Sequential and Adaptive Sampling for Matrix Completion in Network Monitoring Systems

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***Abstract*—End-to-end network monitoring is essential to ensure transmission quality for Internet applications. However, in large-scale networks, full-mesh measurement of network performance between all transmission pairs is infeasible. As a newly emerging sparsity representation technique, matrix completion allows the recovery of a low-rank matrix using only a small number of random samples. Existing schemes often fix the number of samples assuming the rank of the matrix is known, while the data features thus the matrix rank vary over time.**

In this paper, we propose to exploit the matrix completion techniques to derive the end-to-end network performance among all node pairs by only measuring a small subset of end-to-end paths. To address the challenge of rank change in the practical system, we propose a sequential and information-based adaptive sampling scheme, along with a novel sampling stopping condition. Our scheme is based only on the data observed without relying on the reconstruction method or the knowledge on the sparsity of unknown data. We have performed extensive simulations based on real-world trace data, and the results demonstrate that our scheme can significantly reduce the measurement cost while ensuring high accuracy in obtaining the whole network performance data.

***Index Terms*—Matrix Completion, Round-Trip Time Measurement, Sampling Stopping Condition**

I. Introduction

Network monitoring is a central activity in the design, engineering, and operation of a network. To better understand the behaviors of the network and its users, obtaining complete monitoring data of the whole network is critical. However, it is widely recognized that measuring the performance of all nodes and paths in a large network is infeasible due to the high cost involved in taking the measurements and transmitting the monitoring data. In order to reduce the measurement overhead, samples are often taken at a subset of nodes and paths. Despite the lower cost, it is hard to learn the complete network information with partial network measurements.

For natural signals, Nyquist sampling is followed as a fundamental theory in the past several decades to determine the number of samples, where it states that a band-limited signal can be completely recovered if it is sampled at a rate larger than two times its bandwidth. As a new sampling theory, Compressive Sensing (CS) [1]–[4] has attracted a lot of recent attention with its capability of reconstructing sparse signals with the number of measurements much lower than that of the Nyquist sampling rate. Compressive sensing (CS) also serves as a new paradigm for data gathering in WSNs [2], [5]–[9]. Although CS-based approaches can save energy and reduce the sensing cost, they are originally designed to recover the sparse information such as events in the sensor networks. In many practical scenarios, applications do not have clear sparsity features, and we need to get more complete data for system management purpose rather than just detecting events.

With the rapid progress of sparse representation, matrix completion [10], [11], a remarkable new field, has emerged very recently. According to the matrix completion theory, a low-rank matrix can be accurately reconstructed with a relatively small number of entries in the matrix. Taking advantage of the low-rank property of the monitoring matrices, matrix completion brings the benefits of low cost monitoring with a small set of samples and is shown to work in various applications [12]–[17].

Existing work based on matrix completion has mostly been focusing on recovering data in the sensor network environment. Specifically, a raw monitoring matrix can be reconstructed with a small number of samples at a certain accuracy level. The existing work often assumes the matrix rank is known so the necessary number of samples can be determined accordingly. However, it is difficult to know the matrix rank during the on-line monitoring of a practical environment.

In this work, we exploit matrix completion technique to design a sequential and adaptive sampling scheme which enables low-cost and high-accuracy monitoring of the end-to-end network performance. In our proposed scheme, measurements are taken periodically. Within each period, only a subset of end-to-end paths are measured, and the complete path information of the whole network can be derived from the measured data based on matrix completion. Different from the existing work, we consider the practical case that the rank of the monitoring matrix is not known. For each monitoring period, rather than taking all samples together, we propose

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to take samples sequentially within a short duration of time and we develop a novel sampling stopping condition. We propose an adaptive sampling strategy based only on the information derived from the measurements. Our scheme can significantly reduce the traffic and measurement cost while ensuring the high data reconstruction accuracy, without any a priori assumption on the data sparsity and data distribution or depending on the reconstruction algorithm. Our contributions are summarized as follows:

- We first analyze large traces of monitoring data on the Round-Trip Time (RTT), which reveals that there exists the low-rank feature in the data. Taking advantage of these structures, we formulate the on-line end-to-end RTT measurement problem which can be addressed based on matrix completion with a sequential sampling process.
- To minimize the measurement cost, we propose a stopping condition for the sequential sampling, with which the sampling process for a period can stop as soon as the matrix reconstruction reaches a certain accuracy level.
- To increase the reconstruction accuracy while reducing the total number of samples taken, we propose an adaptive sampling strategy to add samples with high information gain based on the knowledge from past measurement data.
- Through comprehensive simulations with real data traces, we demonstrate that our sequential and adaptive sampling scheme can accurately acquire end-to-end performance data with very low cost, which significantly outperforms the competing methods.

To the best of our knowledge, we are the first to study the sampling stopping condition in the area of matrix completion. Based only on the measurement data, this condition is general and does not depend on the reconstruction method or the knowledge of the sparsity of the unknown data matrix. We expect that this stop condition will help for applying the matrix completion to various on-line monitoring.

The rest of this paper is organized as follows. We introduce the related work in Section II. The fundamental of matrix completion is presented in Section III. We present our observations on the features of end-to-end monitoring with real RTT trace data and our problem formulation in Section IV. The proposed sequential and adaptive sampling scheme is presented in Section V. Finally, we evaluate the performance of the proposed scheme through extensive simulations in Section VI, and conclude the work in Section VII.

II. Related Work

Structure and redundancy in data are often synonymous with sparsity. Sparsity and redundancy make "sample a few and infer many" a possible approach to obtain the complete information in the network monitoring system. There exist two typical sparsity representation techniques, compressive sensing and matrix completion. In this section, we review related work and identify the differences of our work from existing work.

Compressive Sensing (CS) is a technique that can accurately recover a vector from a subset of samples given that the vector is sparse [1], [2] with only a few nonzero elements. Compressive sensing has two features, universal sampling and decentralized simple encoding, which makes it a new paradigm for data gathering in sensor networks [2], [5], [6], [18]. Moreover, as a powerful and generic technique for recovering complete information with a subset of data, CS has been applied to estimate the lost data [4], reconstruct network traffic [19], refine localization [20] and improve urban traffic sensing [21], [22]. The majority of works on CS consider vectors of data. A naive approach to deal with matrices by using CS might be to transform these matrices into vectors first and then apply CS to these vectors. However, some matrices have some inherent structure (i.e. the RTT matrix in this paper), low cost data gathering in network monitoring systems has lots of space to improve.

As a newly emerging technique, matrix completion [10], [11] concerns the recovery of a low-rank matrix from incomplete samples of its entries. Candès et al. [10] show that most $n_1 \times n_2$ matrices of rank r ($r \ll \min\{n_1, n_2\}$) can be perfectly recovered with very high probability by solving a simple convex optimization program provided that the number of samples is sufficient. New results show that matrix completion is provably accurate even when the few observed entries are corrupted with noises [23]. Almost all current techniques assume that the sampled data obey uniform distribution to satisfy the incoherence requirement in matrix completion, except the recent work on coherent matrix completion [24], in which Chen et al shows that the incoherence requirement in matrix completion can be eliminated, and propose a sampling strategy according to the local coherence structure of matrix (called coherence sampling in this paper) to further reduce the sampling cost. The progress of matrix completion techniques bring new opportunities for data reconstruction by fully exploiting the low-rank property of the monitoring matrices associated with various applications [12]–[17]. However, existing matrix completion solutions often assume that the data matrix has a known low-rank, and therefore the number of measurements to take is fixed and determined by the rank of the matrix r. However, in the case of on-line monitoring, the rank level is often not known a priori, which makes it difficult to ensure low cost and accurate monitoring using the matrix completion techniques.

Despite the recent interests and progress of sparse sensing techniques, they are mostly applied in the general sensor networks, and there are very limited efforts to apply the sparse sensing in network monitoring. The work in [16] applied matrix-factorization along with Singular Value Decomposition (SVD) and Non-negative Matrix Factorization(NMF) to network latency prediction. Relying heavily on landmark nodes, the proposed schemes depend on a strong assumption that all pairwise measurements among the landmarks and between the hosts and the landmark node are available. In contrast, our measurement scheme is more flexible and can be applied in any sparse monitoring matrices, instead of relying on the measurements associated with fixed pairs of landmark nodes.

Recently, Liao et.al built an inference model to predict the network distance based on distributed Matrix Factorization by Stochastic Gradient Descent (DMFSGD) [12]. The same DMFSGD algorithm was adapted in [15] to classify network performance into binary classes, either "good" or "bad". Matrix Factorization in DMFSGD depends on the rank of the monitoring matrices, and the fix rank applied in DMFSGD makes it not suitable for a practical system with the rank of the monitoring matrix varying over time.

In this work, we propose to apply matrix completion for efficient monitoring of end-to-end network performance. To conquer the challenge of not knowing the rank of the monitoring matrix, we propose a sequential and adaptive sampling scheme which takes measurements in a sequential set of steps until a *stopping condition* we propose is reached. Rather than taking random samples, our information-based adaptive sampling strategy chooses the paths which can provide higher information gain to take samples in each subsequent step. Our performance studies demonstrate that the proposed sampling strategy significantly outperforms current coherence sampling strategy.

III. Fundamentals of Matrix Completion

Matrix completion (MC) is a new technique which can be applied to recover a low-rank matrix from a subset of the matrix entries [10], [11]. That is, MC can recover an unknown matrix $M \in R^{n_1 \times n_2}$ with rank $r << min\{n_1, n_2\}$ while only a subset of its entries $M_{ij}, (i,j) \in \Omega$ are known. The general form of the MC problem is:

$$\begin{array}{ll} \min & rank(X) \\ \text{subject to} & P_\Omega(X) = P_\Omega(M) \end{array} \tag{1}$$

where the sampling operator $P_\Omega : R^{n_1 \times n_2} \rightarrow R^{n_1 \times n_2}$ is defined by:

$$P_\Omega(X)_{ij} = \begin{cases} X_{ij} & (i,j) \in \Omega \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

In (1), $rank(X)$ is defined as the number of nonzero singular values of X, and X is the variable matrix.

However, solving this rank minimization problem in (1) is often numerically expensive because it is NP-hard. Hence people tend to consider its relaxation:

$$\begin{array}{ll} \min & \|X\|_* \\ \text{subject to} & P_\Omega(X) = P_\Omega(M) \end{array} \tag{3}$$

where $\|X\|_*$ stands for the nuclear norm of the matrix X, which is the sum of its singular values $\delta_{\mathrm{k}}(X)$. That is $\|X\|_* = \sum_{i=1}^{\mathrm{r}} \sigma_i(X)$, where $r = rank(X)$. It has been shown in [10], [11], [25], [26] that, under certain reasonable conditions, (1) and (3) share the same solution given that the number of samples obeys the following condition:

$$m \geq C\, n^{6/5} r log\, n, \tag{4}$$

where C is a numerical constant and $n = \max\{n_1, n_2\}$

Different types of algorithms have been proposed to solve (3), such as linearized Bregman method [27], fixed point and Bregman iterative methods [28], and Singular Value Thresholding algorithm (SVT) [27]. Our proposed sequential and adaptive sampling scheme does not depend on the underlying reconstruction algorithm. In this paper, we choose the singular value SVT approach to reconstruct the matrix.

IV. Problem Description

In the networking field, it is important to monitor the performance of an end-to-end path between two end nodes, and the monitoring is essential to ensure the performance expected by Internet applications.

Network paths starting from nearby end nodes often have overlapping path segments or go through some common network nodes. This is especially the case in the Internet core that has simple topology. As a result, data from network measurements often have correlations. For example, the congestion at a certain link would cause higher delay for all paths that traverse this link. These correlations also make the rank of the corresponding monitoring matrix to be low, which in turn enables the use of matrix completion for low cost network monitoring.

To present our proposed sequential and adaptive sampling scheme, we use Round-Trip Time (RTT) monitoring as an example.

In this section, we first analyze a large set of trace data to better understand the structure and characteristics of end-to-end RTT data, and then present our problem formulation and challenges.

A. Low-Rank Monitoring Data Matrix

For a network consisting of N nodes, we define a monitoring matrix, $X_{N\times N}$, to hold the end-to-end RTT data, with the (ij)-th entry, X_{ij}, representing the RTT data from node i to node j. In the matrix, a row corresponds to a source node and a column corresponds to a destination node.

As discussed earlier, end-to-end performance data of different node pairs normally have strong correlation due to the sharing of links or nodes among their paths. We first apply singular value decomposition (SVD) to examine whether the monitoring matrix has a good low-rank structure. A monitoring matrix $X_{N\times N}$ can be decomposed as:

$$X = U\Sigma V^T \tag{5}$$

where U is an $N \times N$ unitary matrix, V is an $N \times N$ unitary matrix, and Σ is an $N \times N$ diagonal matrix with the diagonal elements (i.e. the singular values) organized in the decreasing order (i.e. $\Sigma = diag(\sigma_1, \sigma_2, \cdots, \sigma_r, 0, \cdots, 0)$). The rank of a matrix X, denoted by r, is equal to the number of its non-zero singular values. A matrix is low-rank if its $r \ll N$.

If a matrix has low-rank, its top K singular values occupy the total or near-total energy $\sum_{i=1}^{K} \sigma_i^2 \approx \sum_{i=1}^{r} \sigma_i^2$.

To determine whether X has a good low-rank approximation, we define a metric as

$$g(K) = \frac{\sum_{i=1}^{K} \sigma_i^2}{\sum_{i=1}^{r} \sigma_i^2} \tag{6}$$

Although there are several publicly available RTT datasets [29]–[31], P2PSim [30] and Meridian [31] only contain static RTT measurements taken between Internet DNS servers and between network nodes. The only data set that contains dynamic data is Harvard226, which takes measurements of application-level RTTs between 226 Azureus clients [29] and the measurements are taken every five minutes in 4 hours. To reconstruct the matrix in the presence of rank variation in a dynamic environment, this paper chooses Harvard226 as the data set.

Our algorithm is proposed for efficient online network monitoring. Ideally, we would like to have complete data measurements as reference to evaluate the performance of our scheme. In Harvard226, data measurements are supposed to be taken every 5 minutes, however we find that there are many missing data points in a time duration. To mitigate the problem, we take the average of the RTT values measured within every period of 30 minutes to build the new data trace, so the original four-hour Harvard226 data set can be used to build 8 peer-to-peer RTT measurement matrices.

Fig.1 plots the fraction of the total variance captured by the top K singular values for the first data matrix of Harvard226 [29].

The X-axis presents the top K singular values. The Y-axis presents the total variance captured by the top K singular values, and the variance is calculated by Eq.(6). We find that the top 10% singular values capture 99% variance in the real traces. These results indicate that the end-to-end monitoring matrix X has a good low-rank approximation in the scenario under investigation.

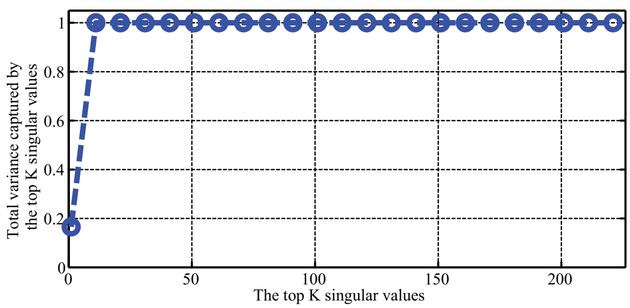

Fig. 1. Fraction captured by the top K singular values

To further investigate the characteristics of rank, we plot the rank of the consecutive eight RTT matrices of Harvard226 in Fig.2. The X-axis represents the sequence number of each RTT matrix, and the Y-axis represents the rank of the corresponding RTT matrix. Obviously, the RTT matrix does not have a constant rank over time, so the number of samples that needs to take should adapt accordingly.

B. Problem Formulation

The low-rank feature satisfies the prerequisite for using matrix completion. For low cost network monitoring, we propose an sequential and adaptive sampling scheme based on Matrix Completion to monitor End-to-End performance, named MC-E2E. In MC-E2E, only a small set of source and destination pairs are sampled (measured) and other items can be accurately inferred through the matrix completion. The monitoring process can be illustrated using Fig.3, where the diagonal entries of the monitoring matrix are empty as the performance of a node to itself is of no interest. We apply the proposed scheme to monitor the RTT of paths.

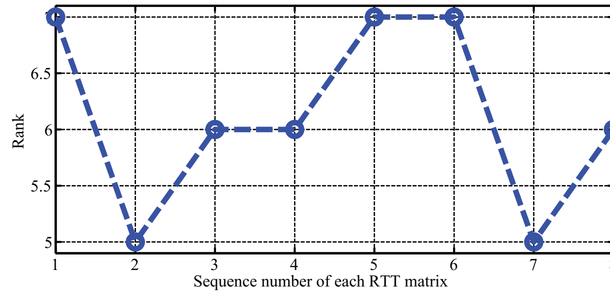

Fig. 2. Rank feature of dynamic RTT data

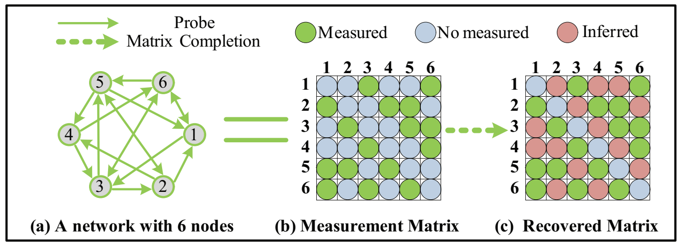

Fig. 3. Sample a few and infer many. Only a small set of source and destination pairs are sampled (measured) and others can be accurately inferred through matrix completion.

We use a Binary Sample Matrix, $B_{N\times N}$, to indicate that whether an end-to-end measurement is taken at the corresponding node pair, a B is defined as

$$B = (B_{ij})_{N\times N} = \begin{cases} 1 & \text{if a node pair is measured.} \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

We define a measurement matrix, $M_{N\times N}$, to record the raw measurement data. Because only a small set of paths (a path corresponds to a node pair) are measured, obviously, $M_{N\times N}$ is an incomplete monitoring matrix and can be represented as

$$M_{N\times N} = X_{N\times N} \bullet B_{N\times N}, \tag{8}$$

where $\bullet$ represents a scalar product (or dot product) of two matrices, $M_{ij} = X_{ij} \times B_{ij}$. If there is no RTT measurement made between a particular pair of nodes, of course, it leaves the corresponding entry in M to be empty. In our study, we use zero as a placeholder to replace the empty entry.

According to the matrix completion technique introduced in Section III, when the number of samples is sufficient, the monitoring matrix $X_{N\times N}$ can be recovered from the measurement matrix $M_{N\times N}$ by solving the following problem

$$\begin{aligned} &\min \ \|X\|_* \\ &\text{subject to } M_{N\times N} = X_{N\times N} \bullet B_{N\times N} \end{aligned} \tag{9}$$

We denote the reconstruction matrix from (9) as $\hat{X}_{N\times N}$.

Obviously, the sampling matrix $B_{N\times N}$ indicates which node pairs need to take samples. To minimize the measurement cost while satisfying the matrix reconstruction requirement, the key problem in our MC-E2E scheme is to identify the optimal $B_{N\times N}$ to schedule the measurement.

C. Challenges

Although the literature work on matrix completion provide some solutions to recovering data from a subset of samples, existing schemes mostly assume the rank of the data matrix is known. However, without obtaining the actual monitoring data, the sparsity level (rank-level) of the monitoring matrix is impossible to know a priori. It is thus very challenging to apply matrix completion theory in the practical monitoring system to obtain the complete performance data. Some practical challenges are as follows.

- To achieve low cost measurement, redundant samples should be minimized. However, according to Eq.(4), it is very hard to identify how many samples are sufficient to recover the complete performance data without knowing the rank level of the monitoring matrix in on-line network monitoring.
- To increase the recovery accuracy, instead of random sampling, we want to design more intelligent sampling strategy to choose the sample node pairs. However, without a prior knowledge of the matrix structure, designing such a sampling strategy is very difficult.

V. MC-E2E MEASUREMENT SCHEME

Generally, the end-to-end performance does not vary significantly within a short time period, so measurements are often taken periodically in the network. Rather than taking measurements from each path in a period, we propose to randomly sample a subset of paths, and infer other data based on the matrix completion theory. As the rank of the measurement matrix is not known and may vary over time, it is difficult to know how many samples are enough. In this work, we propose to adaptively sample the network in each period. We set the initial sampling number to a smaller value based on the measurement performance of the last period, and then determine whether more samples need to be taken based on the recovered data matrix.

In this section, we first present the proposed sampling stopping condition for on-line measurement, and then introduce our adaptive sampling strategy and the complete MC-E2E measurement scheme.

A. Sampling Stopping Condition for On-line Measurement

Without the knowledge of the original data, it is hard to determine whether enough samples have been obtained during the sequential sampling process. For practical on-line monitoring, it is desirable to have a computationally efficient approach to make this decision. To gain an insight on possible ways of finding the sample numbers, we first perform simulations on the real trace data to learn the relationship between the sample number and the reconstruction performance. We define the following metric to measure the error of reconstruction performance.

Definition 1. ***Error Ratio:*** *a metric for measuring the reconstruction error of all entries in the matrix after the interpolation, which can be calculated as*

$$\frac{\sum \left| X_{ij} - \hat{X}_{ij} \right|}{\sum |X_{ij}|} \tag{10}$$

where $1 \leq i \leq N$ and $1 \leq j \leq N$. X_{ij} and $\hat{X}_{ij}$ in (10) denote the raw data and the recovered data at (i, j)-th element of X, respectively.

In the simulation, we increase the sample number by adding more random samples sequentially. After we have the measurement data in each sampling step, we then apply the matrix completion to the measurement data to obtain the recovery data. Finally, we calculate the error rate by comparing the recovered data with the raw data trace.

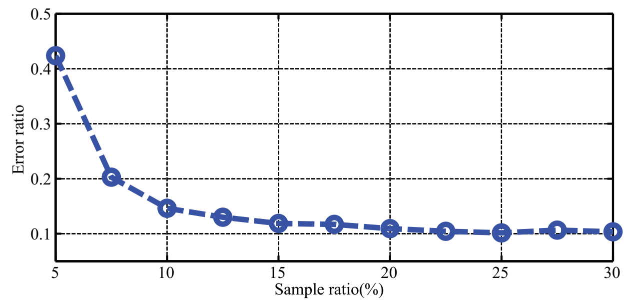

Fig. 4. The reconstruction error and sample number

From Fig.4, we observe when the sample number becomes large, the reconstruction error converges to a small value. Therefore, we have a conclusion that the monitoring matrix can be accurately recovered from the measurement matrix when the sample number is large enough, beyond which adding extra samples will not significantly increase the reconstruction accuracy.

Before we introduce our sampling stopping condition for sequential sampling in matrix completion, the following theorem presents the relationship between the recovered data and the sample number.

Theorem 1. *For a low rank matrix X, given two sequential sampling steps t and $t+1$ with m samples taken at step t while additional C random samples taken at step $t+1$, the recovered matrices obtained in these sequential sampling steps are denoted as $\hat{X}(t)$ and $\hat{X}(t+1)$, respectively. If it holds that $\hat{X}(t) = \hat{X}(t+1)$, then the recovered matrix $\hat{X}(t)$ is exactly equal to the low-rank matrix X.*

Proof: *We define L_M as low sampling bound for the low rank matrix to be correctly recovered. We will prove that $\hat{X}(t) = \hat{X}(t+1)$ holds only under the condition of $m + C > m > L_M$. Our proof includes three parts.*

Part 1) *When $m + C > m \geq L_M$, it is easy to know that $\hat{X}(t) = \hat{X}(t+1) = X$.*

Part 2) *When $m + C \geq L_M > m$, obviously, we have that $\hat{X}(t+1) = X$ and $\hat{X}(t) \neq X$. Therefore, $\hat{X}(t+1) \neq \hat{X}(t)$.*

Part 3) *When $L_M > m + C > m$, by way of contradiction, suppose that $\hat{X}(t) = \hat{X}(t+1)$. Given that $\hat{X}(t) = \hat{X}(t+1)$, the C additional random samples taken at step $t+1$ can be exactly recovered from m samples. We define a recovery function $f(S_m) \rightarrow f(S_m \cup S_C)$ to denote that m samples can exactly recover additional C entries (where $|S_m| = m$ and $|S_C| = C$).*

Let $\Re$ be the whole sampling set that covers all entries in X. Given different random sample sets with C samples $(c_0, c_1, \cdots, c_L)$ and $c_1 \cup c_2 \cup \cdots \cup c_L = \Re - S_m$, as $\hat{X}(t) = \hat{X}(t+1)$, we have $f(S_m) \to f(S_m \cup c_0)$, $f(S_m) \to f(S_m \cup c_1)$, $\cdots$, $f(S_m) \to f(S_m \cup c_L)$.

We can easily obtain that $f(S_m) \to f\left(S_m \bigcup\limits_{i=0}^{L} c_i\right)$. Because $S_m \bigcup\limits_{i=0}^{L} c_i = \Re$, we have $f(S_m) \to f(\Re)$, that is, m samples can recovered all entries in matrix X, which contradicts that $L_M > m$. Therefore, when $L_M > m + C > m$, $\hat{X}(t) \neq \hat{X}(t+1)$. (See Fig.5 for an illustration)

Therefore, combining Part 1, Part 2, and Part 3, we can conclude that $\hat{X}(t) = \hat{X}(t+1)$ holds only under the condition of $m + c > m > L_M$, which completes the proof. ■

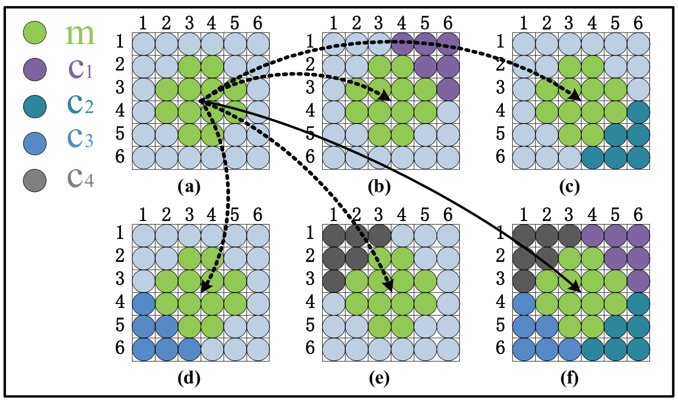

Fig. 5. Illustration of the proof by utilizing the recovery function. Fig.5(a) is the measurement matrix with m samples. Fig.5(b), Fig.5(c), Fig.5(d), Fig.5(e) are the measurement matrices with $m + C$ samples, the size of sample set c_1, c_2, c_3, and c_4 are the same and equal to C. If $\hat{X}(t) = \hat{X}(t+1)$ when $L_M > m + C > m$, then we have $f(S_m) \to f(S_m \cup c_1)$, $f(S_m) \to f(S_m \cup c_2)$, $f(S_m) \to f(S_m \cup c_3)$, and $f(S_m) \to f(S_m \cup c_4)$. Because $S_m \cup c_1 \cup c_2 \cup c_3 \cup c_4 = \Re$, we have $f(S_m) \to f(\Re)$, that is, m sample can recovered all entries in matrix X, which contradicts that $L_M > m$. Therefore, when $L_M > m + C > m$, $\hat{X}(t) \neq \hat{X}(t+1)$.

According to Theorem 1, we propose our sampling stopping condition for practical systems though some relaxations.

Definition 2. *Given two matrices $A_{N\times N}$ and $B_{N\times N}$, we define $A \triangleq B$ if these two matrices satisfy*

$$\frac{\sqrt{\sum (A_{ij} - B_{ij})^2}}{\sqrt{\sum \left(\frac{1}{2}(A_{ij} + B_{ij})\right)^2}} \leq \varepsilon \tag{11}$$

where ε is a small constant.

Definition 3. ***Sampling Stopping Condition.*** *We denote the recovered matrices obtained in sequential sampling steps t and $t+1$ as $\hat{X}(t)$ and $\hat{X}(t+1)$, respectively. If these recovered matrices satisfy $\hat{X}(t) \triangleq \hat{X}(t+1)$, we declare that the monitoring matrix is correctly recovered at step t, and we stop sampling at step $t+1$. (See Fig.6 for an illustration of the proposed sampling stopping condition).*

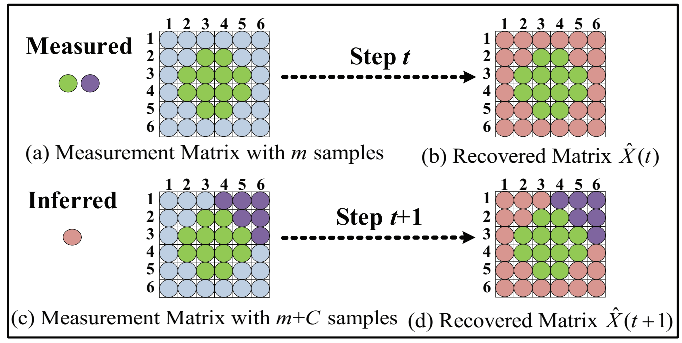

Fig. 6. The illustration of the sampling stopping condition. Fig.6(a) and Fig.6(c) are the measurement matrices at step t and step $t+1$, respectively. Fig.6(b) and Fig.6(d) are the recovered matrices at step t and step $t+1$, respectively. If the recovered matrices satisfy $\hat{X}(t) \triangleq \hat{X}(t+1)$, stop sampling at step t+1; else add more samples in the next sampling step $t+2$.

B. Information Based Adaptive Sampling Strategy

In a practical on-line monitoring system, it is difficult to know the features and the rank of the underlying matrix. Rather than taking random samples, to increase the recovery accuracy while reducing the total number of samples, we propose an *information-based* adaptive sampling strategy where additional samples are taken based on the learning from the past measurement data. We would like to start with a small number of random samples.

Considering two sequential sampling steps t and $t+1$, let $S(t)$ and $S(t+1)$ be the corresponding sets of samples taken in the two steps. Obviously, we have $S(t) \in S(t+1)$. The recovered monitoring matrices are denoted as $\hat{X}(t)$ and $\hat{X}(t+1)$, respectively.

If $\hat{X}_{ij}(t)$ are close to $\hat{X}_{ij}(t+1)$ and $(i,j) \notin S(t+1)$, i.e., the item at (i,j) is inferred data not sampling data, we can conclude that entry (i,j) has been recovered with almost full information. On the other hand, if $\hat{X}_{ij}(t)$ are far from $\hat{X}_{ij}(t+1)$, it indicates that the entry (i,j) is far from being accurately recovered, and it would gain more information by taking a sample at (i,j).

Based on the above analysis, we propose the following information-based metric, INFO, to quantitatively evaluate whether an entry is informative:

$$INFO(i,j) = \frac{\left|\hat{X}_{ij}(t+1) - \hat{X}_{ij}(t)\right|}{\frac{1}{2}\left|\hat{X}_{ij}(t+1) + \hat{X}_{ij}(t)\right|} \tag{12}$$

If an entry (i,j) has larger $INFO(i,j)$, this entry is more informative and should be sampled in the next step. Therefore, our adaptive sampling strategy is designed as follows: taking samples from the paths corresponding to most-informative entries in the next step. Fig.7 illustrates our adaptive sampling strategy.

C. Complete MC-E2E scheme

The complete MC-E2E scheme is shown in Algorithm 1. Initially, to obtain the basic information of the whole network, the measurements are taken uniformly at random among all node pairs. On line 2, the initial number of uniform samples is determined based on the value of β. If β is too small,

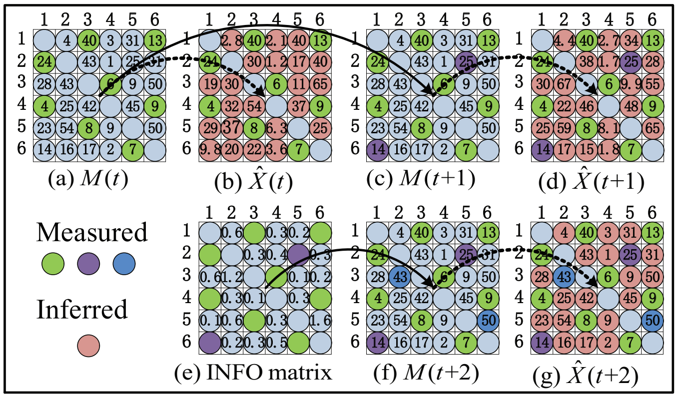

Fig. 7. Illustration of the adaptive sampling strategy. Fig.7(a), Fig.7(c) are the measurement matrices at step t and step $t+1$, respectively. The numbers on the sampled locations are the real measurement values. Fig.7(b), Fig.7(d) are the recovered matrices at step t and step $t+1$, respectively. The data shown on the inferred locations are the inferred values. After computing every entry's INFO metric according to Eq(12), we have an INFO matrix in Fig.7(e) and we find entries (3,2) and (5,6) are the most-informative. According to our sampling strategy, we add samples (3,2) and (5,6) at step $t+2$, as shown in Fig.7(f).

it would require too many additional samples later; while if it is too large, it may take unnecessary measurements and waste network resources. In practice, we can utilize the history RTT data to investigate the impact of β on the sampling cost and identify a β in the future sampling period. In the simulation part, we will investigate the relationship between β and the sampling cost and set an optimal β according to the relationship.

After the uniform sampling, adaptive samples are taken in sequence. By comparing the recovered data matrices obtained in two consecutive steps, the most-informative locations are chosen to sample in the next sampling steps. According to our sampling stopping condition, adaptive measurements will continue until the difference between the recovered matrices $\hat{X}(t)$ and $\hat{X}(t+1)$ is smaller than a threshold ε, i.e., $\hat{X}(t) \triangleq \hat{X}(t+1)$.

On line 12, α is a parameter which has an impact on the number of samples added in each sampling step. To help the sampling algorithm to quickly reach the desired value, α is set as in Eq.(13). Obviously, we have $0 \leq \alpha \leq 1$. If the monitoring matrix is far from being accurately recovered, α is large, more samples are added in one sampling step; otherwise, fewer samples are added in one sampling step to help the algorithm to converge. In this paper, we set $\mu = 0.01$ in Eq.(13). Our adaption design helps to quickly find the desired sample number while reducing the measurement cost.

VI. Performance Evaluations

In this section, we utilize Error Ratio (defined in Eq.(10)) to evaluate the performance of proposal scheme over public RTT data set Harvard226 [29].

Algorithm 1 Matrix Completion Based Network Monitoring

1: Initialize $t = 0$.
2: Apply the uniform sampling to obtain the initial measurement matrix $M(t)$, and the current sample set is denoted by Ω with $|\Omega| = \beta \times N \times N$ where $\beta \in [0,1]$ and $N \times N$ is the total entries in the monitoring matrix.
3: Apply matrix completion to $M(t)$ and obtain the recovered matrix $\hat{X}(t)$.
4: Initialize an extra sample set Ω' with $|\Omega'| = 0.5N \log N$.
5: Add extra Ω' samples to the current sample set Ω, that is $\Omega = \Omega \cup \Omega'$, then obtain the measurement matrix $M(t+1)$ at time $t+1$
6: Apply matrix completion to $M(t+1)$ and obtain the recovered matrix $\hat{X}(t+1)$.
7: **if** $\hat{X}(t) \triangleq \hat{X}(t+1)$ **then**
8: According to the ***Sampling Stopping Condition***, stop sampling.
9: Return $\hat{X}(t+1)$ as the correctly recovered matrix.
10: **else**
11: For entries not sampled before, that is, $(i,j) \notin \Omega$, calculate the entries' INFO according to Eq(12)
12: Sort $INFO(i,j)$ in the descending order, and select the first $\alpha N \log N$ entries into Ω', where α can be calculated as $\alpha = \frac{\sum_{ij} \theta_{ij}}{N \times N}$ where

$$\theta_{ij} = \begin{cases} 1 & \frac{|\hat{X}_{ij}(t+1) - \hat{X}_{ij}(t)|}{\frac{1}{2}|\hat{X}_{ij}(t+1) + \hat{X}_{ij}(t)|} > \mu, (i,j) \notin \Omega \\ 0 & otherwise \end{cases} \quad (13)$$

where μ is a small constant.
13: $t = t+1$, goto step 5.
14: **end if**

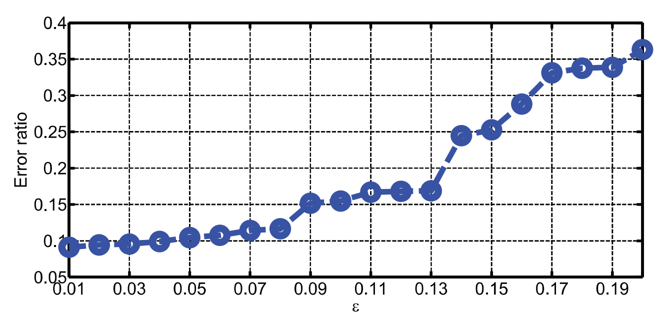

Fig. 8. Impact of ε

A. Parameter study

1) Impact of ε

According to our sampling stopping condition, adaptive measurements will stop when the difference between the recovered matrices $\hat{X}(t)$ and $\hat{X}(t+1)$ is smaller than a threshold ε, i.e., $\hat{X}(t) \triangleq \hat{X}(t+1)$. To investigate how ε impacts on the reconstruction accuracy, we vary ε and run Algorithm 1 for Harvard226. By comparing the recovered data with the raw data trace, we calculate the error ratio. As shown in Fig.8, as expected, the construction error increases as ε increases. Moreover, we observe that when $\varepsilon = 0.03$, the construction error can be controlled to 0.1, a very low value. Therefore, in our following simulations, we set $\varepsilon = 0.03$.

2) Impact of β

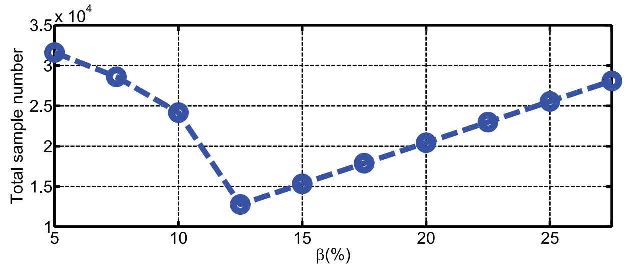

Fig. 9. Impact of β.

In Algorithm 1, β is the fraction of the number of uniform samples. To investigate how β impacts the sampling cost (the number of samples), we vary β and run Algorithm 1 for trace data Harvard226.

Fig.9 plots the number of samples required for successful recovery (y-axis) as β (x-axis) varies. For small values of β, the basic information obtained in the unform sampling phase will be far from reflecting the real information of the data matrix, which makes the sampling cost high. On the other hand, too large a β also leads to a high sampling cost, as some samples are taken unnecessarily. We set $\beta = 12.5\%$ in the following simulations, which allows us to achieve the lowest sampling cost.

B. Performance Comparison

To evaluate the performance of our proposed scheme, we implement two adynamic sampling schemes (MC-E2E and STOP-Coherence) and four static sampling schemes (Uniform-0.1, Uniform-0.2, Uniform-0.3, and Uniform-0.4) which take a fixed percentage of samples in each case. STOP-Coherence is implemented by replacing the information-based sampling strategy of our MC-E2E (in the line 11-12 in Algorithm 1) with the coherence sampling strategy [24].

MC-E2E and STOP-Coherence start with a uniform sampling phase to randomly measure a subset of paths with the sample ratio $\beta = 12.5\%$. Beyond the initiation period, the proposed adaptive sampling strategy in MC-E2E is applied to select the most-informative node pairs in the following sampling steps; In STOP-Coherence, the coherence sampling strategy proposed in [24] is taken to select a fix number of samples in following sampling steps. Sample numbers in both dynamic schemes are added sequentially to better meet the implementation requirement of the practical system. Our proposed stopping condition is applied to end the sampling process as soon as the difference between two consequently recovered data matrices is very small.

Uniform-0.1, Uniform-0.2, Uniform-0.3, and Uniform-0.4 are four static sampling schemes where uniform sampling strategies are applied to satisfy the fix sampling ratios (0.1, 0.2, 0.3, and 0.4).

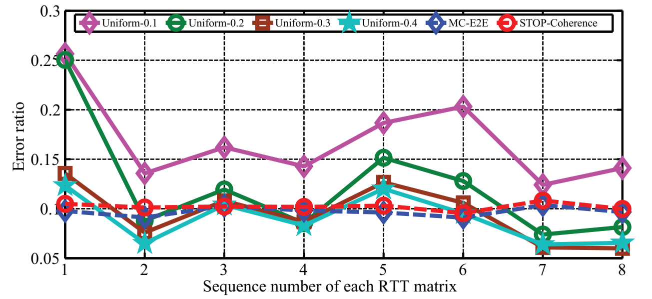

Fig. 10. Error ratio under the proposed matrix completion based scheme

1) Error Ratio

In Fig.10, we compare the reconstruction errors of the six schemes. The reconstruction errors of Uniform-0.1, Uniform-0.2, Uniform-0.3, and Uniform-0.4 fluctuate over the time period simulated, while the errors of STOP-Coherence and our MC-E2E remain low and stable. As the rank of the RTT trace data varies with time, simply sampling with a fixed ratio cannot capture the data changes in a dynamic network.

If the samples are simply taken randomly as done in the literature work with a fixed ratio, even with a high sampling rate 0.4, the corresponding error rate under Uniform-0.4 can still reach 13%. In contrast, the error rate of STOP-Coherence and our MC-E2E can be well controlled to be around 0.1 during the whole testing period.

Although different sampling strategies are adopted in STOP-Coherence and our MC-E2E, the reconstruction errors under different schemes are all close to 0.1. These results demonstrate that, our proposed stopping condition is very effective in determining the number of samples needed for accurate matrix recovery and the stopping condition does not depend on the sampling strategies.

2) Total Sample Number

Fig.11 shows the sampling number under different sampling schemes. In consistence with the results shown in Fig.10, the curves in all the schemes are parallel to the X-axis except STOP-Coherence and our MC-E2E. This is because the other schemes utilize a fixed sampling ratio while STOP-Coherence and our MC-E2E can adjust the sampling ratio according the rank variation to accurately recover the data matrix while reducing the sampling cost.

Although both MC-E2E and STOP-Coherence use our proposed sampling stopping condition, the difference in their sampling strategies result in different sampling cost. Compared with the coherence sampling strategy, our information-based adaptive strategy can further reduce the sampling cost significantly. This demonstrates the effectiveness of our information-based sampling strategy in choosing new samples.

All these results demonstrate that our scheme provides a practical way to apply matrix completion technique in monitoring systems to obtain the whole network performance data with low measurement cost .

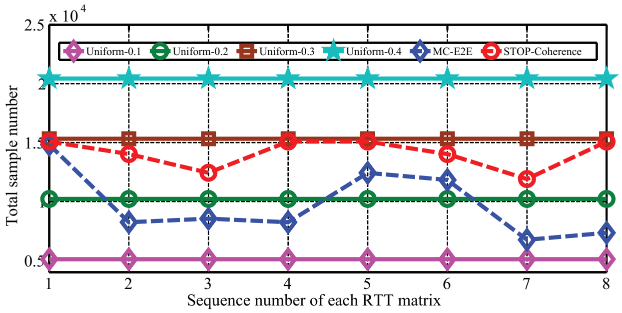

Fig. 11. Total sample number.

VII. Conclusion

For low cost network monitoring, this paper proposes to apply matrix completion for high-accuracy monitoring of the end-to-end network performance. To conquer the challenge of not knowing the rank of the monitoring matrix, we propose to take samples sequentially within a short duration of time and develop a novel sampling stopping condition. In addition, we propose an information-based adaptive sampling strategy to determine where to take additional samples. The sampling stopping condition and the adaptive sampling strategy are based only on the measurement observed, which makes our scheme suitable for on-line monitoring. We demonstrate through real-world trace data that our stopping condition and information-based adaptive sampling scheme are very effective in ensuring accurate and low cost network monitoring.

Although this paper focuses on RTT measurement, our scheme is flexible to apply in various networked monitoring systems including the monitoring of smart grid and other infrastructure.

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