again always possible to provide a connection path through the network to satisfy the request, but other existing connections may be rearranged to some other paths. In an all-optical WDM network, it is desirable for a WDM switch to be nonblocking, since it is difficult to buffer, or store the optical signal. Therefore, unlike the electronic case, we only consider the nonblocking WDM multistage network in this paper. Similar to the notation of $v(m, n, r)$ for an electronic three-stage network, we use $v_k(m, n, r)$ to represent a k-wavelength WDM three-stage network. Notice that there is

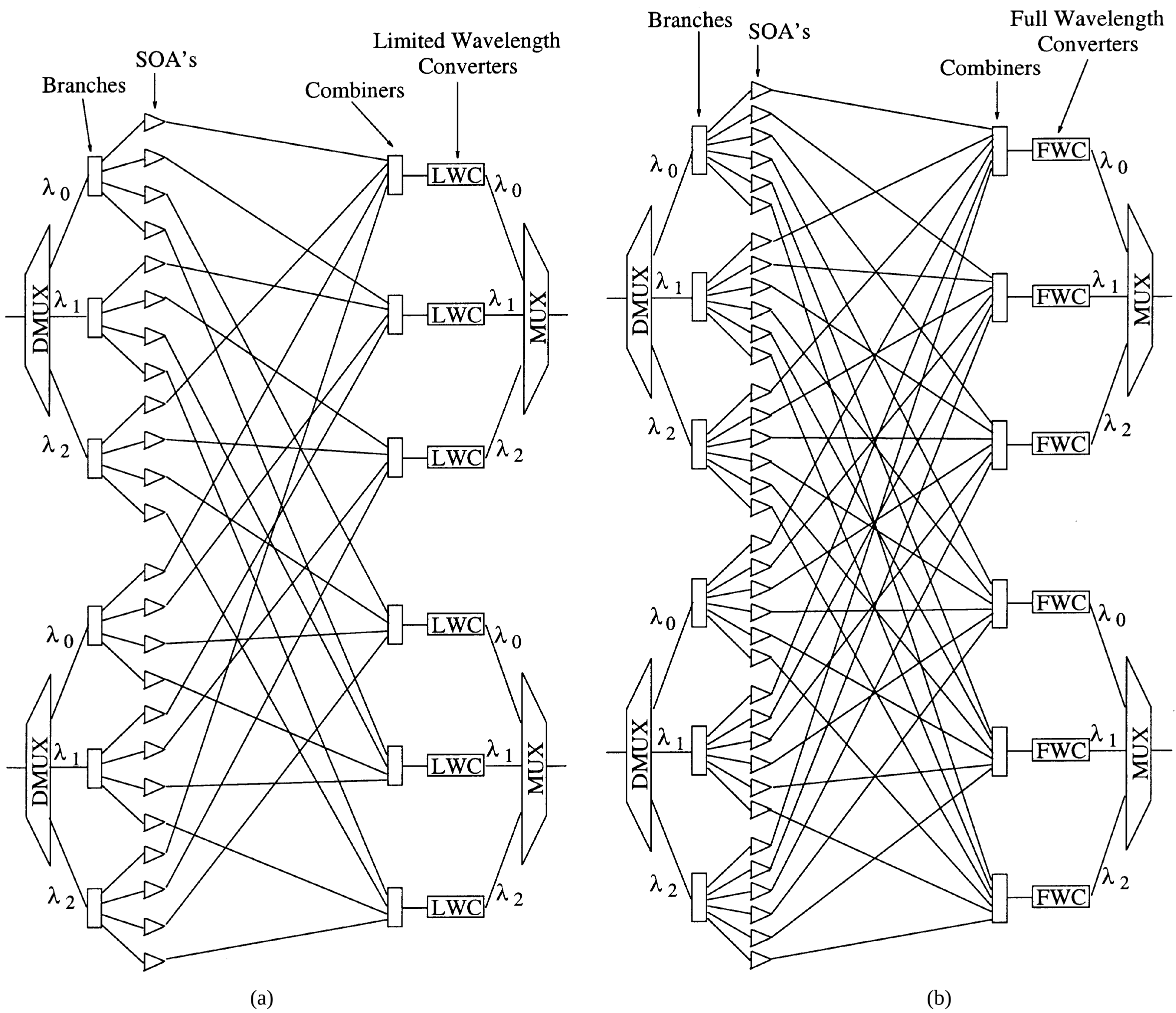

Fig. 4. Example of an $N \times N$ k-wavelength WDM network when $N = 2$ and $k = 3$. (a) PLW model. (b) PAW model.

TABLE I
COMPARISON OF WDM SWITCHING NETWORKS UNDER DIFFERENT MODELS

Model	Permutation Capacity	# Crosspoints	# Converters
PSW	$(N!)^k$	kN^2	0
PLW (λ_i to λ_{i+1})	$(N!)^k \sum_{s=0}^{N} \binom{N}{s}^k$	$2kN^2$	kN LWC's
PAW	$(Nk)!$	k^2N^2	kN FWC's

exactly one link between every two consecutive stage switches, but such links are fiber links with k channels carrying on each link. Notice also that while the switches in an electronic multistage network are all based on crossbar switches, those in a WDM multistage network can be different (i.e., under different models such as PSW, PLW, and PAW), which introduces a major challenge in the analysis.

A. Construction Methods of Multistage WDM Networks

For an $N \times N$ k-wavelength WDM crossbar-like switching network, we have proposed three different models, i.e., PSW, PLW, and PAW models, in Section II. When constructing a multistage WDM network, the overall network can be any one of the three models, but the switches inside the multistage network do not always have to choose the same model as that of the whole network. Thus, there are many different ways to build

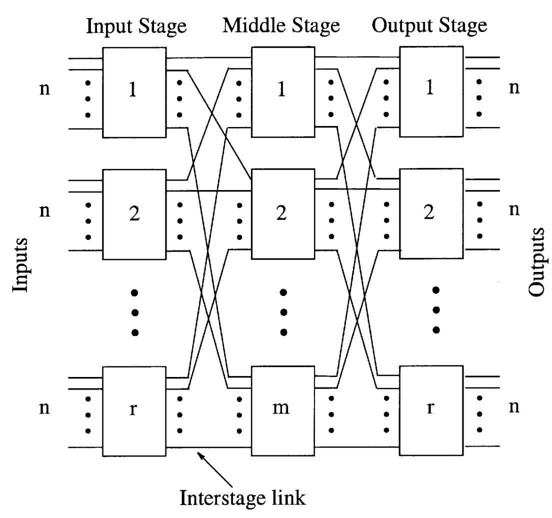

Fig. 5. Three-stage switching network.

a multistage WDM network. However, based on the results obtained for the crossbar-like WDM networks, the PSW model has the lowest network cost and the smallest permutation capacity, while the PAW model has the highest network cost but greatest permutation capacity. Therefore, when constructing a

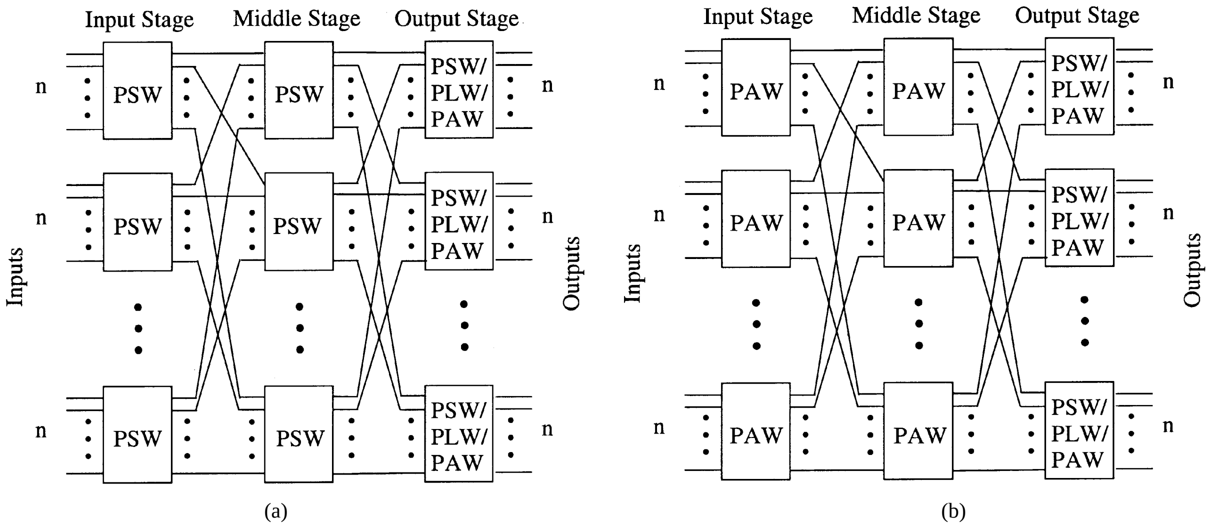

Fig. 6. Two construction methods for a $v_k(m, n, r)$ network. (a) PSW-dominant construction. (b) PAW-dominant construction.

multistage WDM network, it is natural to choose these two extreme models for the switches. Now we propose two construction methods for a $v_k(m, n, r)$ network.

1) For a PSW-dominant construction method, the input-stage and middle-stage switches adopt the PSW model, and the switches in the output stage adopt one of the PSW, PLW, or PAW models, which in turn determines the overall model of the $v_k(m, n, r)$ network. This construction method is illustrated in Fig. 6(a).
2) For a PAW-dominant construction method as shown in Fig. 6(b), the switches in both input and middle stages adopt the PAW model and output stage switches use the PSW, PLW, or PAW model according to whatever model that the $v_k(m, n, r)$ network will be.

It should be noticed that these two construction methods for a three-stage WDM network can also be used to implement a general multistage WDM network, which can have any odd number of stages with the middle-stage switches being built in a recursive fashion as the three-stage network. For such a general multistage WDM network, all switches except those in the last stage adopt the PSW or PAW model according to its corresponding construction method. In the following, we analyze the nonblocking condition as well as the network cost for both construction methods in a three-stage WDM network.

B. Nonblocking Condition for the PSW-Dominant Construction

In a three-stage WDM switching network under either the PSW, PLW, or PAW model and adopting the PSW-dominant construction method, a connection with any input wavelength λ_i can be realized using the same wavelength λ_i in the first two stages, and then realized in the third stage under the PSW, PLW, or PAW model, respectively. Therefore, we can simply ignore other wavelengths and consider permutation routing using only wavelength λ_i. Consequently, the results of nonblocking condition of a WDM network is reduced to the case of a traditional electronic switching network. We can establish the following theorem.

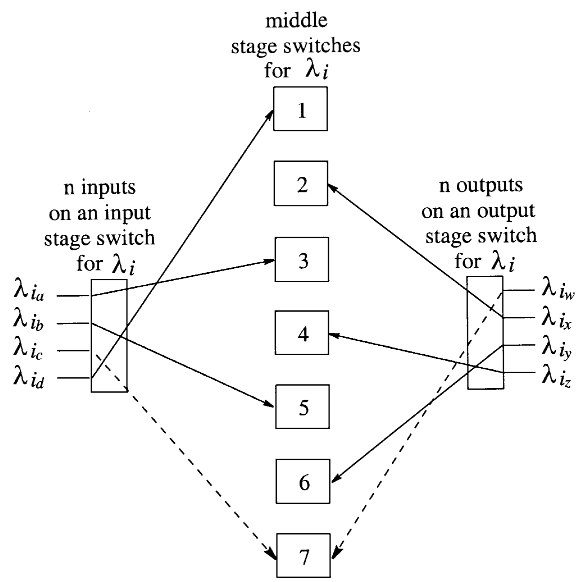

Fig. 7. Connection request from input wavelength λ_{i_c} to output wavelength λ_{i_w} in a three-stage network.

Theorem 2: A $v_k(m, n, r)$ network adopting the PSW-dominant construction method is nonblocking if $m \geq 2n-1$.

Proof: We consider a worst-case connection request. As shown in Fig. 7, suppose that wavelength λ_{i_c} (one of the n input λ_i's) of an input-stage switch asks for connection to wavelength λ_{i_w} (one of the n output λ_i's) of an output-stage switch. Note that, at most, $(n-1)$ wavelengths other than λ_{i_c} on the particular input stage switch for λ_i can be busy, i.e., such $(n-1)$ input wavelengths are being connected to some $(n-1)$ output wavelengths. Also note that, at most, $(n-1)$ wavelengths other than λ_{i_w} on the particular output-stage switch for λ_i can be busy, i.e., such $(n-1)$ output wavelengths are being connected by some $(n-1)$ input wavelengths. In the worst case, each of these $2(n-1)$ existing connections uses a separate middle-stage switch. Therefore, we need one more middle-stage switch to make the new connection from λ_{i_c} to λ_{i_w}, i.e., the number of middle stage switches needed for nonblocking is $m \geq (n-1)+(n-1)+1 = 2n-1$. $\square$

C. Nonblocking Condition for the PAW-Dominant Construction

For a multistage WDM network under either PSW, PLW, or PAW model and using the PAW-dominant construction method, we have the following theorem. Surprisingly, the nonblocking condition is the same as that of the PSW-dominant construction method.

Theorem 3: A $v_k(m,\, n,\, r)$ network adopting the PAW-dominant construction method is nonblocking if $m \geq 2n - 1$.

Proof: Under the PAW-dominant construction, there is no restriction on how a wavelength can be connected in the first two stages. Thus, an input wavelength can be converted to any output wavelength on input and middle-stage switches. While such a construction method increases complexity, it also gives us flexibility to utilize the middle-stage switches. This leads us to the following definition.

A middle-stage switch is unavailable if and only if all the k wavelengths of the fiber link, which connects an input-stage switch (or an output-stage switch) to a middle-stage switch, are being used by k existing connections. Consider connecting a wavelength λ_{p_i} of an input fiber at input-stage switch i to a wavelength λ_{q_j} of an output fiber at output-stage switch j. Note that at most $(nk - 1)$ wavelengths on input-stage switch i can be busy, i.e., at most

$$\left\lfloor \frac{(nk-1)}{k} \right\rfloor = \left\lfloor \left(n - \frac{1}{k}\right) \right\rfloor$$

middle-stage switches are not available for λ_{p_i}. Also note that, at most, $(nk-1)$ wavelengths on the output-stage switch j can be busy, i.e., at most

$$\left\lfloor \frac{(nk-1)}{k} \right\rfloor = \left\lfloor \left(n - \frac{1}{k}\right) \right\rfloor$$

middle-stage switches are not available for λ_{q_j}. In the worst case, those unavailable middle-stage switches for λ_{p_i} and λ_{q_j} have no common switches in between. Thus, to ensure the new connection from λ_{p_i} to λ_{q_j} unblocked, we need one more middle-stage switch beyond those unavailable switches, i.e.,

$$m \geq 2 \left\lfloor \left(n - \frac{1}{k}\right) \right\rfloor \; + 1 = 2(n-1) + 1 = 2n - 1.$$

□

D. Network Cost and Comparison

Since the permutation capacity of a given model remains the same under either a crossbar-like construction or a multistage construction, we will focus on the analysis of multistage WDM network cost in this subsection. Similarly, we argue that given an overall WDM switching network model (PSW, PLW, or PAW), although it can be constructed under either the PSW-dominant method or the PAW-dominant method, the permutation capacity does not change at all. From *Theorem 2* and *Theorem 3*, the numbers of middle-stage switches required for nonblocking in the PSW-dominant construction and the PAW-dominant construction are the same. Since a switch under the PAW model has more crosspoints than that of a switch under the PSW model, a WDM switching network using the PAW-dominant construction method under any one of the three models costs more than that using the PSW-dominant construction method, in terms of the number of crosspoints. Furthermore, the PAW-dominant method needs extra WCs, which are very expensive. Therefore, we conclude that it is better to construct a multistage WDM network using the PSW-dominant method.

In the following, we calculate the cost for a $v_k(m,\, n,\, r)$ network adopting the PSW-dominant construction method to see how much we can save compared to that of a crossbar-like $N \times N$ k-wavelength WDM network. The cost is analyzed in terms of the number of crosspoints (SOAs) and WCs required under each model. In calculating the number of crosspoints, we use the optimal values for n and r to minimize the cost. For a PSW $v_k(m,\, n,\, r)$ network, all the three stages are now under the PSW model. The number of crosspoints is given by

$$\begin{aligned} \#\text{crosspoints} &= r \cdot knm + m \cdot kr^2 + r \cdot kmn \\ &= km\left(2nr + r^2\right) \\ &= k(2n-1)\left(2N + \frac{N^2}{n^2}\right). \end{aligned} \tag{9}$$

For a given value of $N = nr$, the minimum number of crosspoints occurs when $d(\#\text{crosspoints})/dn = 0$, which gives

$$2n^3 - (n-1)N = 0. \tag{10}$$

As N approaches large values, (10) can be approximated by

$$2n^2 - N \approx 0. \tag{11}$$

This equation gives the optimal value for n as $n = \sqrt{N/2}$. Replacing n by its optimal value in (9), we get

$$\begin{aligned} \#\text{crosspoints} &= 4kN\left(2\sqrt{N/2} - 1\right) = k\left(4\sqrt{2}N^{3/2} - 4N\right) \\ &= O\left(kN^{3/2}\right). \end{aligned} \tag{12}$$

For a PLW $v_k(m,\, n,\, r)$ network, the first two stages are under the PSW model and the last stage is under the PLW model. For convenience of comparison, we adopt the same model as we used in the previous section, i.e., wavelength degree $w = 2$. The number of crosspoints for such a PLW $v_k(m,\, n,\, r)$ network is given by

$$\begin{aligned} \#\text{crosspoints} &= r \cdot knm + m \cdot kr^2 + r \cdot 2kmn = km\left(3nr + r^2\right) \\ &= k(2n-1)\left(3N + \frac{N^2}{n^2}\right). \end{aligned} \tag{13}$$

Similar to the case of PSW $v_k(m,\, n,\, r)$ networks, when N approaches large values, we can obtain the optimal value for n as $n = \sqrt{N/3}$. From (13), we calculate the number of crosspoints as

$$\begin{aligned} \#\text{crosspoints} &= 6kN\left(2\sqrt{N/3} - 1\right) = k\left(4\sqrt{3}N^{3/2} - 6N\right) \\ &= O\left(kN^{3/2}\right). \end{aligned} \tag{14}$$

TABLE II
COST COMPARISON OF THREE-STAGE AND CROSSBAR WDM NETWORKS UNDER DIFFERENT MODELS (CB: CROSSBAR, TS: THREE-STAGE, PLW: WITH WAVELENGTH DEGREE TWO)

Model	Permutation Capacity	# Crosspoints	# Converters
PSW/CB	$(N!)^k$	kN^2	0
PSW/TS	$(N!)^k$	$O\left(kN^{3/2}\right)$	0
PLW/CB	$(N!)^k \sum_{s=0}^{N} \binom{N}{s}^k$	$2kN^2$	kN LWC's
PLW/TS	$(N!)^k \sum_{s=0}^{N} \binom{N}{s}^k$	$O\left(kN^{3/2}\right)$	kN LWC's
PAW/CB	$(Nk)!$	k^2N^2	kN FWC's
PAW/TS	$(Nk)!$	$O\left(k^{3/2}N^{3/2}\right)$	kN FWC's

For a PAW $v_k(m, n, r)$ network, the input stage and middle stage are under the PSW model, and the output stage is under the PAW model. Similarly, the calculation of the number of crosspoints is given by

$$\begin{aligned}\#\text{crosspoints} &= r \cdot knm + m \cdot kr^2 + r \cdot k^2mn \\ &= km\left[(k+1)nr + r^2\right] \\ &= k(2n-1)\left[(k+1)N + \frac{N^2}{n^2}\right].\end{aligned}$$

When N approaches large values, the optimal value for n under the PAW model is $n = \sqrt{N/(k+1)}$. Hence, we obtain the number of crosspoints under this model as the following:

$$\begin{aligned}\#\text{crosspoints} &= 2k(k+1)N\left[2\sqrt{N/(k+1)} - 1\right] \\ &= O\left(k^{3/2}N^{3/2}\right).\end{aligned} \tag{15}$$

As for the number of WCs under the PSW-dominant construction method, since there are no converters in the first two stages, we only need to consider the switches in the last stage. For a PSW $v_k(m, n, r)$ network, the output stage is also under the PSW model, thus, no converters are needed. For a PLW $v_k(m, n, r)$ network, the output stage is under the PLW model, and the number of LWCs required is $r \cdot kn = kN$, since the LWCs can be placed at the output side [see Fig. 4(a)]. Similarly, the number of FWCs required for a PAW $v_k(m, n, r)$ network is also kN.

We summarize these three different network models under either crossbar (CB) or three-stage (TS) construction in Table II. Clearly, a multistage WDM network under any of the three models has significantly fewer crosspoints than that of a crossbar-like WDM network, yet the permutation capability of a $v_k(m, n, r)$ network remains the same as that of a crossbar network. In Fig. 8, we plot the permutation capacity for $v_k(m, n, r)$ networks under the PSW, PLW, and PAW models for network size of 8×8 and the number of wavelengths from three to eight.

From Table II and Fig. 8, we see that among the three different multistage WDM networks, a PSW $v_k(m, n, r)$ has the lowest network cost but the least permutation capacity, while a PAW $v_k(m, n, r)$ network has the highest network cost but the greatest permutation capacity. This represents the cost–performance tradeoffs among these network designs. Fig. 8 also

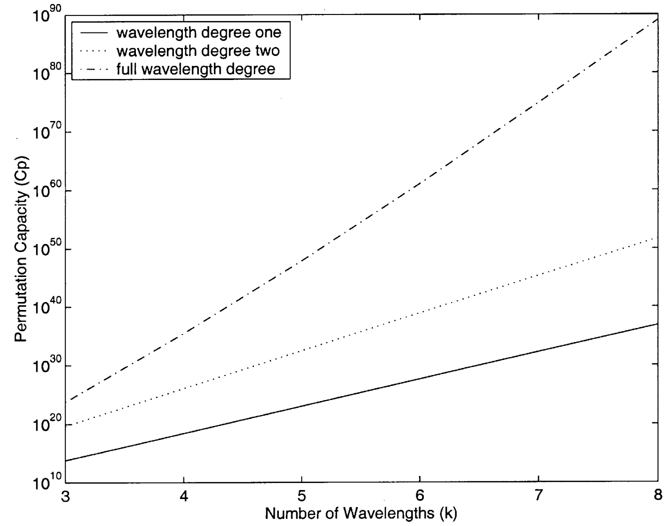

Fig. 8. Comparison of permutation capacity for a WDM switching network with eight nodes and three to eight channels.

shows that, compared to the PSW model (i.e., with no wavelength conversion), the network performance is significantly improved with the limited-range wavelength conversion. For example, for an $N = 10$, $k = 4$ WDM network under the PLW model with wavelength degree $w = 2$, the permutation capacity is $\sum_{s=0}^{10} \binom{10}{s}^4 \cong 8.35 \times 10^9$ times as that of the PSW model.

IV. CONCLUSION

The objective of this paper was to provide efficient designs for nonblocking WDM permutation switching networks. We have presented three crossbar-like WDM network models, PSW, PLW, and PAW. We have analyzed the performance of nonblocking WDM networks in terms of permutation capacity as well as the network cost measured by the number of optical cross-connect elements (e.g., SOAs) and the number of optical WCs under these proposed network models. In particular, we have given a systematic approach to analyzing the permutation capacity under the more complex PLW model. This approach can be extended to study other PLW models with higher wavelength degrees as well. We have, furthermore, proposed two construction methods, namely, PSW-dominant and PAW-dominant methods, to build nonblocking multistage WDM networks. We have obtained the nonblocking conditions in terms of the number of middle-stage switches for the three-stage WDM switching networks $(v_k(m, n, r))$ under both construction methods. Using the PSW-dominant construction method, we have demonstrated that the network cost has been greatly reduced compared to crossbar-like WDM networks. Our results have also indicated that the PSW and PAW models represent cost–performance tradeoffs in designing both crossbar-like and $v_k(m, n, r)$ networks. In addition, we have shown that, even with the very limited wavelength conversion capability (e.g., wavelength degree two), the performance of a WDM network has been significantly improved. Finally, we believe that PSW-dominant is a better choice to implement $v_k(m, n, r)$-type networks.

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