# Directional Beam Alignment for Millimeter Wave Cellular Systems

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***Abstract*—Transmission in millimeter wave (mmW) band has a big potential to provide orders of higher wireless bandwidth. To combat the high channel loss in high frequency band, beamforming is generally taken to transmit along the direction that provides the maximum transmission gain. This requires the MAC protocol to facilitate the finding of the optimal beamforming direction. Exiting protocol suggests the rotational channel measurement which may introduce high measurement cost, and compromise the transmission capacity.**

**This paper presents a comprehensive design for more efficient directional beam alignment in mmW cellular networks. Instead of exhaustively searching all possible beamforming directions at the transmitter (TX) and the receiver (RX), our proposed scheme selects only a fairly small number of TX and RX beam pairs to facilitate effective beam alignment. To avoid long and resource-consuming exhaustive search, our scheme not only takes advantage of the low rank characteristics of the channel to estimate the full channel information with a small number of measurements, but also further exploits the channel estimation from initial measurements to guide the selection of future beam pairs for more effective measurements later. These strategies help to speed up the process of finding satisfactory beam pairs. We perform extensive simulations to evaluate the performance of our proposed schemes, and our results demonstrate our scheme can significantly outperform other schemes in terms of measurement effectiveness and cost efficiency.**

***Keywords*-millimeter wave, beamforming, beam alignment, directional antenna, matrix completion.**

## I. Introduction

The popularity of wireless devices and applications changes the world around us and also introduces the significant need of wireless resources. Millimeter wave (mmW) communication is gaining increased attentions from both the industry and academia as a promising candidate for next-generation cellular networks [1]–[3]. The mmW frequencies, ranging from 30 to 300 GHz, show the attractiveness of larger bandwidths along with further gains of beamforming and spatial multiplexing via multi-element antenna arrays.

A key challenge faced by millimeter wave (mmW) cellular networks (a typical example shown in Figure 1) is its low signal range. According to Frii's Law, the high frequencies of mmW signals result in large isotropic path loss (the free-space path loss grows with the frequency polynomially). Therefore, transmissions over fairly distant range can be a big issue in mmW cellular networks. Fortunately, the small wavelengths of mmW signals also enable large number of antenna elements to be placed in a space with small dimensions (e.g. at the base station, in the skin of a cellphone, or even within a chip), which can provide high beamforming gain that can compensate for the increase in isotropic path loss.

However, the development of mmW networks faces significant technical obstacles. Taking the initialization and synchronization of base stations (BSs) and mobiles as an example, using only omni-directional transmissions of synchronization signals would be problematic in the mmW range: the availability of high gain antennas would bring a discrepancy between the range at which a cell can be detected (when signaling messages are transmitted omni-directionally before the correct beamforming directions are found) and the range at which reasonable data rates can be achieved (after the beamforming is applied). On the other hand, although a beamed transmission from the base station provides a larger footprint and allows for higher data rate, it is difficult for a mobile to find the base stations (BSs) initially without knowing the correct beamforming directions. To address these issues, a cell search phase is introduced in the mmW range where a base station beams towards different directions to facilitate a mobile to find a direction that maximizes its receiving rate.

The antenna gains of the transmitter (TX) and the receiver (RX) have significant impacts on the transmission quality. Simply transmitting signaling messages rotationally along each direction as suggested by the existing standard [4] would introduce very high delay and cost for finding the optimal beamforming direction with the maximum gain. An example is when TX and RX each has 64 beam directions (in practical mmW networks this number can be even larger), to exhaustively measure every beam pair, $64 \times 64 = 2^{12}$ measurements are required. The finding of optimal beam direction may take long time to complete. As the channel conditions are dynamic, the direction finding may need to be performed constantly before transmissions, which would significantly compromise the transmission capacity. Therefore, it is in great need to have efficient beam searching schemes or beam alignment methodologies that can be incorporated into the MAC protocol design in mmW networks.

Instead of exhaustively training all possible beam pairs, we are motivated to perform beam searching with much lower number of measurements. It is noted in many literature studies (e.g. [5]–[7]) that the spatial covariance matrix of wireless mmW channels often present low-rank characteristics, which indicates that only a small number of paths are dominant so most of the energy of the channel is concentrated in a low-dimensional space. The initial efforts, however, focus on the modeling of mmW channel and verifying the possibility of sparse modeling without giving detailed schemes to enable the transmission direction finding. In this paper, we investigate the low-rank property of mmW channel and then propose a learning-based algorithm which exploits the mmW channel estimation to enable more effective channel measurements. This in turn allows for more efficient beam alignment and thus supports a higher network transmission performance.

The rest of this paper is organized as follows. After briefly reviewing related work in Sec. II, we provide the system model in Sec. III, and describe our beam alignment algorithm in Sec. IV. Finally, we present and analyze the simulation results in Sec. V and conclude the paper in Sec. VI.

## II. Related Work

MmW communications require high beamforming gain to compensate for the high path loss in the mmW spectrum range. Thus joint beamforming (BF) protocol is required to select the best transmission and reception beam directions according to some metric, e.g. signal-to-noise ratio (SNR) [8].

IEEE 802.15.3c [4], the first wireless IEEE standard proposed in the 60 GHz (mmW) band, provides specifications on wireless Medium Access Control (MAC) Layer and Physical Layer (PHY) for high rate transmissions in wireless personal area networks (WPANs). An optional beam-codebook-based BF protocol [9], [10] is included in IEEE 802.15.3c.

In [11], S. Hur *et al.* propose the use of outdoor millimeter wave communications for backhaul networking between cells and mobile access within a cell. To overcome the outdoor impairments found in millimeter wave propagation, this paper studies beamforming using large arrays. The authors propose an efficient beam alignment technique using adaptive subspace sampling and hierarchical beam codebooks. To perform the initial directional cell search in mmW cellular networks for the mobile and base station to jointly search over a potentially large angular directional space to locate a suitable path to initiate communication, C. N. Barati *et al.* in [12] propose a directional cell search procedure where base station periodically transmits synchronization signals in randomly varying directions, detectors are derived for both analog beamforming and digital beamforming.

An efficient beam switching technique for the emerging 60GHz wireless personal area networks is proposed in [13]. B. Li *et al.* formulate the problem of finding the best beam-pair for data transmissions as a global optimization problem and adopt a numerical approach to implement the beam searching through divide and conquer in a small region. In [14], J. Singh *et al.* investigate the feasibility of employing multiple antenna arrays to obtain diversity/multiplexing gains in mmW systems. To overcome the complexity of jointly optimizing the beamforming directions across multiple arrays, complementary approaches are proposed to restrict the attention to a small set of candidate directions based on the sparse multi-path feature of mmW channel. Although the methods in [12]–[14] reduce the direction search overhead by selecting a subset of directions to measure, their beam-pairs are only selected from the measured directions, which compromises their directional gains thus performance.

The studies in [5]–[7] provide promising evidence that the covariance matrix of mmW wireless channel is typically low-rank. However, there still remain some questions to be addressed: How to exploit this low-rank property? How to design efficient beam pair training process so that it can be incorporated into mmW MAC design to facilitate better network transmissions? Different from existing efforts, the focus of this paper is to design a beam alignment algorithm that can better take advantage of the low-rank properties of mmW channel to reduce the training cost, thus allowing for more transmission time for higher transmission capacity. The proposed efficient beam alignment scheme is also expected to be incorporated into the MAC protocol design in mmW networks. To be more specific, by exploring the low-rank property of mmW channel, instead of blindly or exhaustively training, we design a scheme that learns from the already trained beam alignment results to guide the selection of better beam pairs to train, which will improve beamforming performances.

The goal of this work is to enable intelligent and efficient beam alignment scheme in mmW cellular networks. Rather than simply and exhaustively measuring all possible beam directions, we can measure a smaller part of the beam pairs to achieve comparable performances, thus saving resources like time and power.

Some important issues we consider include: (a) Why does mmW channel have low-rank property? (b) How to take advantage of the low-rank property of mmW channel to perform channel estimation in order to guide beam

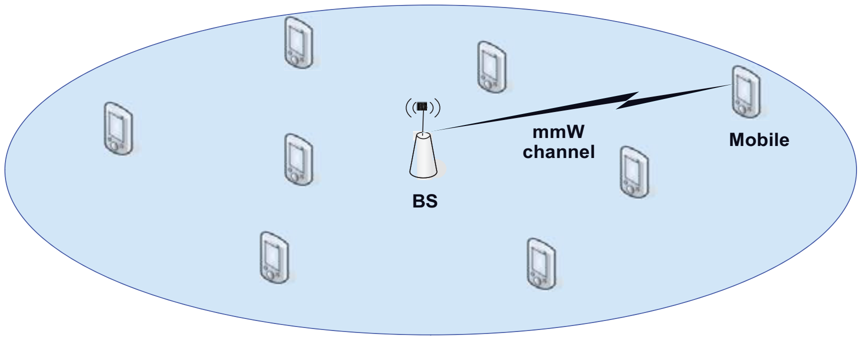


Figure 1. Millimeter wave cellular network.

alignment? (c) How to comprehensively design beam alignment schemes to achieve better performances?

To answer these questions, we'll first present our system model in the next section.

## III. System Model and Motivation

### A. System Structure

In this paper, we focus on the low complexity analog beamforming in the mmW system, where TX or RX can "look" in only one direction at a time.

The structures of transmitter (TX) and receiver (RX) antennas are shown in Figure 2, where TX has $M$ antennas and RX has $N$ antennas. Typical placement options for TX and RX antennas can be 1-dimensional uniform linear array or uniform planar (2-dimensional) array (e.g. $8 \times 8$, $16 \times 16$). We denote the beamforming vectors for TX and RX as $\mathbf{u} \in \mathbb{C}^M$ ($\mathbf{u} = [u_1 \ \ u_2 \cdots u_M]^T$ and $\mathbf{v} \in \mathbb{C}^N$ ($\mathbf{v} = [v_1 \ \ v_2 \cdots v_N]^T$) respectively, where $u_i$ and $v_j$ are determined by antenna steering direction and correspond to the complex weights for TX antenna $i$ and RX antenna $j$, respectively. We assume $\mathbf{u}$ and $\mathbf{v}$ to be unit norm, i.e., $\|\mathbf{u}\| = \|\mathbf{v}\| = 1$. The mmW channel from TX to RX is characterized by a channel matrix $\mathbf{H} \in \mathbb{C}^{N \times M}$.

To ensure high-gain communications between TX and RX, a beam pair $(\mathbf{u}, \mathbf{v})$ needs to be selected. Unless otherwise stated, a beam pair $(\mathbf{u}, \mathbf{v})$ means transmission from TX (with antenna weights $\mathbf{u}$) to RX (with antenna weights $\mathbf{v}$). We will also talk about transmission from RX to TX later in this paper. In order to find the best (or at least, near best) TX-RX beam alignment, TX and RX need to make some measurements from different beam pairs along different directions and further decide which pair is the most appropriate in terms of some metric $R(\mathbf{u}, \mathbf{v})$, e.g. signal-to-noise ratio (SNR).

In order to discover the best beam pair, a straightforward way is to let TX and RX exhaustively scan through (e.g. in spatially adjacent order) all the possible beam pairs. We denote $\mathcal{U}$ and $\mathcal{V}$ as the sets of ALL possible antenna steering vectors for TX and RX, respectively. Therefore, the total number of beam pairs is

$$T = \text{card}(\mathcal{U})\text{card}(\mathcal{V}). \tag{1}$$

The best beam pair $(\mathbf{u}_{opt}, \mathbf{v}_{opt})$ is the one that can maximize a certain performance metric $R(\mathbf{u}, \mathbf{v})$:

$$(\mathbf{u}_{opt}, \mathbf{v}_{opt}) = \arg\max_{(\mathbf{u}, \mathbf{v})} R(\mathbf{u}, \mathbf{v}), \tag{2}$$

$$\text{s.t. } \mathbf{u} \in \mathcal{U} \text{ and } \mathbf{v} \in \mathcal{V}, \tag{3}$$

In mmW communications, due to the large number of narrow beams made possible through the large antenna array that is used to compensate for high path loss, $T$ can be very large. For example, if $\text{card}(\mathcal{U}) = \text{card}(\mathcal{V}) = 64$, $T$ will be as large as 4096. In practice, this exhaustive scheme will significantly reduce the temporal efficiency of the beamforming scheme.

In this work, we aim to reduce the overhead of searching for the beam pairs and also speed up the process of finding better beam pairs by taking advantage of information from mmW channel estimation. To achieve this, TX and RX need to *selectively* measure the link quality for some beam pairs.

In the system we consider, we assume that TX is allowed to dwell in a time slot (TX-slot) with a fixed $\mathbf{u}$, as indicated in Figure 3. To reduce the search overhead, we select a subset of directions to measure the channel, and the number of TX-slots taken is denoted as $I$, and $I < \text{card}(\mathcal{U})$. In a slot $i$ ($i = 1, 2, ..., I$), TX takes the beam direction $\mathbf{u}_i$.

Figure 4 shows that within each TX-slot, the RX can observe $J$ measurements from $J$ different RX beam directions, where $J < \text{card}(\mathcal{V})$. Therefore within $I$ TX-slots, the RX can obtain a total of $L = I \cdot J$ measurements. In a TX-slot $i$, the beam pairs selected will be $(\mathbf{u}_i, \mathbf{v}_j)$ with $j = 1, 2, ..., J$. The selection of $\mathbf{v}$ can be specifically designed (e.g. randomly, or according to a predefined order).

We denote the selected subset of possible TX beams and RX beams as $\mathcal{U}_I \subset \mathcal{U}$ and $\mathcal{V}_J \subset \mathcal{V}$, respectively.

The number of beam pairs we need to test is denoted as $L = \text{card}(\mathcal{U}_I)\text{card}(\mathcal{V}_J) = I \cdot J < T$.

By selectively measuring a small subset of possible beam directions, we can significantly reduce the overhead to find the best beam pair. One of the remaining issues we need to address is how to guarantee the quality of beam direction finding. A more detailed design is given in Section IV.

### B. Signal Representation

Without loss of generality, we now discuss the signal representation in the $j$-th measurement in each TX-slot $i$. For ease of presentation, we may omit the index of TX-slot $i$.

Before transmission on the mmW channel, TX signal is weighted by a steering factor $\mathbf{u}_i \in \mathbb{C}^M$. Then the TX signal transmitted over the mmW channel for the $j$-th measurement is expressed as:

$$\mathbf{x}_j(t) = \mathbf{u}_i \cdot s_i(t), \tag{4}$$

where $\mathbf{u}_i(t)$ is the BF weight for TX antennas in TX-slot $i$ and $s_i(t)$ is the signal to transmit in the $i$-th TX-slot before applying the antenna steering factor with BF weighting. Since $\mathbf{u}_i$ and $s_i(t)$ remain the same within a TX-slot, $\mathbf{x}_j(t)$ also doesn't change within a TX-slot. For $\mathbf{x}_j$, $j$ is only noted to indicate the index of the measurement.

We assume a standard correlated Rayleigh fading model for the mmW channel,

$$\mathbf{H}_j \sim \mathcal{C}N(0, \mathbf{Q}), \tag{5}$$

$$\mathbf{Q} = \mathbb{E}(\mathbf{H}_j \mathbf{H}_j^*), \tag{6}$$

where $\mathbf{H}_j$ is the instantaneous channel matrix ($N$-by-$M$) and $\mathbf{Q}$ is the spatial covariance matrix ($N$-by-$N$).

The receiver cannot directly observe $\mathbf{H}_j$, instead, it observes a noisy version of signals after beamforming and channel effects. For the RX antennas, using a beamforming vector $\mathbf{v}_j$, the receiver has access to

$$y_j(t) = \mathbf{v}_j^* \mathbf{H}_j \mathbf{x}_j(t) + \mathbf{e}_j(t) \tag{7}$$

$$= \mathbf{v}_j^* \mathbf{H}_j \mathbf{u}_i s_i(t) + \mathbf{e}_j(t), \tag{8}$$

where $\mathbf{e}_j(t)$ is the noise in $j$-th measurement slot.

The RX adopts a matched filter to obtain the following measurement:

$$z_j = \frac{1}{\sqrt{E_s}\|\mathbf{u}_i\|\|\mathbf{v}_j\|} \int s_i^*(t) y_j(t) dt, \tag{9}$$

$$E_s = \int |s_i(t)|^2 dt, \tag{10}$$

where $E_s$ is the energy of the transmitted signal $s_i(t)$.

### C. Motivation and Problem

As discussed above, exhaustive search of all beam pairs can be very inefficient and resource-consuming, which motivates us to think: can we just intelligently search/measure a very small part of the set of ALL the possible beam pairs and use the information to help further select better beam pairs? The benefits are two-fold: on the one hand, within the same time provided, the intelligent scheme can find better beam pairs; on the other, in order to achieve a certain beamforming performance level (e.g. SNR larger than a threshold), the intelligent scheme is expected to require less time, cost and overhead. Therefore we propose to measure $L$ beam pairs instead of all the beam pairs and then extract the useful information from measurements obtained to help further search for better beam pairs.

Within each TX-slot, RX will steer its antenna array to different angles to measure the signal strength of different beam pairs. In order to facilitate the efficient beam alignment, there are two major remaining questions to be answered: (1) How to select beam pairs to perform more effective measurement? (2) How to extract more information from the measurements obtained to select a beam-pair with a higher gain? We will further discuss these in the next section.

## IV. Beam-Alignment Design

From the previous discussion, we now need to solve two major problems: (1) How do we choose the beam pairs to measure; (2) After we measure the selected beam pairs, how do we extract the information from the measurements to improve the beam alignment performance? To answer these questions, we will first present the estimation of low-rank mmW channel.

### A. Low-Rank mmW Channel Estimation

In this paper, we use the obtained measurements to estimate low-rank mmW channel to further guide beamforming. Before presenting the details of our design, we need to first look at the low-rank property of mmW channel.

*1) Low-Rank Property of MmW Channel:* In [3], spatial statistical models of mmW channels are derived from real-world measurements at 28 and 73 GHz in New York City. It indicates that at the micro-cell level, receivers in typical measurement locations experience a small number of path clusters, two to three being dominant. Moreover, within each path cluster, the angular spread is relatively small. The covariance matrix of the mmW channel is low-rank in the sense that the paths are clustering into relatively small and narrow beam clusters. The authors in [3] also studied the distribution of energy fraction in spatial directions and the results show that for 28GHz NYC channel, 3 dimensions of spatial directions

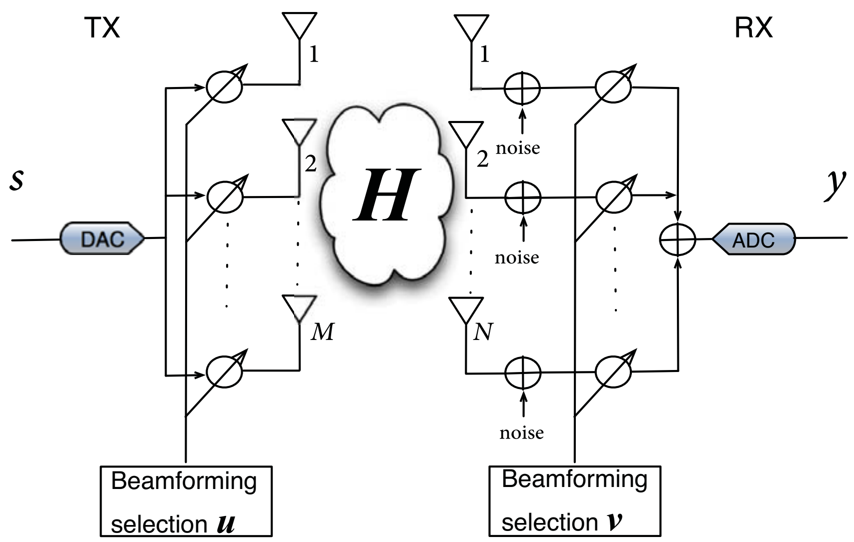


Figure 2. Beam alignment between TX and RX.

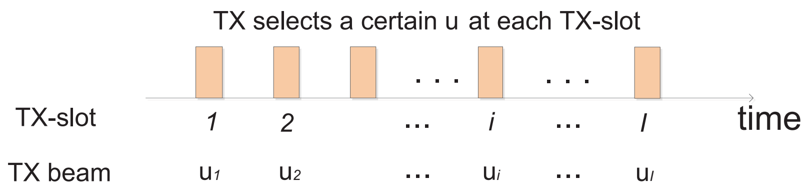


Figure 3. TX dwells with $\mathbf{u}_i$ in TX-slot $i$ $(i = 1, 2, ..., I)$.

can capture 95% of the channel energy for a $4 \times 4$ uniform planar array (which has a dimension of 16).

The low-rank property of mmW channel can be exploited to reduce the number of measurements needed to estimate the spatial covariance $\mathbf{Q}$.

*2) Channel Estimation:* In a given time slot, channel estimation is performed by an RX after it obtains certain number of measurements. As we will discuss in Section IV-B1, the channel information learnt from the first $(J-1)$ measurements will be applied to determine the best RX beam direction of the $J$-th measurement. Specifically, in the channel estimation, the covariance matrix $\mathbf{Q}$ will be estimated from the measurements $z_j$ $(j = 1, 2, ..., J-1)$ based on Equation (9).

Under the assumptions that noise level $N_0$ can be measured and channel gains $\mathbf{H}_j$ are independently faded across different transmissions $(j = 1, 2, ..., J-1)$, we have the following sufficient statistic for the unknown parameter $\mathbf{Q}$:

$$w_J = \sum_{j=1}^{J-1} |z_j|^2. \tag{11}$$

$w$ is a random variable conforming to the following distribution:

$$W_J = \frac{\sum_{j=1}^{J-1} \lambda_j K_j}{2}, \tag{12}$$

where $K_j$ is a chi-squared random variable with 2 degrees of freedom, since channel is assumed to be independently faded across $j$, we can omit the $j$ for $K_j$. Then

$$W_J = \frac{\sum_{j=1}^{J-1} \lambda_j}{2} K, \tag{13}$$

$\lambda_j$ is the energy

$$\lambda_j(\mathbf{Q}) = \mathbf{v}_j^* \left[\mathbf{Q} + \gamma^{-1}\mathbf{I}\right] \mathbf{v}_j, \tag{14}$$

$$\gamma = \frac{E_s}{N_0}. \tag{15}$$

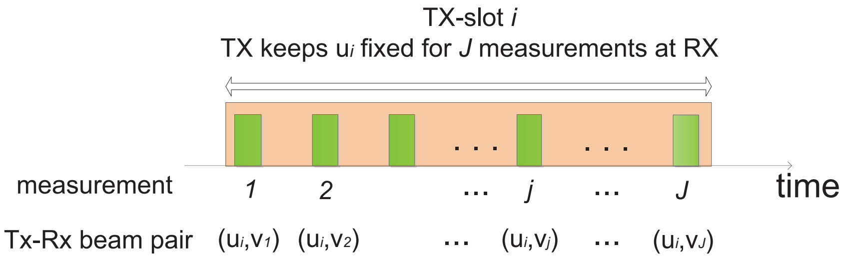


Figure 4. RX makes $J$ measurements with $\mathbf{v}_j$ $(j = 1, 2, ..., J)$ in a TX-slot.

Thus, we have the Maximum likelihood (ML) estimation of $\mathbf{Q}$ as

$$\widehat{\mathbf{Q}} = \arg\min_{\mathbf{Q}} \left[-\log p(w_J|\mathbf{Q})\right], \tag{16}$$

$$\text{s.t. } \mathbf{Q} \geq \mathbf{0}, \tag{17}$$

It can be verified that (16) is equivalent to the following:

$$\widehat{\mathbf{Q}} = \arg\min_{\mathbf{Q}} \left[\log(\sum_{j=1}^{J-1} \lambda_j(\mathbf{Q})) + \frac{w_J}{\sum_{j=1}^{J-1} \lambda_j(\mathbf{Q})}\right], \tag{18}$$

$$\text{s.t. } \mathbf{Q} \geq \mathbf{0}, \tag{19}$$

which can be expressed as

$$\widehat{\mathbf{Q}} = \arg\min_{\mathbf{Q}} J(\mathbf{Q}), \tag{20}$$

$$\text{s.t. } \mathbf{Q} \geq \mathbf{0}, \tag{21}$$

where

$$J(\mathbf{Q}) := \log(\sum_{j=1}^{J-1} \lambda_j(\mathbf{Q})) + \frac{w_J}{\sum_{j=1}^{J-1} \lambda_j(\mathbf{Q})}. \tag{22}$$

To take advantage of the low-rank property of mmW channel, we adopt the increasingly popular technique called matrix completion (MC) [15]. The basic concept of matrix completion is that one can fully recover an unknown low-rank matrix $A \in \mathbb{R}^{n_1 \times n_2}$ with $\text{rank}(A) \ll \min\{n_1, n_2\}$ as long as a small subset of its entries are known. More details can be found in [16], [17].

In this design, to estimate $\mathbf{Q}$, MC cannot be used directly because we can't directly obtain entries of channel covariance matrix $\mathbf{Q}$. Instead, we can get statistic for $\mathbf{Q}$ as in Equation (11), based on which we can estimate $\mathbf{Q}$ through ML estimation, as discussed earlier. A closer look at Equation (14) reveals that $\lambda_\ell$ is a linear function of the unknown matrix $\mathbf{Q}$. Since $w_J$ has an average value that contains $\lambda_j$, it can be considered as a noisy linear measurement of the original $\mathbf{Q}$ matrix. Our objective becomes to recover a low rank $N \times N$ matrix $\mathbf{Q}$ from $J$ noisy linear measurements of it. Due to the low-rank property of $\mathbf{Q}$, the relation between our estimation problem and matrix completion can reduce the requirement of $J$. In order to exploit the benefits of MC, we will place low-rank constraints on the optimization problem in (20).

Some matrix completion methods place low-rank constraints by sparsity regularization [18]–[20]. We then have the regularized optimization problem as follows:

$$\widehat{\mathbf{Q}} = \arg\min_{\mathbf{Q}} J_\mu(\mathbf{Q}), \tag{23}$$

$$\text{s.t. } \mathbf{Q} \geq \mathbf{0}, \tag{24}$$

where

$$J_\mu(\mathbf{Q}) := J(\mathbf{Q}) + \mu||\mathbf{Q}||_1, \tag{25}$$

and $\mu > 0$ is a regularization parameter.

We will use the algorithm presented in [18] to solve (23).

### B. Determination of Beam Measurement Direction

With the low rank feature of mmW channel, in a given time slot, beam measurements may be taken randomly from a subset of directions and MC-based algorithms can be applied to find the complete channel Information with the aforementioned model. In order to increase the transmission capacity, however, there is a need to find the direction with the optimal beamforming gain to transmit, and find the optimal transmission direction with the number of measurements as few as possible.

Rather than simply taking random measurements as done in the conventional sparse sensing studies, in this work, we propose an adaptive measurement scheme with which the following beam directions to measure are determined based on the channel estimation results using the previous measurement data.

In this section, we first introduce the beam selection strategy for a given measurement slot, and then then present how to select the beams to measure in the following time slots.

*1) RX Beam Direction Decision in the Current TX-slot:* In conventional communication scheme involving beamforming, with the channel matrix known, the optimal beamforming direction is determined based on the maximal eigenvector of the estimated covariance matrix of the channel. In this work, we exploit this strategy to select the next beam direction to measure. More specifically, after $(J-1)$ measurements are obtained by the RX, $\mathbf{Q}$ is estimated according to (23). The RX beamforming direction in the following $J$-th measurement is aligned according to the maximal eigenvectors of the estimated covariance matrix $\widehat{\mathbf{Q}}$ [3].

For the discussions above, since $\mathbf{Q}$ is estimated at the RX, we denote the estimation as $\widehat{\mathbf{Q}}_{rx}$. The RX will update its receiving beamforming vector as follows:

$$\widehat{\mathbf{v}} = \arg\max_{\mathbf{v}} \left[\mathbf{v}^* \widehat{\mathbf{Q}}_{rx} \mathbf{v}\right], \tag{26}$$

$$\text{s.t. } \mathbf{v} \in \mathcal{V}, \tag{27}$$

where $\mathcal{V}$, the set of possible directions, is from (1).

We denote the calculated $\widehat{\mathbf{v}}$ in TX-slot $i$ as $\widehat{\mathbf{v}}_i$. Therefore, in the $J$-th measurement of the $i$-th TX-slot, the receiver will align its antennas according to $\widehat{\mathbf{v}}_i$ in (26).

Thus, by taking measurements in TX-slot $i$, RX can estimate the performances of beam pairs tested, $R(\mathbf{u}_i, \mathbf{v}_j)$, $j = 1, 2, ..., J$ with $R(\mathbf{u}_i, \mathbf{v}_J)$ being $R(\mathbf{u}_i, \widehat{\mathbf{v}}_i)$. With $J$ measurements, the best beam pair selected in TX-slot $i$ is

$$(\mathbf{u}_{oi}, \mathbf{v}_{oi}) = \arg\max_{(\mathbf{u},\mathbf{v})} \left[\bigcup_{j=1}^{J} R(\mathbf{u}_i, \mathbf{v}_j)\right] \tag{28}$$

$$= \arg\max_{(\mathbf{u},\mathbf{v})} \left[\bigcup_{j=1}^{J-1} R(\mathbf{u}_i, \mathbf{v}_j) \cup R(\mathbf{u}_i, \widehat{\mathbf{v}}_i)\right]. \tag{29}$$

Once TX has visited $I$ TX slots, the best beam pair of the system can be claimed as:

$$(\mathbf{u}_o, \mathbf{v}_o) = \arg\max_{(\mathbf{u},\mathbf{v})} \left[\bigcup_{i} R(\mathbf{u}_{io}, \mathbf{v}_{io})\right]. \tag{30}$$

In a practical infrastructure, in order for TX and RX to communicate better, TX can attach its direction information in the data transmitted to RX and RX can also transmit some feedback messages as specified in IEEE 802.15.3c [4] (e.g. its best receiving direction, and the quality of the best beam pair) to TX so that TX can know what is the best beam direction for itself so far.

*2) RX Beam Direction Selection in the Next TX-slot:* We have previously introduced how to process the obtained measurements to help select beam direction in the TX-slot where channel estimation is performed. The remaining question is how to determine which beam pairs to measure for each TX-slot because the quality of selected beam pairs will also affect significantly the channel estimation accuracy thus the beam alignment efficiency.

We will randomly select TX beam direction in each TX-slot and focus on the selection of RX beam direction in each measurement.

Based on the theory of eigenvector beamforming, as the mmW channel is estimated at the receiver and the channel covariance matrix $\mathbf{Q}$ from (5) doesn't change dramatically between consecutive TX-slots (though $\mathbf{H}_j$ varies for different $j$ and different TX-slots), we propose to use the channel information from the previous TX-slot to help select the RX beam directions in the current TX-slot (special case: for the very first TX-slot, RX beam directions can be randomly selected). To be more specific, RX will select the beam directions based on the eigenvectors of the estimated covariance matrix $\widehat{\mathbf{Q}}$ [3] from last TX-slot. The procedure for RX to select $(J-1)$ beam directions in each TX-slot can be briefly explained as follows:

Step 1) Obtain the estimated covariance matrix $\widehat{\mathbf{Q}}_{rx}$ of the previous TX-slot.

Step 2) For all $\mathbf{v} \in \mathcal{V}$, where $\mathcal{V}$ is the whole possible RX beam direction set, calculate $\left[\mathbf{v}^* \widehat{\mathbf{Q}}_{rx} \mathbf{v}\right]$.

Step 3) Choose the $(J-1)$ beam directions that give the $(J-1)$ largest value of $\left[\mathbf{v}^* \widehat{\mathbf{Q}}_{rx} \mathbf{v}\right]$.

The proposed beam direction selection scheme aims to select RX beam directions with better quality, therefore it helps make the channel estimation for the current slot more accurate and benefit the RX beam direction decision ($J$-th measurement) presented in Section IV-B1 and also the RX beam direction selection (first $J-1$ measurements) in the next TX-slot.

### C. Integrated Design of Beam Alignment

We have introduced how channel estimation can help RX beam decision in Section IV-B1 and RX beam selection in Section IV-B2. Now we present how these two schemes work in our comprehensive beam alignment design.

The major procedures of the proposed beam alignment scheme in each TX-slot can be summarized as follows:

- Forward transmission.
  For a TX-slot $i$, the transmitter selects TX antenna direction $\mathbf{u}_i$ and transmits to the receiver over the mmW channel. Note that $\mathbf{u}_i \in \mathcal{U}_I$ and $\mathcal{U}_I \subset \mathcal{U}$.
- Receiver beam direction selection.
  Receiver will select the first $(J-1)$ receiving beam directions according to estimated channel information from the previous TX-slot, as discussed in Section IV-B2.
- Receiver measurement.
  The receiver collects $(J-1)$ measurements for the selected $(J-1)$ RX antenna directions $\mathbf{v}_j$ ($j =$

$1, 2, ..., J-1$), as discussed in Section III. Note that $\mathbf{v}_j \in \mathcal{V}_J$ and $\mathcal{V}_J \subset \mathcal{V}$.

- Receiver updates and measurement.
  The receiver estimates the mmW channel ($\mathbf{Q}$) from the previous $(J-1)$ measurements (Section IV-A2) and updates its RX antenna direction according to (26) and make a measurement for the $J$-th measurement.
- Continue to the next TX-slot till termination (all $I$ TX-slots are visited).
- System beam pair determination.
  After $I$ TX-slots, the best TX and RX antenna directions found so far are as (30), the quality of which will show the effectiveness and efficiency of the beam alignment scheme.

Since our design selects only a small part of possible beam pairs to measure, the quality of selected beam directions will significantly affect the performance of the beam alignment. To achieve an effective TX-RX beam direction matching, channel estimation in a TX-slot helps in two aspects: 1) It benefits the decision of RX beam for $J$-th measurement in the current TX-slot; 2) It assists in determining the first $(J-1)$ RX beam directions to be measured in the next TX-slot.

Our design can be described as shown in Algorithm 1.

**Data**: TX-slot index $i$, maximal TX-slot index $I$, RX measurement index $j$, maximal RX measurement number $J$
**Result**: Best estimated beam pair
initialization $i = 1,\ j = 1$;
**while** $i \leq I$ **do**
    RX obtains the estimated covariance matrix $\widehat{\mathbf{Q}}_{rx}$ of the previous TX-slot and chooses the $(J-1)$ receiving beam directions $\mathbf{v}_j (j = 1, 2, ...J-1)$ that give the $(J-1)$ largest value of $\left|\mathbf{v}^* \widehat{\mathbf{Q}}_{rx} \mathbf{v}\right|$, where $\mathbf{v} \in \mathcal{V}$, $\mathcal{V}$ is the whole possible RX beam direction set;
    TX selects direction $\mathbf{u}_i$ and transmits to RX;
    **if** $j \leq J-1$ **then**
        RX collects measurements using direction $\mathbf{v}_j$;
    **else**
        RX estimates the mmW channel ($\mathbf{Q}$) from the previous $(J-1)$ measurements (Section IV-A2) and updates its $J$-th receiving direction according to (26);
        RX makes the $J$-th measurement using the updated direction;
    **end**
**end**
The best beam pair are found according to (30).

**Algorithm 1:** Beam Alignment

## V. Simulations and Results

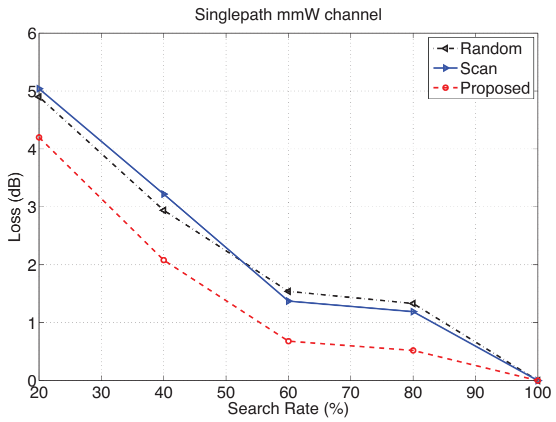


Figure 5. Search effectiveness for singlepath channel.

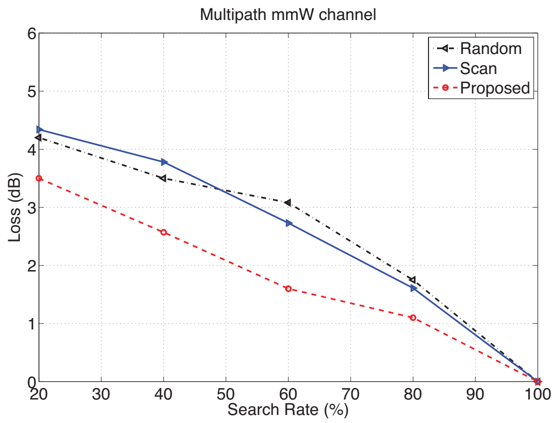


Figure 6. Search effectiveness for multipath mmW channel.

In this section, we will perform simulations to show the effectiveness and efficiency of the proposed design. We will show the results in two major scenarios: singlepath mmW channel and multi path mmW channel derived from NYC measurements [3] .

Some straight-forward beam alignment schemes have been used in existing studies and protocols, include randomly selecting beam pairs or exhaustively scan all possible beam pairs. In practical networks, purely random schemes can be very ineffective and exhaustive schemes can be very time-consuming. To validate the efficiency of our proposed scheme, we will compare the performances of the proposed scheme with those of two other schemes, *Random* and *Scan* defined as follows:

- *Random*
  For each measurement, $\mathbf{u}_i$ and $\mathbf{v}_j$ are randomly

selected.

- *Scan*
  At the beginning of the scheme a starting beam pair is selected, and then for each following measurement, the next $\mathbf{u}_i$ and $\mathbf{v}_j$ can only be chosen from the beam direction that is spatially adjacent to the previous beam direction.
  Instead of randomly hopping from direction to direction as in *Random*, this scheme can only *scan* the directions according to spatial order.

Note that in the above two schemes and our proposed methodology, beam pairs won't be measured repetitively. That is, if a beam pair has already been measured, it will no longer be measured.

Each scheme, *Random*, *Scan* and the proposed one, will search beam pairs according to its rules and then find one pair with the highest SNR.

### A. Simulation Settings

In the simulations, we assume the TX has $4 \times 4$ $\lambda/2$ uniform 2-dimensional planar arrays, and the RX has $8 \times 8$ $\lambda/2$ uniform 2-dimensional planar arrays. The mmW channel is generated from the model derived from NYC measurements in [3].

The metric we will use to evaluate the performance of a beam pair is the SNR degradation compared with the SNR value obtained at the optimal beam pair, as mentioned in Section III. We define the optimal SNR as $R_{opt} = R(\mathbf{u}_{opt}, \mathbf{v}_{opt})$ and the actual SNR obtained for beam pair $(\mathbf{u}, \mathbf{v})$ as $R(\mathbf{u}, \mathbf{v})$. Then the SNR loss for this beam pair in decibels is defined as:

$$\text{Loss}(dB) = 10 \log_{10} \left[ \frac{R(\mathbf{u}, \mathbf{v})}{R_{opt}} \right]. \tag{31}$$

The smaller the loss, the better the beam pair selected.

We also evaluate the *Search Rate*, which is defined as the number of measured beam pairs ($L$) normalized to all the possible beam pairs $T$, that is:

$$\text{Search Rate} = \frac{L}{T}. \tag{32}$$

### B. Search Effectiveness

We will now see the performances of different schemes in terms of SNR Loss for different search rates, which shows the search effectiveness of each scheme.

Figure 5 and 6 show the search effectiveness of various beam alignment schemes for singlepath and multipath channel, respectively. We can see that for the same Search Rate, our proposed scheme always outperforms *Random* and *Scan* (approximately 1 dB) with lower SNR Loss. The reason is that our proposed scheme takes advantage of the information from the beam pairs that have been already measured to further guide the selection of future beam pairs for measurement. To be more specific, we adopt the measurements to first estimate the mmW channel by exploiting its low-rank feature and then use the estimated channel information to suggest better beam pair measurement.

When the Search Rate is $100\%$, all three schemes reduce to exhaustive scan, which can find the optimal beam pair at the cost of large search delay.

### C. Cost Efficiency

For a targeted SNR Loss, each scheme will continue searching beam pairs until the obtained Loss is smaller than the targeted SNR Loss threshold. Therefore a certain Search Rate requirement will need to be met. This required Search Rate for a specific Loss indicates the cost efficiency of the scheme. When more beam pairs are searched, the overhead will become higher (e.g. time, energy, computational complexity).

Figures 7 and 8 show the cost efficiency of different schemes in singlepath and multipath channel, respectively. We observe that our proposed scheme requires smaller number of beam pairs in order to find a beam pair as good as the other two schemes, *Random* and *Scan*. For a target Loss, our scheme generally requires up to $25\%$ less the number of total possible beam pairs, which can be a huge cost saving under the circumstances that the number of all possible beam directions is large.

Note again that for Search Rate of $100\%$, all three schemes reduce to the exhaustive scan, which can find the optimal beam pair (no loss) at the cost of large search delay ($100\%$ search rate).

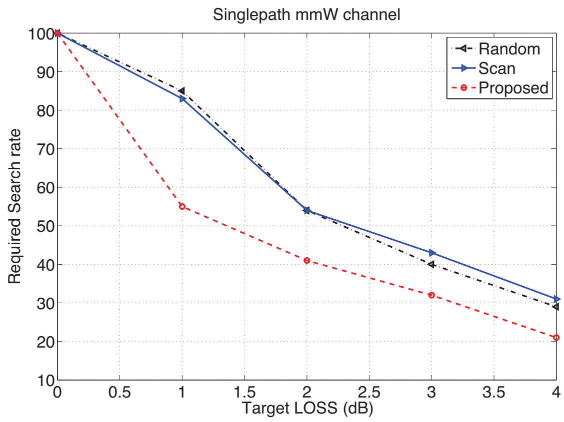


Figure 7. Cost efficiency for singlepath channel.

## VI. Conclusion

This paper presents a beam alignment scheme to efficiently perform directional beam direction matching in mmW cellular networks, where the transmitter and receiver need to jointly decide the beam directions to

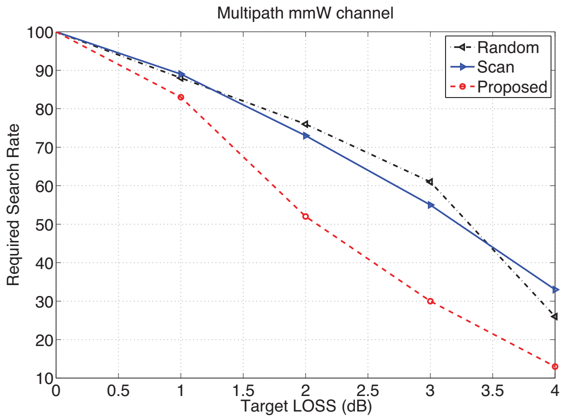


Figure 8. Cost efficiency for multipath mmW channel.

combat the large path loss in mmW range. Unlike the exhaustive search scheme which searches all the possible beam pairs at the cost of severe delay and overhead, we propose to measure only a small subset of the beam pairs to enable efficient beam alignment. To achieve this goal, we estimate the mmW channel based on its low-rank property, and then exploit the channel information to help guide more effective future beam pair measurement and selection. Simulation results demonstrate the significant advantages of our design in search effectiveness and cost efficiency.

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