CONCURRENT EXPLORATION OF MIMO RADAR AND CO-PRIME ARRAY FOR FASTER AND HIGHER RESOLUTION SCANNING

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ABSTRACT

There are a lot of recent research interests on MIMO radars. Transmitting orthogonal waveforms, the extra degrees of freedom (DoFs) provided by MIMO radar can increase the detection and estimation performance. Besides, the recent work on nested array and co-prime array also bring encouraging results. But whether these two ideas can be employed together remains an open problem. In this work, we investigate the potential benefit of using the two techniques together. Our analysis indicates that the use of the two techniques can speed up the scanning process. Compared to conventional Uniform Linear Array (ULA), our performance studies through simulations further demonstrate the use of co-prime scanning can significantly improve the detection probability and resolution, and reduce the number of antennas needed for a given number of targets.

Index Terms— MIMO radar, co-prime array, degrees of freedom (DoFs), scanning.

1. INTRODUCTION

In conventional phased array technique, all the transmitting antennas transmit coherent signals, and the array is controlled to generate a beam that can illuminate a $2\pi/N_t$ space at a time, where N_t is the number of transmitting antennas. The phases of different antenna elements can be controlled so the beams can be generated along different directions to scan the whole space of monitoring.

Rather than transmitting the same signals from all antennas, the recent advance of MIMO radar technique allows the transmission of orthogonal waveforms from different antennas or antenna groups to increase the degree of freedom, which further helps to improve the detection and estimation performance. The idea of MIMO virtual array [1] exploits the use of co-located radar with the element spacing of transmitting antennas N_r times that of the receiving antennas. This allows the establishment of a virtual MIMO array at the receiver with N_tN_r degrees of freedom(DoFs) using only $N_t + N_r$ elements, and also allows the formulation of sharper beams compared to the conventional transmission of coherent signals using beam-forming.

As an alternative technique, the concept of co-prime array has recently been introduced in [2], where an array is formed with a pair of uniform linear arrays (ULA), each with spacing $M\lambda/2$ and $N\lambda/2$ separately, with λ the wavelength of an impinging narrow band wave, M and N co-prime numbers. It has been shown the difference Co-Array generated from the two sub-arrays has more than MN degree of freedom. Compared to MIMO radar, as M and N are generally larger than 1, the minimum distance between antenna elements in co-prime arrays is bigger which helps reduce the mutual coupling between elements. On the other hand, the total radar aperture size is smaller which helps to reduce the radar occupancy space and cost. The co-prime array could also be explored to generate narrow beam patterns without using multiple waveforms and matched filter banks. Some example application of co-prime array are the finding of direction of arrivals (DoAs) and the scanning of targets. In either case, the co-prime feature allows the generation of narrower beam to improve the resolution.

In existing studies on the application of co-prime array [2], each or both sub-arrays of a co-prime array work in coherent mode where all the antennas of an array transmit the same signal. In this work, we would like to investigate the potential benefit of transmitting orthogonal waveform like that done MIMO radar case using a co-prime array. In particular, we apply the co-prime array for signal scanning.

The remainder of the paper is organized as follows. In Section 2, we introduce our system model and formulate our problem. Section 3 presents our scanning method with orthogonal waveforms and co-prime array. Several supporting simulation results are provided in Section 4 and conclusions are drawn in Section 5.

2. PROBLEM FORMULATION

We consider a co-located radar where the antennas can serve as transmitters or receivers at different time, and we have $N_T = N_R = N$. Instead of using ULAs, the total number of antennas are divided into two groups with N_1 and N_2 antennas respectively, where N_1 and N_2 are co-prime numbers. That is, the two groups of antennas form a co-prime array which has N_1 and N_2 number of antennas and the interval between elements are N_2d and N_1d. Fig. 1 shows the configuration of the two groups of antennas.

To explore the benefits of using multiple waveforms with the co-prime array, we let each array transmit a signal with the superposition of more than one orthogonal waveform. Thus the two groups of transmitted signals are

$$r_{n_1}(t) = \sum_{k=0}^{K-1} \phi_{n_1,k} u_k(t) e^{j\omega_c t},\ n_1 = 0, 1, ..., N_1 - 1 \quad (1)$$

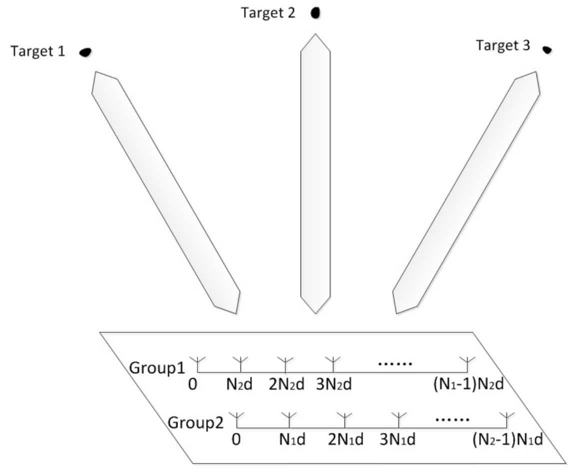

Fig. 1: Antennas configuration

$$r_{n_2}(t) = \sum_{k=0}^{K-1} \phi_{n_2,k} u_k(t) e^{j\omega_c t},\ n_2 = 0, 1, ..., N_2 - 1 \tag{2}$$

where K is the number of orthogonal waveforms; $\phi_{n_1,k} and \phi_{n_2,k}$ are weights. The received signal at a receiver m can be represented as:

$$\begin{aligned} r_m(t) = (&\sum_{n_1=0}^{N_1-1}\sum_{k=0}^{K-1} \phi_{n_1,k} u_k(t-\tau) e^{j\omega_c(t-\tau-\overrightarrow{\tau_{n_1}})} + \\ &\sum_{n_2=0}^{N_2-1}\sum_{k=0}^{K-1} \phi_{n_2,k} u_k(t-\tau) e^{j\omega_c(t-\tau-\overrightarrow{\tau_{n_2}})}) e^{j\omega_c \overrightarrow{\tau_m}} + v(t) \end{aligned} \tag{3}$$

where τ is the round-trip distance between the array and the target and τ_{n_1}, τ_{n_2} are the delay parameters between the reference antenna and the antenna n_1 and n_2 respectively; $v(t)$ is the Gaussian noise.

In [2], co-prime array is exploited to scan the target, with one group serving as the transmitter and the other group serving as the receiver. With the proposed technique, N_1 scans are needed to scan the whole monitoring space. In this work, we will exploit use of orthogonal waveforms to take full advantage of all available antennas to speed up the scanning process.

3. SCANNING WITH ORTHOGONAL WAVEFORMS

To take advantage of both multi-waveform transmission and coherent beam pattern, we use only two orthogonal waveforms and assign each of them to the co-prime array. We set the first antenna array interval to $N_2 d$ and the second antenna array interval to $N_1 d$, where $d = \lambda/2$. Therefore the signal at a receiver m becomes:

$$\begin{aligned} r_m(t) = (&\sum_{n_1=0}^{N_1-1}\sum_{k=0}^{K-1} \phi_{n_1,k} u_k(t-\tau) e^{j\omega_c(t-\tau-n_1 N_2 d c^{-1} sin\theta)} + \\ &\sum_{n_2=0}^{N_2-1}\sum_{k=0}^{K-1} \phi_{n_2,k} u_k(t-\tau) e^{j\omega_c(t-\tau-n_2 N_1 d c^{-1} sin\theta)}) e^{j\omega_c \overrightarrow{\tau_m}} + v(t) \end{aligned} \tag{4}$$

After going through the demodulation and matched filter (Here we actually have two matched filters for two waveforms), the signal received can be divided into two parts plus Gaussian noise:

$$\begin{aligned} r_m &= (\sum_{n_1=0}^{N_1-1}\sum_{k=0}^{K-1} \phi_{n_1,k} e^{j\omega n_1 N_2} + \sum_{n_2=0}^{N_2-1}\sum_{k=0}^{K-1} \phi_{n_2,k} e^{j\omega n_2 N_1}) e^{j\omega_c \overrightarrow{\tau_m}} + V \\ &= (\sum_{n_1=0}^{N_1-1} \phi_{n_1,1} e^{j\omega n_1 N_2} + \sum_{n_2=0}^{N_2-1} \phi_{n_2,2} e^{j\omega n_2 N_1}) e^{j\omega_c \overrightarrow{\tau_m}} + V \end{aligned} \tag{5}$$

where $\omega = \pi sin(\theta)$. The two orthogonal signals can utilize the in-phase quadrature signals in communication which is easy to implement.

Now the next step is to design $\phi_{n_1,k}, \phi_{n_2,k}$ so that the beam can cover the scanning space. This can be done similarly to that in [2]. We configure the two sub-arrays as two ideal low-pass filters:

$$H_i(e^{j\omega}) = \sum_{n_i=0}^{N_i-1} \phi_{n_i,1} e^{j\omega n_i},\ i = 1, 2 \tag{6}$$

Assume $H_1(e^{j\omega})$ and $H_2(e^{j\omega})$ have passbands $[-\frac{\pi}{N_1}, \frac{\pi}{N_1}]$ and $[-\frac{\pi}{N_2}, \frac{\pi}{N_2}]$ respectively. Then $H_1(e^{j\omega N_2})$ has N_2 passbands with each passband having width $\frac{2\pi}{N_1 N_2}$; $H_2(e^{j\omega N_1})$ has N_1 passbands with each passband having width $\frac{2\pi}{N_1 N_2}$. Thus, the received signal becomes:

$$r_m = (H_1(e^{j\omega N_2}) + H_2(e^{j\omega N_1})) e^{j\omega_c \overrightarrow{\tau_m}} + V \tag{7}$$

The central frequency of these bandpass filters are at:

$$\begin{aligned} \frac{2\pi n_2}{N_2} + \frac{2\pi l_1}{N_1 N_2} &= (N_1 n_2 + l_1)\frac{2\pi}{N_1 N_2},\ 0 \le n_2 \le N_2 - 1 \\ \frac{2\pi n_1}{N_1} + \frac{2\pi l_2}{N_1 N_2} &= (N_2 n_1 + l_2)\frac{2\pi}{N_1 N_2},\ 0 \le n_1 \le N_1 - 1 \end{aligned} \tag{8}$$

l_1, l_2 are phase shift parameters that are controlled by transmitting weight.

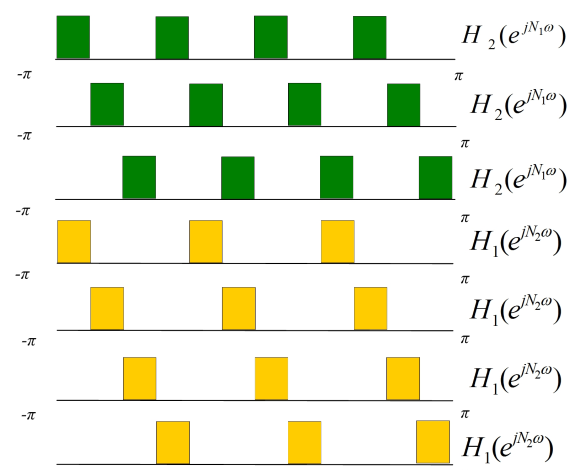

Fig. 2: An example of filter passbands in which case $N_1 = 4, N_2 = 3$

When using one sub-array as transmitter and the other has the receiver, there is one and only one overlapped passband (as shown in Fig. 2). Taking advantage of orthogonal waveform transmission, we can form two transmission and receiving pairs, one with sub-array1 as transmitter and sub-array2 as receiver, and another using sub-array2 as transmitter and sub-array1 as receiver. Thus the total number of different passbands during this scanning process is $N_1 + N_2 - 1$. To cover the whole monitoring space, we would need $N_1 N_2$ such different sharp passbands. The number of scanning times will be at least $\lceil \frac{N_1 N_2}{N_1+N_2-1} \rceil$. Compared to using only one waveform which needs N_1 scans [2], we only need $\frac{N_2}{N_1+N_2-1}$ of the scanning time.

In order to detect the targets, we need to detect these $N_1 + N_2 - 1$ possible bandpass signals. This needs the design of receiver beam patterns. We multiply each output signal a weight. Since we have matched filters for two waveforms, we can separate two different waveform signals. So the two extracted signal vectors at receivers are:

$$\begin{aligned} \overrightarrow{r_{M_1}} &= [r_1^1, r_1^2, ..., r_1^{N_1-1}]^T \\ \overrightarrow{r_{M_2}} &= [r_2^1, r_2^2, ..., r_2^{N_2-1}]^T \end{aligned} \tag{9}$$

$$r_m^n = \psi_m^n H_m(e^{j\omega N_{1,2}} e^{2\pi l_m/N_m}) + V_m \tag{10}$$

To fully explore the advantages of co-prime array, since this is a co-located radar, we can let the second sub-array output the first waveform signal while the first sub-array output the second waveform signal. If ψ_m^n is the weight assigned to each output for beamforming, the two output beams will be:

$$\sum_{n_2=0}^{N_2-1} H_1(e^{j\omega N_2} e^{j2\pi l_1/N_1}) e^{j\omega n_2} \psi_1^{n_2} \tag{11}$$

$$\sum_{n_1=0}^{N_1-1} H_2(e^{j\omega N_1} e^{j2\pi l_2/N_2}) e^{j\omega n_1} \psi_2^{n_1} \tag{12}$$

As in [2], if we treat $\psi_1^{n_2}, \psi_2^{n_1}$ as the coefficients of two passband signals, the signals thus the targets can be detected.

In summary, to exploit the use of co-prime array for scanning, the active sensing process consists of the following procedures:

1. Design and transmit two proper phase controlled beams so that they can cover $N_1 + N_2 - 1$ out of $N_1 N_2$ passbands; the total number of scanning slots will be $\lceil \frac{N_1 N_2}{N_1+N_2-1} \rceil$.

2. The first sub-array uses the matched filter to intercept the second waveform signal while the second sub-array applies the matched filter to intercept the first waveform signal. If both signals have been received, we know the signal is only possible transmit from one passband, from which we can find the direction of the target.

3. If the signal is detected or the whole space is scanned, terminate and output; otherwise go back to 1 to scan another direction.

4. PERFORMANCE EVALUATION

Besides the benefit of improving the scanning speed based on our analysis in the previous section, in this section, we further evaluate the performance of our proposed signal scanning method using co-prime array through simulations over matlab. We use Maximum-Likelihood algorithm to estimate the directions of a group of randomly distributed targets. We compare the performance with the traditional method using uniform linear array (ULA).

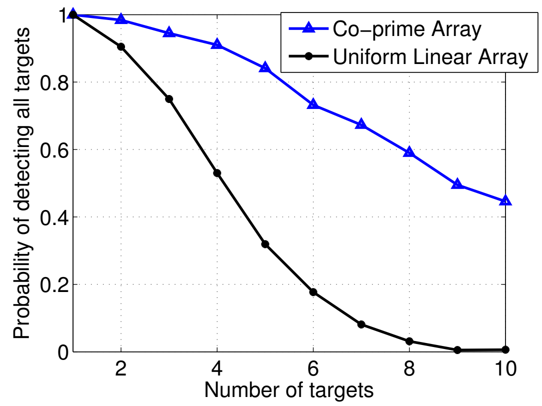

Fig. 3: Probability of detecting all targets versus number of targets

In the first study, we consider the impact of different number of targets, with the targets randomly distributed in the range $[-2\pi, 2\pi]$. We use our scanning method with a co-prime array configuration of $N_1 = 10$ and $N_2 = 11$, i.e., the total number of physical antennas is $N_1 + N_2 = 21$. We compare the probability of successfully detecting all targets with the traditional uniform linear array with the same number of antennas, 21 in this case. We vary the number of targets from 1 to 10. From the result in Fig. 3, we can see that our method using co-prime array has a much better chance of capturing all targets. When the number of targets increases, the probability of two targets locate in the same angular space will increase, which makes it difficult to distinguish two targets. Taking advantage of co-prime scanning, the maximum available DoF in our method is $N_1 N_2 = 110$ and the theoretical resolution is $\frac{2\pi}{N_1 N_2} = \frac{2\pi}{110}$, which is significantly higher than that of the traditional uniform linear array method whose resolution is $\frac{2\pi}{N_1+N_2} = \frac{2\pi}{21}$.

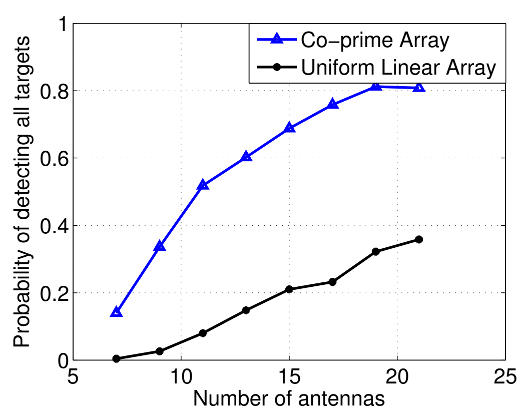

Fig. 4: Probability of detecting all targets versus number of antennas

We then consider adding the number of antennas when the number of targets is fixed. Adding more antennas will enhance DoFs and improve the resolution as well. Our goal is to show that by applying our method, we can take advantage of the extra DoFs provided by the co-prime array to achieve a better detection performance. We have the number of targets set to 5. From the result in Fig. 4, we can see that with increase of the number of antennas, the probability of detecting all 5 targets using co-prime array method is more than 2 times that using traditional uniform linear array method, which demonstrates that co-prime scanning technique could use antennas more efficiently.

Finally, we focus on the number of antennas required to detect different number of targets given a detection probability threshold which is 0.8 in our case. In practice, we always want to use as few antennas as possible to obtain a smaller radar aperture and lower energy consumption. We can see from Fig. 5 that our method with co-prime array can save more than 90% number of antennas when the number of targets is greater than 10. We can also conclude that with higher number of targets, the antenna saving will be larger.

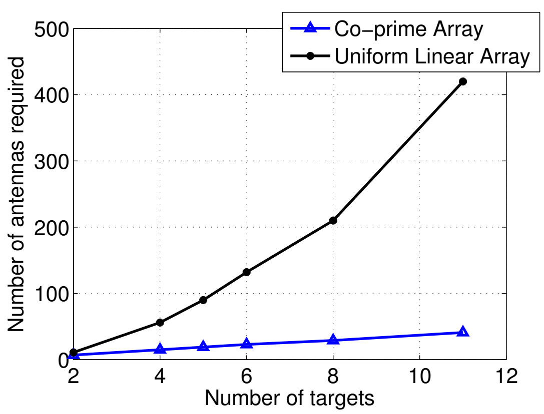

Fig. 5: Probability of detecting all targets versus number of targets

5. CONCLUSION

MIMO radars have been heavily investigated in the past several years. On the other hand, co-prime array has been proposed lately to demonstrate its potential of increasing the degree of freedom and the resolution of detection and estimation at lower cost. In this work, we investigate the potential of concurrently exploiting both array techniques to improve detection performance. Our analysis indicates that the scanning time can be significantly reduced by taking advantage of the orthogonal waveforms and co-prime features of the array. Compared to ULA, our performance studies through simulations further demonstrate the use of co-prime scanning can significantly improve the detection probability and resolution while reducing the number of antennas needed thus the radar cost for a given number of targets to detect.

6. REFERENCES

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