DYNAMIC FORMULATION OF CO-PRIME ARRAY FOR DOA ESTIMATION

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ABSTRACT

Among various sparse array techniques, co-prime array is found to be more attractive because its higher DoF with a smaller number of sensing elements. However, because of its specific physical non-uniform linear structure, it would be inconvenient and costly to implement in the presence of varying detection scenarios. In this paper, we propose to exploit the current ULA to dynamically formulate a co-prime array structure, which takes advantage of the properties of co-prime array while reducing the energy consumption and operational cost. Rather than being constrained with the specific physical structure, our scheme can be cost effectively applied in existing systems.

***Index Terms*—** Co-prime array, dynamic array, difference co-array, DOA estimation.

1. INTRODUCTION

Array signal processing techniques are often applied to estimate the direction-of-arrival (DOA) of sources. Generally, a uniform linear array (ULA) with $N+1$ elements can identify N sources, and has a degree of freedom (DoF) of N. To detect a large number of sources, it requires the number of array elements $N+1$ to be big, which would incur a high system cost and energy consumption. The estimation accuracy also reduces when there are a big number of sources.

Recently, sparse array constructions such as minimum redundancy arrays (MRAs) [1] [2], nested arrays [3] and co-prime arrays [4] have attracted a lot of attentions. These sparse arrays use their difference co-arrays to generate a larger-size virtual array to increase DoF. Because there is no closed-form expression for the geometry configuration and no approximate DoF for MRAs, it is hard to design the MRA system in most cases. Co-prime array becomes more attractive because of its high efficiency and simplicity. Spatial smoothing technique [5] [6] and sparse reconstruction technique [7] are usually used with co-prime array for DOA estimation. Despite the potential of co-prime array, the physical construction of an array based on co-prime infrastructure needs time and will not likely to be seen very soon. In addition, there is a tradeoff between constructing a large array at high cost and meeting the changing need of DoF in different application scenarios with minimum cost.

Rather than depending on physical co-prime array, we propose to exploit the co-prime method to select array elements in ULA to form co-prime arrays. This not only helps to achieve the largest DoF, but also allows for energy conservation from the selective use of array elements while deactivating RF chains corresponding to the remaining elements.

The co-array method that is usually used in co-prime array will generate holes in the virtual array, that is, i.e., some virtual array elements are missing. To solve this problem, Pal *et. al* in [8] consider using an extended co-prime construction, which doubles the number of sensors of one sub-array to achieve a longer hole-free virtual array. Other works use proportional frequencies [9], compressive sensing [10] and temporal signal coherence (TCP) in moving co-prime arrays [11] to fill holes in the virtual array. These schemes require different specific non-uniform physical array structures. In contrast, we apply two methods to address the problem: the use of complement sub-array along with co-prime sub-array, and the intelligent adaptive operation that can select fewest elements in ULA to meet variable detection requirements.

The aim of our work is to find an approach that can dynamically activate sensors in the fixed basic ULA configuration to formulate the longest hole-free virtual array and therefore obtain the largest achievable DoF for higher implementation flexibility and performance quality. Specifically, based on a physical N_0-ULA, we can achieve up to N_0 DoFs while keeping only $O(\sqrt{N_0})$ sensors active in the detection period. Our method may be extended to form different co-prime arrays based on the actual detection need while minimizing the total number of elements activated thus reducing the cost.

The remainder of the paper is organized as follows. In section 2, we review the concept of traditional co-prime arrays. Section 3 introduces our proposed dynamic array formulation including the basic array configuration, the sensor selection scheme and the adaptive operation mode. We provide performance studies with simulation results in Section 4 and conclude the work in Section 5.

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2. CO-PRIME ARRAY STRUCTURES

A conventional co-prime array [4] shown in Fig. 1 consists of two uniform linear sub-arrays with the separation Md and Nd respectively. There are N sensors in the first sub-array and M sensors in the second sub-array. M and N are co-prime integers, i.e., $gcd(M, N) = 1$, and d is the unit of inter-element spacing. To avoid spatial aliasing, d is typically set to $\lambda/2$, where λ is the wavelength of impinging narrowband signals. Since the first sensors of the two uniform linear sub-arrays are co-located, the total number of sensors in conventional co-prime array is $M + N - 1$.

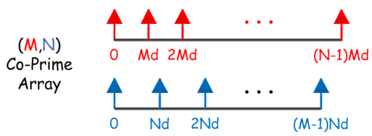


Fig. 1: Conventional co-prime array

However, one major problem of the conventional co-prime geometry is that there exist missing elements which are often called "holes" in the difference co-array. For example, the difference co-array of a $M = 3$, $N = 4$ form the set $[-9d, +9d]$ except $\pm 7d$. These holes will decrease the length of the ULA segment in the virtual array and therefore effect the size of manifold matrix in spatial smoothing. As a result, a (M, N) conventional co-prime array usually can not obtain MN freedoms.

To deal with the hole problem, extended co-prime array was proposed in [8]. As shown in Fig. 2, the number of sensors in the second ULA sub-array is doubled to enlarge the size of ULA segment in the difference co-array, and thus the virtual array has a ULA segment with the length of $(2MN + 2M - 1)$. Then after the spatial smoothing [5], the (M, N) extended co-prime array can still offer MN DoFs.

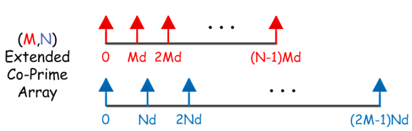


Fig. 2: Extended co-prime array

3. DYNAMIC ARRAY FORMULATION

In this section, we first provide the motivation why we want to explore the use of dynamic array (DA). We then introduce the basic configuration of our proposed dynamic array, the co-array method and the scheme we apply to select the sensors in ULA to activate dynamically. Finally we discuss an improved operation mode that can be implemented under different circumstances.

3.1. Motivation

Recent studies on co-prime arrays show very promising results. However, there does not exist physical co-prime array currently. Also, as different co-prime structures will form arrays with different DoFs and sizes, it would be inconvenient and costly to build specific physical non-uniform linear arrays in the presence of varying detection scenarios. Departing from existing studies which assume the existence of non-uniform linear array and work under a specific co-prime array structure, we propose to dynamically form co-prime arrays based on the universal ULA configuration. This allows for the harvesting of gains from co-prime array techniques to significantly improve the performance of the current sensing systems without additional cost and waiting of new array hardware. Our studies indicate that the maximum possible DoF of a dynamic array under a given ULA configuration can maintain the same as that of ULA. Furthermore, as only a small part of the sensors and their RF chains in the ULA stay active during the detection, it can potentially save a lot of energy. Especially this energy cost would be huge if a large sensing array is applied for continuous environment monitoring.

3.2. Basic Configuration

Our proposed dynamic array is formed based on the universal N_0-ULA configuration. Sensors in the ULA can be dynamically selected to be activated based on the application need and detection quality.

The active sensors form two sub-arrays, a (M, N) extended co-prime sub-array and a complement sub-array. The extended co-prime sub-array has the same structure as we introduced in the previous section. The complement sub-array is a short dense ULA located at the end of array. It is applied to fill all remaining "holes" in the virtual array generated by the (M, N) extended co-prime sub-array so that the continuous part in the virtual array can be maximized which is from $-(2M - 1)N$ to $(2M - 1)N$. It can be proved that the minimum number of required sensors in the complement sub-array is $M - 1$. Therefore, the number of selected active sensors in the universal N_0-ULA is $2M + N - 1$ from co-prime sub-array plus $M - 1$ from complement sub-array which is $3M + N - 2$ in total.

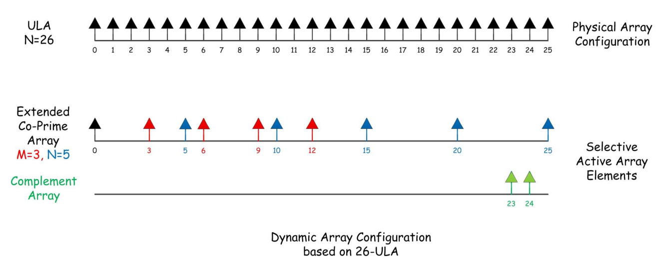


Fig. 3: basic dynamic array configuration

Fig. 3 shows an example of dynamic array based on 26-

ULA. It consists of an extended co-prime array with $M = 3, N = 5$ and a complement array with 2 extra sensors. As a result of applying the co-array method, its DoF can reach 25 with only 12 sensors active, which conserves more than half of energy consumption.

3.3. Difference Co-array Method and DOA Estimation

In the sparse array signal processing, to achieve a given number of DOFs with fewer physical sensors, a virtual array generated from the difference co-array is usually applied to substitute for the physical array. The difference co-array can be achieved from the correlation of the received data.

Assuming D narrowband sources with powers $[\sigma_1^2 \cdots \sigma_D^2]$ impinge on the array from directions $[\theta_1\ \theta_2\ \cdots\ \theta_D]$, the signals received at the array elements can be expressed as

$$\mathbf{x}[k] = \mathbf{A}\mathbf{s}[k] + \mathbf{n}[k] \tag{1}$$

where $\mathbf{A}$ is the array manifold matrix of the form

$$\mathbf{A} = [\mathbf{a}(\theta_1)\ \mathbf{a}(\theta_2)\ \cdots\ \mathbf{a}(\theta_D)] \tag{2}$$

and

$$\mathbf{a}(\theta_i) = [e^{j\frac{2\pi d}{\lambda} l_1 sin\theta_i}, \cdots, e^{j\frac{2\pi d}{\lambda} l_{3M+N-2} sin\theta_i}]^T \tag{3}$$

where $[l_1, l_2, \cdots, l_{3M+N-2}]$ represents the locations of $3M+N-2$ selected active sensors. $\mathbf{s}[k] = [\mathbf{s}_1(k)\ \cdots\ \mathbf{s}_D(k)]^T$ denotes the k^{th} snapshot of the source signal vector, and the noise vector $\mathbf{n}[k]$ is assumed to be temporally and spatially white and uncorrelated from the source.

In array signal processing, the difference co-array is formed naturally in the computation of the correlation of the received signal,

$$\mathbf{R}_{xx} = E[\mathbf{x}(k)\mathbf{x}(k)^H] = \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \sigma_D^2\mathbf{I}, \tag{4}$$

where $\mathbf{R}_{ss}$ is the source autocorrelation matrix, with

$$\mathbf{R}_{ss} = diag([\sigma_1^2\ \sigma_2^2\ \cdots\ \sigma_D^2]. \tag{5}$$

In practice, the autocorrelation matrix can be computed by the following sample average

$$\widehat{\mathbf{R}}_{xx} = \frac{1}{L}\sum_{k=1}^{L} \mathbf{x}(k)\mathbf{x}(k)^H, \tag{6}$$

where L is the total number of snapshots. In order to build the new model using the difference co-array as the new array manifold matrix, we vectorize the autocorrelation matrix and get

$$\mathbf{z} = vec(\mathbf{R}_{xx}) = \mathbf{B}\cdot\mathbf{p} + \sigma_n^2 vec(\mathbf{I}), \tag{7}$$

where $\mathbf{B} = [\mathbf{B}_{\theta_1}\ \mathbf{B}_{\theta_2}\ \cdots \mathbf{B}_{\theta_D}] = \mathbf{A}^* \odot \mathbf{A}$ (Khatri-Rao product of $\mathbf{A}^*$ and $\mathbf{A}$) and $\mathbf{p} = [\sigma_1^2\ \sigma_2^2\ \cdots\ \sigma_D^2]^T$.

We consider the vector $\mathbf{z}$ to be the new received data, $\mathbf{B}$ to be the new array manifold matrix and $\mathbf{p}$ to be the new source signal.

As there exist redundant and out-of-order elements in the vector, we have to drop and reorder some elements to rebuild $\mathbf{z}$ to form a new vector $\mathbf{z}'$ so that its corresponding $\mathbf{B}'$ has the same expression as the manifold of the ULA segment in the virtual array. The rebuilt vector $\mathbf{z}'$ can be expressed as

$$\mathbf{z}' = \mathbf{B}'\cdot\mathbf{p} + \mathbf{n}'. \tag{8}$$

Since the new source signals $\mathbf{p}$ are no longer incoherent, we use spatial smoothing technique [5] to build the rank of a positive semi-definite matrix from this new model. We divide the new received data vector $\mathbf{z}'$ into multiple vectors $\mathbf{z}_i^{'}$ so that its corresponding virtual ULA array is divided into multiple overlapping sub-arrays. Then we compute the autocorrelation-like matrix of each divided received data vector $\mathbf{z}_i^{'}$

$$\mathbf{R}_{zi} \triangleq \mathbf{z}_i'\mathbf{z}_i'^H \tag{9}$$

Taking the average of the autocorrelation matrices of all sub-arrays, we can get the final spatial smoothed matrix $\mathbf{R}_{zz}$ as

$$\mathbf{R}_{zz} = \frac{1}{DOF}\sum_{i=1}^{DOF}\mathbf{R}_{zi} \tag{10}$$

where DOF equals the number of sub-arrays and denotes the maximum number of detectable sources.

Finally, we can accomplish DOA estimation by applying MUltiple SIgnal Clasification (MUSIC) algorithm [12] on $\mathbf{R}_{zz}$.

3.4. Selection of (M,N)

By applying the difference co-array method and spatial smoothing technique, the length of available virtual array segment can approach to the length of the physical ULA. However, to achieve the largest DoF with minimal number of sensors active in our dynamic array, the selection of appropriate co-prime array factors (M, N) is a critical and crucial challenge. In this section, we provide a scheme to find the optimal (M, N) from a given N_0-ULA structure.

In our proposed dynamic array, since there exist no holes in the virtual array, the number of DoF equals to the array aperture. To achieve the largest DoF, we need to select a pair of co-prime number M and N so that the position of the last sensor in the extended co-prime array can be as close to $(N_0 - 1)d$ as possible. Meanwhile, to save more energy consumption, we want the number of selected active sensors to be as small as possible. The problem can be formulated as a two-objective optimization problem:

$$\begin{aligned} \max_{M,N\in N^*} \quad & A = (2M-1)N \\ \min_{M,N\in N^*} \quad & |T| = 3M + N - 2 \\ \text{subject to} \quad & (2M-1)N \leq N_0 - 1 \\ & gcd(M,N) = 1 \end{aligned} \tag{11}$$

We first do not take the integrality and the coprimality of (M, N) into consideration, then (11) becomes:

$$\begin{aligned} \min_{M,N\in R^+} \quad & |T| = 3M + N - 2 \\ \text{subject to} \quad & (2M-1)N = N_0 - 1 \end{aligned} \tag{12}$$

Equation (12) has optimum solution where

$$\begin{aligned} M &= \sqrt{\frac{N_0-1}{6}} + \frac{1}{2} \\ N &= \frac{\sqrt{6(N_0-1)}}{2} \end{aligned} \tag{13}$$

and the minimal number of sensors $|T|$

$$|T| = 3M + N - 2 = \sqrt{6(N_0-1)} - \frac{1}{2} \tag{14}$$

We can then test the coprimality of the integers around the optimum M and N in (13) and find the optimal solution to the original problem (11).

Finally, our dynamic array can be formulated as the union of the optimal (M, N) co-prime sub-array and the complement sub-array. To complete the DOA estimation using MUSIC algorithm, we just need to activate the selected sensors in the universal N_0-ULA during the detection period.

3.5. Adaptive Operation Mode

Our scheme above can maximize the DoF of the virtual array with a given ULA. Rather than being restricted by the physical array structure, a benefit of dynamic array formulation is that we can select sensors and generate different sizes of virtual arrays based on the application need.

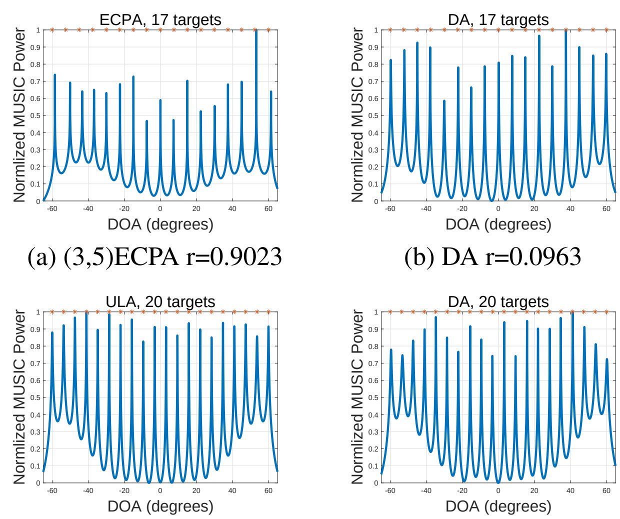
We can adjust the number of active sensors dynamically according to the actual application needs and detection quality. In a long-term monitoring scenario, once suspecting with targets, we can use our sensor selection scheme to get an array with the largest DoF at the beginning. If we find that the number of targets is far smaller than our current DoF, we reduce the number of sensors to activate in the tracking mode. This could be realized by reducing N_0 in (11). This process can continue until the detection performance approaches the predetermined threshold. Later when the number of targets increases or a higher quality detection is required, we can increase the number of active sensors. The flexibility of dynamic array formation allows for appropriate detection performance while minimizing the cost.

4. PERFORMANCE EVALUATION

We evaluate the performance of our proposed dynamic array through simulations over matlab. We apply MUSIC algorithm to detect the DOAs of a group of uniformly distributed sources. We compare the performance of dynamic array (DA) with some other reference methods, including the extended co-prime array (ECPA) and uniform linear array (ULA).

In our study, we consider a universal ULA structure with 26 physical sensors. By utilizing the selection scheme, we can get $(M, N) = (3, 5)$ and $\{C\} = \{23, 24\}$. The optimal dynamic array configuration is as the previous example shown in Fig. 3. Although the total number of physical sensors is 26 which forms an ULA, the number of active sensors can be decreased to 12, which can save more than half of the energy and operational cost. To make the results comparable, all the reference methods are also applied to select sensors from the same 26-ULA physical structure. Specifically, ECPA uses the same co-prime factor $(3, 5)$ but without the complement sub-array.

Fig. 4 shows the MUSIC spectrum in different cases. The target sources are uniformly distributed within the range $-60°$ to $60°$. The covariance matrix is estimated by using 2000 snapshots. SNR is set to 0dB. We can see that our proposed DA has much clearer spectrum compared with ECPA and the root mean squared error (RMSE) can decrease by 90%. When compared with the ULA with all 26 sensors active, our proposed DA with only 12 sensors active doesn't sacrifice detection quality too much. It can still successfully identify all targets. This demonstrates the feasibility and effectiveness of our proposed method in forming the dynamic array within an ULA configuration.



(a) (3,5)ECPA r=0.9023 (b) DA r=0.0963
(c) 26-ULA r=0.1247 (d) DA r=0.2109

Fig. 4: MUSIC Spectrum

We then compare the root mean squared error (RMSE) of the DOA estimation between different methods, with 10 target sources. Fig. 5 shows the impact of SNR. Compared to ECPA, our proposed DA method reduces RMSE over 50%. Compared to the universal 26-ULA, it has similar detection quality with only 12 sensors active. Fig. 6 shows the impact of the number of snapshots instead of SNR. We can see that our method reduces RMSE over 50% compared to ECPA. In other word, if we want to achieve the same RMSE, our method can save over 75% snapshots, thus can perform much faster estimation. Further more, the reduction of scanning time also reduces the cost.

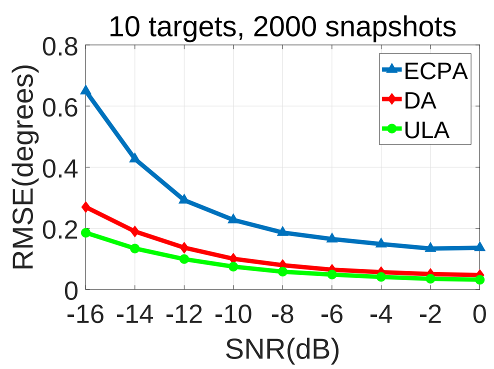


Fig. 5: RMSE versus SNR

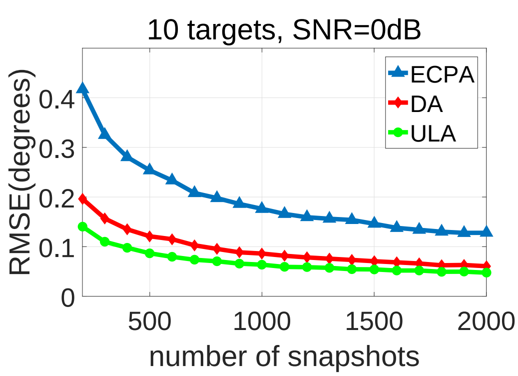


Fig. 6: RMSE versus snapshots

5. CONCLUSION

Rather than being constrained with the lack of physical co-prime array building and the inflexibility of specific co-prime array structure, we propose to exploit the current basic physical ULA configuration to flexibly formulate dynamic co-pirme linear arrays based on the detection need to keep the same DoF while reducing the cost. Our performance results demonstrate that, given a fixed physical ULA configuration, our scheme can keep comparable detection quality compared with the original ULA. More significantly, it conserves a large amount of energy since most of the sensors and RF chains can stay inactive during the scanning periods. In addition, the adaptive dynamic array allows high flexibility and efficiency in different detection scenarios.

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