A Hybrid Model For Short-Term Traffic Volume Prediction In Massive Transportation Systems

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Abstract—The prediction of short-term volatile traffic becomes increasingly critical for efficient traffic engineering in intelligent transportation systems. Accurate forecast results can assist in traffic management and pedestrian route selection, which will help alleviate the huge congestion problem in the system. This paper presents a novel hybrid DTMGP model to accurately forecast the direction of passenger flows multi-step ahead with the comprehensive consideration of factors from temporal, origin-destination spatial, and frequency and self-similarity perspectives. We first apply discrete wavelet transform to decompose the traffic volume series into an appropriate component and several detailed components. Then we propose a more efficient tracking model to forecast the appropriate component and a novel Gaussian process model to forecast the detailed components. The forecasting performance is evaluated with real-time passenger flow data in Chongqing, China. Simulation results demonstrate that our hybrid model can achieve on average 20%–50% forecasting accuracy, especially during rush hours.

Index Terms—Passenger flow prediction, wavelet decomposition, Gaussian process (GP).

I. INTRODUCTION

The accelerated urbanization process and urban population explosion bring great pressure to the urban traffic management. In order to withstand the high traffic pressure especially during the morning rush hours or holidays, many cities have been planning and building rail transit systems. In addition, smart cards are promoted to speed up the traffic flow in railway stations and facilitate traffic management.

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A well-designed transportation system is a key element in the economic welfare of major cities [1]. For the efficient operation of urban transit system, planners must have a quantitative understanding of traffic patterns, and be able to predict and prevent any disruptions, either planned or unplanned [2]. More specifically, to enable real-time traffic management of subway operations and support functions such as regulating station passengers, it is important to accurately forecast the volume of station passenger flows. Passengers enter the light rail system from the origin station and leave the system from the destination station. Given an origin-destination (OD) station pair in a light rail system, the key questions that need to be answered are: (1) How to predict the number of passengers that exit from the destination station at time t? (2) How to improve the average prediction accuracy per day? (3) How to improve the prediction accuracy at rush hours so as to find the crowded moment that passenger flows reach the peak and become much greater than the historical average?

Traffic volume forecasting is fundamental to the performance of many components in intelligent transportation systems. Great efforts have been devoted to improve the forecasting accuracy and the transportation efficiency. Designed based on purely spatial or purely temporal information, the forecasting performance of most known approaches [3]–[24] is often low. The traffic patterns in the massive transportation systems are affected by various factors. Several recent studies [25]–[31] try to improve the traffic forecasting accuracy by decomposing the traffic data into different frequency components to understand the flow evolution from the frequency perspective. Hybrid methods are also proposed to identify the daily traffic patterns and some other flow patterns to improve the forecasting accuracy [32]–[34].

Although the approaches based on the frequency analysis present good performance on capturing the overall traffic trend, most of them fail to track the local fluctuation of passenger flows. High frequency sub-signals extracted by the frequency domain analysis are still fitted by traditional non-linear models, which can’t extract enough information to make accurate prediction. In addition, very limited studies are made on the relationship between an OD flow between a pair of stations and the overall passenger flow leaving from the same origin. Our preliminary studies indicate that the OD flow often has a high volatility when the corresponding passenger flow exhibits intense oscillations or steeply changes.
In this work, we propose a novel hybrid model to more accurately forecast the volume of passenger flows multi-step ahead with the comprehensive consideration of factors from temporal, OD spatial, frequency and historical probabilistic distribution perspectives. Our contributions are summarized as follows:

- To extract flow features from both the temporal and frequency domains, we apply discrete wavelet transform (DWT) to decompose the traffic volume series into a set of sub-signals.
- To deal with the chaos and uncertainty of traffic flow, we apply the state space reconstruction to preprocess the detailed components extracted by DWT. Our analysis of real passenger flow data reveal that traffic time series belong to a type of chaotic signals, which are commonly handled with the state space reconstruction method.
- To capture temporal and spatial OD features, we propose a more efficient tracking model to timely adapt the predicted value of exit passenger flow according to the entrance passenger flow.
- To better follow the dynamic patterns of flows, we propose a novel Gaussian Process (GP) model to forecast the detailed components from the probabilistic perspectives. We exploit the simultaneous predictions, with each based on the state space reconstructed with a different time delay between two adjacent components of the state vector.
- To evaluate the performance of our proposed hybrid model, we have performed extensive simulations based on real passenger flow data. Simulation results demonstrate that the proposed hybrid model could significantly improve the prediction accuracy, especially in rush hours.

The rest of this paper is organized as follows. Section II summarizes the related work on the recent popular techniques for the prediction of traffic volume. The introduction of the background and the analysis of data are given in Section III and Section IV, respectively. Section V presents the technical details of our hybrid prediction model. We evaluate the performance of our proposed model through simulations driven by the real-world data sets in Section VI, and conclude our work in Section VII.

II. RELATED WORK

In this section, we review the related work on traffic volume forecasting, and identify the differences of our work from the existing work.

Existing traffic forecasting techniques can be mainly divided into two categories, parametric and non-parametric [3], [4]. The common parametric techniques include the classical Autoregressive Moving Average (ARIMA) [5]–[8] and Kalman filtering model [9], [10]. Because of the fast and easy operation, they are widely used in transportation systems in the early stages. Due to the stochastic and nonlinear nature of traffic flow, researchers have also paid much attention to nonparametric methods in the traffic flow forecasting field. Wu et al. [14] utilize support vector regression (SVR) with a radial basis function kernel to predict the travel time in a transportation network. In [15] and [16], an online version of SVR is employed to solve the same problem. Oswald et al. [17] use the nearest neighbor approach to forecast the traffic flow. Yu et al. [18] apply Gaussian mixture model and expectation-maximization to estimate the density functions of transition probability and predict traffic flow. Lv et al. [19] propose the deep-learning-based traffic flow prediction, which considers the spatial and temporal correlations inherently. Methods proposed in [20]–[24] attempt to capture the process dynamics and improve forecasting accuracy based on GP model. These methods can generally capture the long-term trend of the traffic flow, but fail to capture the traffic short-term fluctuations while the overload is one major factor that impacts the performance of transportation systems.

In recent years, a number of approaches are proposed to better capture the traffic features by decomposing the traffic data into different components. Hu and Wang [25] and Agarwal et al. [26] apply discrete wavelet transform (DWT) to make short-term wind speed prediction and detect traffic incidents. Wei and Chen [27] develop a hybrid EMD-BPN forecasting approach that combined empirical mode decomposition (EMD) with back-propagation neural networks (BPN) to predict short-term passenger flow in metro systems. Wang et al. [28] propose a least squares support vector regression ensemble learning model based on the seasonal decomposition for the forecast of the Chinese hydropower consumption. Xing et al. [29] introduce the Robust Principal Component Analysis for passenger flow decomposition. This method can more accurately and robustly capture the underlying temporal and spatial characteristics of passenger flow in the presence of all kinds of fluctuations. Zhang et al. [30], [31] present a hybrid model for multi-step ahead passenger flow forecasting based on spectral analysis technique, ARIMA model and generalized autoregressive conditional heteroskedastic model. Decomposing traffic data into different components can help more accurately track the temporal evolution of the traffic flow at different time resolutions and improve traffic forecasting accuracy. However, the performance of existing approaches is often low and not stable. Most existing studies use the same method to handle different components without considering their difference, and simply fit the high frequency components with traditional non-linear models that cannot follow well the sophisticated behaviors of transportation systems.

Some recent studies attempt to apply hybrid methods that can take advantage of different models to identify both the common daily traffic trend and some variations in flow patterns to improve the forecasting accuracy. Wang and Shi [32] propose a traffic speed forecasting hybrid model based on support vector machine (SVM), DWT and phase space reconstruction. In [33], a hybrid modeling approach which combines artificial neural networks with a simple statistical is studied to make prediction for traffic flow. Silva et al. [34] propose a new approach to analyze massive transportation systems that used data obtained from smart cards in the London transport system to predict future passenger flow. These methods can achieve higher overall forecasting accuracy, but they often fail to identify the time of congestion or fail to accurately predict the
congestion degree in rush hours. This is because these models still cannot extract enough information from traffic flow data for forecasting.

In this paper, we design a novel hybrid method with the comprehensive consideration of several factors, including temporal and frequency signal features, spatial relationship among OD pairs, and probabilistic evolution of passenger flows. Therefore, we can extract more valuable information from data to improve the forecasting accuracy. Different from previous studies, we deal with the approximation component and detailed components generated by DWT in different ways. To capture temporal and OD spatial features, we apply a more effective tracking model to predict the approximation component. We further propose a mixed time delay Gaussian process (MTGPR) model to predict detailed components with the intention of achieving the least predicted variance. Gaussian Process models are nonparametric kernel-based probabilistic models.

III. PRELIMINARIES

Before presenting our detailed design for more accurate forecast of passenger flows, we provide some background knowledge on related techniques.

A. State space reconstruction

The number of customers in the transportation system often varies over time due to the impact of factors such as climate, weather, holidays, peak hours and events. These factors, either expected or uncertain, could lead to complicated changes of the customer flows. These changes are hard to be tracked by a conventional linear system.

For the analysis of non-linear time series, Froehling et al. [35] introduce the concept of phase space reconstruction and the chaos theory. Based on the theory, all the dynamical information needed for determining a system state is included in the time series of any a system variable. A state trajectory constructed from a single-variable time series \( \{y_i, i = 1, 2, \cdots, N\} \) of the length \( N \) maintains the foremost characteristics of the state trajectory in the original space. The reconstructed phase space can be represented as

\[
x_i = (y_i, y_{i+\tau}, \cdots, y_{i+(d-1)\tau}), \quad i = 1, 2, \cdots, N \quad -(d-1)\tau
\]

where \( x_i \) is a state in the reconstructed phase space, \( d \) is the embedding dimension, and \( \tau \) is the delay between adjacent elements of a state in the reconstructed phase space.

According to the Takens Theorem, if \( d \geq 2D + 1 \), where \( D \) is the dimension of the dynamic system, the reconstructed dynamic system and the original one are topologically equivalent. The system state of the next moment \( x_{i+1} \) can be predicted from the current state of the system. That is, acquiring the predication value for the next instant of the time series provides a basis for predicting chaos time series.

Determining an acceptable minimum embedding dimension \( d \) and the time delay \( \tau \) between measurements for the state space reconstruction is crucial [36], [37]. Given an appropriate value of \( d \), it is difficult to ensure that a chosen value of the delay \( \tau \) is optimal all the time. As reported from past studies, if \( \tau \) is too small, the system’s dynamical characteristics will not be revealed; if it’s too big, a simple trajectory will be made complicated, which can reduce the number of effective data points. We will discuss in Section VI how to make use of multiple delay values to increase the prediction accuracy.

B. Gaussian process regression (GPR)

Gaussian process regression is an applicable and practical probabilistic approach based on statistical learning theory and Bayesian theory, which is widely used for state and performance prediction in a variety of fields. A Gaussian process model seeks to establish a mapping of the form

\[
y = f(x)
\]

between the predictor (output) \( y \in \mathbb{R} \) of a complex dynamical system and the input vector \( x \in \mathbb{R}^d \). It assumes that any finite set of function values have a joint Gaussian distribution. Let \( f \) be the known function values of the training cases, and let \( f_* \) be a set of function values corresponding to the test set inputs \( X_* \). We write out the joint distribution as:

\[
\begin{bmatrix} f \\ f_* \end{bmatrix} \sim N \left( \begin{bmatrix} \mu \\ \mu_* \end{bmatrix}, \begin{bmatrix} K & K_* \\ K & K_*^T \end{bmatrix} \right)
\]

where \( \mu \) and \( \mu_* \) are the means for the training set and test set, \( K \) is the covariance for the training set, \( K_* \) the covariance for the training-test set, and \( K_{**} \) the covariance for the test set. Since we know the values for the training set \( f \), we are interested in the conditional distribution of \( f_* \) given \( f \), which is expressed as:

\[
f_* | f \sim N(\mu_* + K_*^T(K + K_*^T K_*)^{-1}(f - \mu), K_{**} - K_*^T K_*^{-1} K_*)
\]

C. Tracking Model

A tracking model can be established to follow the evolution of the traffic flow of an OD pair based on the flow features extracted from the traffic data. Silva et al. [34] developed the general model to forecast the traffic volume of the London transportation system, where the traffic flows are for fast train commuting service within the Greater London area. Tracking model was designed to predict three unknowns: (i) entering counts (entering process), (ii) the rate at which passengers...
and exits (tapping-out) from the station process). In Figure 1, exit given the number of passengers inside the system and remain inside the transportation system given these counts.

Fig. 2. Schematic of multi-step-ahead (MS) prediction with a GPR model.

D. Multi-Step Strategy

In many scenarios, there is a need to predict the system condition multiple steps ahead, and the Equation (5) shows the prediction for h steps. Recursive and Direct strategies [24], as shown in Figure 2, are two main approaches used for the multi-step ahead (MS) prediction. In the Recursive strategy, a single-step prediction model is trained by minimizing the square sum of the prediction errors to perform the first step prediction. Then the window of historical data moves ahead to include the newly predicted value and exclude the oldest sample. The process will continue until the step size of the multi-step prediction is reached. As the predicted values from past steps are always taken into the model as input to get the next prediction value, the prediction error is accumulated, thus largely affecting the prediction accuracy.

\[ y_{t+h} = f_h(y_t, \cdots, y_{t-d+1}) + \varepsilon \]  

(5)

Unlike the Recursive strategy, the window of historical data in the Direct strategy doesn’t need to slide in every step of the process. However, to perform h-step ahead prediction, there is a need to train h models, each corresponding to an i-step (i ∈ [1, h]) prediction. This will incur a high computation overhead when h is large. Furthermore, because the prediction tasks for different ahead steps are run independently, the direct method may result in broken curves, especially when the system is nonlinear and dynamic.

IV. EMPIRICAL STUDY WITH PASSENGER FLOW DATA

A dynamic system is considered to be chaotic if the distance between two system evolution trajectories starting from very close initial position grows exponentially apart. Before presenting our model for prediction, we perform a set of experiments to investigate and identify the traffic patterns, especially the chaotic characteristics hidden in passenger flow series. From previous studies, we find that the chaotic characteristics often exist in the signals generated by the complex dynamic systems, such as the transportation system. As a result of the chaotic characteristics, signals can be computational unpredictable and sensitive to initial conditions. The dimension of the saturation correlation and the largest Lyapunov exponent are two fundamental measures of chaotic characteristics. If the chaotic characteristics really exist in traffic flow data, we need to carefully consider their impacts in our new model.

We analyze the smart-card readings of light rail system in Chongqing, China. The data were collected over 150 days, from March 2014 to July 2014. Each reading consists of a card ID, a “tap-in” time stamp, a “tap-in” location code, a “tap-out” time stamp and a “tap-out” location code. A location code uniquely identifies one of 142 active stations of the system. To preserve the customer privacy, card IDs are anonymized and ignored in our analysis. The time resolution of the recorded time stamps is 1 min. Each day is composed of 1,200 mins, starting from 5:00 AM to 1:00 AM of the next calendar day. Weekdays are assumed to be exchangeable [34].

1) Calculation of the Correlation Dimension: Grassberger P [38] is a common method used for calculating the correlation dimension of the time series attractor. With the increase of the embedding dimension, the correlation dimension of random sequences increases but does not reach the saturation, while the correlation dimension of chaotic sequences approaches the saturation. Therefore, the chaotic sequence and the random sequence can be distinguished based on whether the correlation dimension has saturation phenomenon. The correlation dimension can be calculated as

\[ D_2 = \lim_{d \to 0, r \to 0} \left( \frac{\ln C_d(r)}{\ln(r)} \right). \]  

(6)

where \( C(r) \) is the correlation function, \( r \) is the radius of the sphere in the reconstructed state space, \( d \) is the embedding dimension of the reconstructed state space. As shown in Figure 3, the curve \( \ln C(r) \) versus \( \ln(r) \) gradually becomes parallel (i.e., the slope of \( \ln C(r) / \ln(r) \) becomes constant), with the increase of the embedded dimension \( d \), which indicates that the correlation dimension gradually gets saturated and the passenger flow sequence has chaotic characteristics.

2) Calculating the Largest Lyapunov Exponents From Small Data Sets: The detection of the presence of chaos in a

Fig. 3. \( \ln C(r) / \ln(r) \) curve of passenger flow series.
The quick growth of light rail networks and urban population explosion creates great pressure to the scheduling of subway operations. To make effective operation plans and alleviate the huge congestion problem in a light rail system with the regulation of passenger flows, planners need to know the time variations of short-term passenger flow between stations and between areas. A key problem to solve is to ensure the accurate prediction of the volume of the passenger flow between a pair of OD stations, based on which we can aggregate OD pairs to predict the total exit counts for a particular station or the total passenger flow between urban areas. In this section, we first introduce our problem system with the regulation of passenger flows, planners need to know the time variations of short-term passenger flow between stations and between areas. A key problem to solve is to ensure the accurate prediction of the volume of the passenger flow between a pair of OD stations, based on which we can aggregate OD pairs to predict the total exit counts for a particular station or the total passenger flow between urban areas. In this section, we first introduce our problem system framework, and then detail our design of the prediction model.

A. Problem and System Framework

We would like to predict the traffic volume between an OD pair corresponding to passengers entering from the subway station $i$ and leaving from the station $j$. Let $L_{it}$ be the number of passengers entering station $S_i$ at time $t$ (i.e., entering tap-in counts), $N_{ijt}$ be the number of passengers entering from $S_i$ and exiting from $S_j$ at time $t$ (i.e., tapping-out count). Given two sequences of observed passenger flow data $\{L_{it}, t = 1, 2, \ldots, T\}$ and $\{N_{ijt}, t = 1, 2, \ldots, T\}$, the prediction problem is to predict the value of $N_{ij(t+\Delta)}$, where $\Delta$ denotes the prediction horizon. According to the duration of $\Delta$, we can divide the forecasting task into short, medium and long term. In this paper, we only discuss the short-term forecasting tasks, and set up the duration for each step of prediction to 1 min. The prediction for multiple steps ahead can be applied to predict passenger flows in the future, where we set up the ahead of time steps to 15, 20, 25, and 30 minutes respectively.
For more accurate passenger flow forecasting, we propose a hybrid model, named DTMGP. As shown in Figure 5, our prediction approach mainly contains three stages: (i) data preprocessing and decomposition, (ii) approximation component forecasting, and (iii) detail component forecasting.

In our model, we first apply discrete wavelet transform (DWT) to decompose raw traffic flow data into an approximation component and multiple detailed components. Then we apply a more efficient tracking model to forecast the approximation component. As the system is shown to have the chaotic and dynamics, rather than using the original time series data, we exploit the phase space method. Specifically, we reconstruct phase space from the detailed components and further propose a mixed time delay Gaussian process (MTGPR) model with a recursive strategy to forecast the detailed components. As the last step, we generate the final result with these forecasted components.

### B. Data Preprocessing and Decomposition

A general traffic flow is mixed with smooth and continuous component and sudden changes due to rush hours or events. To better capture the dynamics of traffic flow, we exploit discrete wavelet transform to translate the data into frequency domain components, so that we can more accurately track the temporal evolution of the traffic flow at different time resolutions. The time resolution increases with the increase of the frequency level. Given a proper mother wavelet \( \psi(t) \) and the decomposition level \( l \), the discrete wavelet transform of a series \( f(t) \) can be expressed as [40]

\[
W(l, k) = \int_{-\infty}^{+\infty} f(t) \psi_{l,k}(t) dt, \quad \phi_{l,k}(t) = 2^{-l/2} \phi(2^{-l} t - k)
\]

(9)

where \( k \) is a time translation factor, \( \psi_{l,k}(t) \) is the complex conjugate of \( \psi_{l,k}(t) \), and \( W(l, k) \) is the discrete wavelet coefficient at the level \( l \) and time \( k \).

Now the original time series is represented as:

\[
f(t) = \mathcal{C} + \sum_{l=1}^{L} W_l(t)
\]

(10)

where the first term, \( \mathcal{C} \), is the approximation component that captures the low frequency variation of the time series. The second term contains detailed components at levels \( l = 1, 2, \ldots, L \). A higher level represents lower frequency and captures the remaining details left from the previous levels.

The input data of our hybrid model are two sequences of observed passenger flow data \( \{L_{it}, t = 1, 2, \ldots, T\} \) and \( \{N_{ijt}, t = 1, 2, \ldots, T\} \). We choose the Daubechies (Db) mother wavelet function and choose three levels of detailed decompositions. We first apply DWT to decompose the sequence \( \{N_{ijt}, t = 1, 2, \ldots, T\} \) into one appropriation component \( \{a_1\} \) and three detailed components \( \{d_1, d_2, d_3\} \), as shown in Figure 6. The red curve represents the sequence \( \{N_{ijt}, t = 1, 2, \ldots, T\} \) within one day. The approximation component \( a_1 \) tracks the slow-varying long trend of the traffic volume. The curves \( d_1, d_2 \) and \( d_3 \) correspond to the high frequency data that fluctuate more dramatically in short-time scales, with \( d_1 \) representing the shortest time scale change. Then we take each of the three detailed components as the input to a MTGPR model for detailed component prediction. The sequence \( \{L_{it}, t = 1, 2, \ldots, T\} \) and the appropriation component \( a_1 \) will be taken as the input to the tracking model for the appropriation component prediction. Finally, we sum up all these predicted components to the final result. Figure 7 illustrates the detailed data processing procedure in our model.

### C. The More Efficient Tracking Model and Approximation Component Forecasting

We divide the traffic flow in a light rail system into three processes: the entrance process, the negotiation process and the exiting process. The entrance process tracks how many passengers enter a station to start a journey at a specific time. The negotiation process captures the number of passengers that enter from a specific origin and have stayed inside the system for a time duration. The exiting process captures the number of passengers that exit from a specific origin and exit from a destination station to end their journey, given the number of passengers inside the system and the length of their stay.

**The Model for the Entrance Process:** Silva et al. [34] model the expected value of \( L_{it} \) using \( \theta_{L_{it}} \), and the history values \( L_{i(t-w)} \), where \( \theta_{L_{it}} \) represents an unconditional time-dependent mean that captures most of the variation of the data. However, our simulation results prove that this method may lead to large error. To better capture the dynamics of flow volume, we instead model the expected values of \( L_{it} \) under
the condition \( t \geq 1 \) and the set of given past entries as

\[
E(L_{it} | PAST, L_{it} > 0) = (\theta L_{it} + \sum_{w=1}^{W} \beta_{L_{it}} (L_{i(t-w)} - \theta L_{i(t-w)}))^+, \quad (11)
\]

where \((x)_+ = \max(x,0)\), and

\[
P(L_{it} > 0 | PAST) \equiv \pi_{L_{it}} \quad (12)
\]

In the above representation of the expected value of \( L_{ij} \), we modify \( \theta_{L_{it}} \) by adding in the accumulative fluctuations of the history tap-in counts, with the difference at the time \( t - w \) represented as \((L_{i(t-w)} - \theta L_{i(t-w)})\). For simplicity, we define \( L_{it} \equiv 0 \) for \( t < 1 \).

The Model for the Negotiation Process: For each station \( S_i \) and time \( t \), we provide a concise presence table to represent the number of passengers who enter the system via \( S_i \) and have not left the system by time \( t - 1 \). The presence table captures the empirical distribution of passengers through the amount of time they have stayed inside the system. This empirical distribution is given in seven coarse time periods: [1, 10] min, [11, 20] min, ... , [50, 60] min, and more than 60 min. For each station and time \( t \), we have the vector \( M_{it} \equiv (M_{it}^1, \ldots, M_{it}^7) \) representing the counts at these seven levels.

We model the temporal evolution of the entries of \( M_{it} \) through a cascade of nonparametric binomial regression models. We find that, for a given time \( t \), the variation on \( M_{it}^k (1 \leq k \leq 7) \) depends on \( M_{i(t-1)}^k + H_{it}^k - H_{it}^{k+1} \), where \( H_{it}^k \) represents the number of passengers who enter the system via the station \( S_i \) at time \( t - 1 - (k - 1) \times 10 \) and have not left the system by time \( t - 1 \). For example, for a time \( t \) of interest, \( M_{it}^3 \) represents the number of passengers who have stayed inside the system in the time bracket [21, 30]. It is not difficult to understand that these passengers are part of the ones who have been in the system in the time bracket [20, 29] at \( t - 1 \), which is represented by \( M_{i(t-1)}^3 + H_{it}^3 - H_{it}^4 \).

So \( M_{it}^k \) in a given day can be modeled as

\[
M_{it}^k | PAST \sim Binomial(M_{i(t-1)}^k + H_{it}^k - H_{it}^{k+1}, p_{it}^k),
\]

\[
H_{it}^k = L_{i(t-1)-(k-1)\times10} \times q_{it}^k \quad (13)
\]

where we define \( M_{i(t-1)}^k \equiv L_{i(t-1)}^k \). The parameter \( p_{it}^k \) represents the probability of entering the system via the station \( S_i \) during the period \([t - 1 - (k - 1) \times 10, t - k \times 10]\) and staying in the system at time \( t \). It is different for each station \( S_i \) and the time \( t \) of the day. The parameter \( q_{it}^k \) represents the probability of entering the system via the station \( S_i \) at \( t - 1 - (k - 1) \times 10 \) and staying in the system for more than \((k - 1) \times 10\) minutes, and it is fitted by the cubic spline smoothing.

The Model for the Exiting Process: According to historical observations, we can calculate the median of the traveling time from the station \( S_i \) to the station \( S_j \). The value of \( N_{ij} \) will be impacted by the passenger numbers in the previous, current and next time brackets. For instance, given that the median of the traveling time from \( S_i \) to \( S_j \) is 35 minutes, the corresponding brackets are \([21, 30], [31, 40], [41, 50]\). The model for \( N_{ij} \) is then

\[
N_{ij} | PAST = M_{it}^{k=2} \times q_{ijt}^{k=2} + M_{it}^{k=3} \times q_{ijt}^{k=3} + M_{it}^{k=4} \times q_{ijt}^{k=4} \quad (14)
\]

where \( q_{ijt}^{k=2} \) is the ratio of passengers from \( M_{it}^{k=2} \) that leave from the station \( j \) at time \( t \) and contribute to \( N_{ij} \). In the Equation (14), we assign a different \( q_{ijt}^k \) to each \( M_{it}^k \). To identify the traffic patterns, we conduct many experiments. Parameters \( q_{ijt}^k \) for a station pair \( i - j \) are fitted using cubic spline smoothing. However, our experiment results demonstrate that the value of \( q_{ijt}^k \) varies largely for the same values of \( i, j, t \). Figure 8 provides a visualization of \( q_{ijt}^k \) values, with \( k = 2, 3, 4 \). It is obvious that \( q_{ijt}^{k=3} \) is often far greater than the other two values at a specific time \( t \).

D. The MTGPR Model and the Forecast of Detailed DWT Components

As identified in Section IV, traffic flow data have strong chaotic characteristics. When handling this type of signals, traditional forecasting methods are often hard to meet the accuracy requirements. In addition, we usually need to reconstruct the state space first when making the prediction for the time series generated by a complex dynamic system. However, the passenger flow pattern in the complex dynamic system won’t stay unchanged, and we cannot ensure that time delay selected for reconstructing a state space is optimal at any time.

Considering these factors, we propose an MTGPR model for more accurate traffic flow prediction. The schematic of multi-step-ahead (MS) prediction with an MTGPR model is shown in Figure 9, where the area between blue and green lines represent the forecasted values based on the Gaussian Process (GP) model. The larger the distance between the two lines, the greater the predicted variance. Given the embedding dimension \( d \), we first select the total number of different time delays we will use in the prediction, \( M \). We then reconstruct \( d \)-dimensional state space \( M \) times with different delay values, varying from 1 to \( M \). For each space reconstructed, we make the prediction based on the GP model. Therefore, for the
prediction of \( y_{t+1} \), we can obtain \( M \) pairs of predicted mean \( \tau_i \) and predicted variance \( \tau_i \). We put them in the set \( \{(\text{mean}_i, \text{variance}_i)|\tau \in [1, M]\} \). Then we select the pair with the least predicted variance as the final predicted result. For the prediction based on a phase space with a specific time delay, a red circle marks the smallest predicted variance. For \( \tau \) delays, \( \tau \) is the time delay set \( \{1, 2, \cdots , t\} \). The MTGPR model for \( y_{t+1} \) is then:

\[
\begin{align*}
\hat{\tau} &= \hat{\text{arg min}}_{\tau \in [1, M]} \text{var}(y_{t+1}|X_\tau, x_{t+1, \tau}), \\
Y &\sim N(0, K(X_\tau, X_\tau) + \sigma_{\text{noise}}^2).
\end{align*}
\]

where \( E() \) and \( \text{var()} \) represent the predicted mean and variance of forecasted values respectively based on the GP.

To reduce the overall processing time, we can use multi-thread or parallel computing to realize MTGPR. Suppose there is a \( h \)-step-ahead forecasting task, the algorithm can be executed iteratively \( h \) times with the recursive strategy. Though there exist dependency among different steps of looking ahead, parallel computing is just executed within each step. Given the historical observations \( \{y_i, i = 1, 2, \cdots , t\} \), the time delay set \( \{1, 2, \cdots , M\} \) and the embedding dimension \( d \), there are four phases in each iteration:

1) Initialization: We initialize \( M + 1 \) processors/threads, with \( M \) processors responsible for making the prediction in parallel and the remaining one responsible for summarizing the results.

2) Forecasting: Each of the \( M \) processors makes one-step-ahead prediction based on the Gaussian process model and outputs a pair of predicted mean and predicted variance.

3) Summarization: The remaining one processor summarizes all the \( M \) results. The pair with the smallest predicted variance will be selected as the final result.

4) Updating: We update the input vector with the new predicted mean \( \hat{\tau} \), where \( i \in \{t+1, t+2, \cdots , t+h\} \). Then the program will continue to the next iteration.

**VI. EXPERIMENTS**

We randomly select time series data from 100 pairs of OD stations to evaluate the performance of our prediction algorithm. As shown in Figure 10, the traffic patterns on weekdays and weekends are completely different. In order to reduce the impact of data difference on the experimental results, we divide the historical observations into two parts, one part from the weekdays and the other from weekends. The training process is independent in each part. We use data of the first 4 months as the training set and the later 1 month as the testing set.

To evaluate the performance of our hybrid model, we have performed two groups of extensive simulations. In the first group, we first evaluate the accuracy of our MTGPR model on making prediction for detailed components, and in the second group we evaluate the final forecasting performance of our hybrid model based on the original time series. We calculate the forecasting accuracy for each OD station pair and take the average to obtain the final forecasting accuracy.

**A. Evaluation Metrics and Reference Models**

We use four metrics to evaluate the forecasting performance of our model:

- DAY-MAE, the mean absolute error during the whole day.
- MR-MAE, the mean absolute error during morning rush hours.
- MPT-MAE, the mean absolute error during peak traffic forecasting during morning rush hours.
- DAYPT-MAE, the mean absolute error during peak traffic forecasting during the whole day.

MPT-MAE and DAYPT-MAE reflect the performance of our model on forecasting the congestion degree in a transportation...
These metrics are defined as follows:

\[
MAE = \frac{\sum_{t=t_{\text{start}}}^{t_{\text{end}}} |N_{ijt} - \hat{N}_{ijt}|}{t_{\text{end}} - t_{\text{start}} + 1}
\]

where \(N_{ijt}\) represents the number of passengers that have entered the system from the station \(S_i\) and exits (tapping-out) from the station \(S_j\) at time \(t\). \(\hat{N}_{ijt}\) represents the forecasted value. For the metric DAY-MAE, \(t_{\text{start}} = 1\) and \(t_{\text{end}} = 1200\) (from 5:01 AM to 1:00 AM on the next calendar day). For the metric MR-MAE, \(t_{\text{start}} = 141\) and \(t_{\text{end}} = 240\) (from 7:21 AM to 9:00 AM). For the metric MPT-MAE, \(t\) include the moments at which the observed number of passengers soar to top30 during morning rush hours. For the metric DAYPT-MAE, \(t\) include the moments at which the observed number of passengers soar to top200 during the whole day.

We implement four reference forecasting models for the performance comparison.

1) **AR [41]**: AutoRegressive (AR) models are most widely studied because of their flexibility in modeling many stationary processes. In this study, we set the time lag \(I = 2\), order \(p = 30\).

2) **SVR [42]**: This approach reduces the over-fitting and computational costs inherent in traditional SVR, with the basic assumption that the most recent data samples provide more relevant information for forecasting.

3) **BPNN**: It is the most representative learning model for the ANN. BPNN in this study used 30 input neurons, 25 hidden nodes (three-layer) and 1 output neuron.

4) **Tracking [34]**: This model was designed to keep track of the number of passengers inside the transportation system.

**B. Performance Results**

1) **Performance Comparison of Detailed Components Forecasting**: In Figure 11(a-c), our MTGPR model is shown
to be able to control the MAE on the $d_1$ component, $d_2$ component, and $d_3$ component at a very low level, while MAE under conventional Gaussian process model with direct strategy or recursive strategy are much higher. Even when raising the forecasting steps to 35, MAE values of our MTGPR model have very small increase, while the MAE of the other two models constantly increase with the prediction steps. This demonstrates the effectiveness of our model in minimizing the prediction error by constructing the phase space using the time delay with the minimum predicted variance in each step.

2) Performance Comparison of the Original Signals Forecasting: We carry out four series of simulations: 15-step ahead forecasting, 20-step ahead forecasting, 25-step ahead forecasting and 30-step ahead forecasting to evaluate the performance of our hybrid model. Figure 12 shows the forecasting performance of different models on weekdays and weekends. As shown in Figure 12, the forecasting using our hybrid model achieves the lowest error under all prediction steps studied.

With the number of prediction steps increased, the forecasting accuracy of the AR, SVR, BPNN and Tracking models deteriorate dramatically, while the forecasting accuracy of our hybrid model has very small increase. This indicates that our model does not have cumulative error and can better predict the traffic in advance, which will allow more time for efficient traffic scheduling. In addition, under the same steps, our hybrid model achieves on average 10%—30% higher accuracy compared to other four models. It is worth noticing that, the DAPT-MAE and MPT-MAE values by using the hybrid DTMGP model are only about 20%—45% those of using other four models. This fact indicates that our model can better predict the overload degree in a transportation system, which will help alleviate the congestion. With the comprehensive consideration of spatial relationship among OD pairs, the temporal and frequency features of traffic, and the correlation among signals in the reconstructed phase space, our model can sense the small changes of external factors and timely adjust the predicted result.

Figure 12 (e-h) present the prediction accuracy by taking different methods on weekends. It can be easy to find that the forecasting accuracies of different methods are overall higher than those in Figure 12 (a-d). The reason for this phenomenon is that the passenger flow data on weekends are more stable compared with those on weekdays.

VII. Conclusion

In this paper, we propose a novel hybrid model to more accurately forecast the volume of passenger flows multi-step ahead. To the best of our knowledge, this is the first hybrid model to follow the evolution of traffic flow simultaneously from the temporal, frequency, OD spatial and historical probabilistic distribution perspectives. As one important part of the hybrid model, the MTGPR model proposed in this article can also effectively capture the process dynamics and make prediction with a smaller predictive variance compared with state of the art Gaussian process models. The simulation results demonstrate that our hybrid model can achieve on average 20%—50% higher accuracy compared to other models. In the future work, we also plan to explore methods to further improve forecasting accuracy by fully employing the graph structure of the road networks or light rail network.

REFERENCES


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